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Fintech Companies Valuation Using Real Options: Fawry Empirical Study

Yassmen Abou Elwafa
yassmeen@aucegypt.edu

THE AMERICAN UNIVERSITY IN CAIRO
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Fintech Companies Valuation Using Real Options: Fawry Empirical Study

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Yassmen Mohamed Ali Elsayed Abou Elwafa

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Abstract:

This thesis presents an improved valuation approach for Fintech companies, which are high-growth technology-based firms. We first identify the shortcomings of traditional valuation methods in valuing high-growth technology firms. Then, an alternative valuation technique based on real options theory is discussed to value fintech companies. The relevance and validity of using real options valuation to value fintech firms are also discussed. We propose a new flexible framework for valuing fintech companies by extending the Schwartz-Moon (2001) model to include various types of real options. To validate the practical application of the Extended Schwartz-Moon model, we conduct an empirical study to value Fawry for Banking Technology and Electronic Payment (S.A.E), which is a premier fintech company in Egypt. This thesis contributes to the field of valuing fintech companies both academically and practically. By offering an enhanced valuation methodology for fintech companies as a high-growth technology-based firm, we provide a valuable tool for investors, analysts, and managers to make more informed decisions when valuing such companies.

Keywords: Real options, American Style Options, Dynamic Programming, Fintech Valuation, High-growth tech firms, Fawry.

INTRODUCTION

In the last few decades, the financial sector has been disrupted by a wave of innovative technologies forming a new phenomenon known as fintech. Gomber et al. (2017) define fintech, characterizing it as a "neologism" that emerged from the combination of the terms "financial" and "technology." They further elaborate that fintech integrates contemporary internet-based technologies, including cloud computing and mobile internet, with traditional business operations of the financial services industry. The emergence of fintech has been driven by digitalization and digitization, which refers to the increased use of digital technologies to transform business models and operations, leading to increased efficiency and innovation (Gobble, 2018). Fintech is increasingly embedded in everyday economic transactions, supported by its ability to cut transaction costs, improve service quality, and create a more diverse and stable financial landscape.

The innovation of smartphones and internet-related technologies (e.g., cloud computing, blockchain, and mobile internet) leads the development of fintech, which offers efficient financial services combined with cost reductions (Gomber et al.,2017). Diemers et al. (2015) have identified an ecosystem that drives fintech innovation, as shown in Figure 1. This ecosystem consists of five integrated elements. The interactions among the ecosystem elements are essential for creating a dynamic environment for the fintech industry development and growth. The first element of the fintech ecosystem is the fintech companies themselves. These companies are responsible for providing innovative financial services, such as electronic payment, peer-to-peer lending, and crowdfunding, which meet customers' changing needs. They are the pillar stone for developing and introducing new financial products and services. The system's second element is the availability of technology developers and financial expertise. Financial experts advise and support fintech companies, especially in their initial stages. Meanwhile, technical experts use technologies like cloud computing, blockchain, and big data analytics to create new technology-based financial services. The government and regulatory environment represent the third element in the ecosystem, with legislation and regulations in place to govern the establishment and operations of fintech companies, protect customers' rights, and ensure that fintech companies comply with regulations governing traditional financial services operations. The customer acceptance of technology-based financial services offered by fintech companies is a critical driver in having an active market for fintech products and services. Thus, the fourth element in the fintech ecosystem is customers, either individuals or corporations, as they are the end-users of fintech products and services. Technology-based financial services are developed due to a niche in customers' need for customized services and innovative solutions delivered by fintech companies, setting fintech companies apart from their peers' traditional financial services (Lee & Shine, 2018). Traditional financial institutions, such as banks, insurance companies, and venture capital firms, represent the last element in the ecosystem. While these institutions initially viewed fintech companies as disruptors, they now recognize the benefits of adopting fintech services rather than competing with these startups. Banks and other traditional financial services providers have embraced technology-based financial services to offer customers more customized and convenient services and products. In order to benefit from fintech, traditional financial institutions start investing in technology-based financial services either by direct investment or by acquiring fintech companies with high-growth prospects.

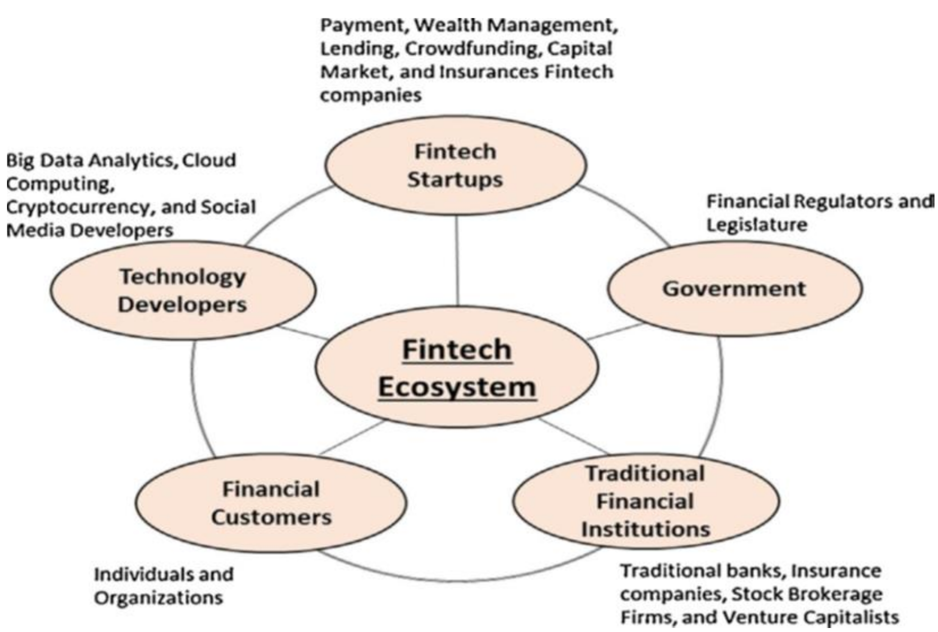

Figure 1:Fintech: Ecosystem, business models, investment decisions, and challenges. (Lee and Shine, 2018)

Customers' strong appetite to use digital technology that offers them personalized financial services using their mobile devices or via the internet, combined with the growing demand for efficient and transparent services delivered through digital platforms, increases their reliance on fintech services (Lee & Shine, 2018). This change in customers' appetite attracts the attention of investors toward the potentials embedded in fintech investments, who start looking for profitable investment opportunities in fintech's innovative financial solutions. Nevertheless, it is challenging for investors to identify promising innovative fintech projects from among numerous ones. The unique hybrid business model of FinTech companies, which combines elements of traditional financial/banking institutions and internet-related technology companies, presents a significant challenge for investors seeking to value these companies using traditional valuation methods. Compared to traditional financial institutions, fintech companies are fast-growing technology firms providing financial products/services, making it challenging to apply the same valuation methodologies used to value traditional financial institutions. One of the most significant challenges when valuing fintech companies is the high risk and uncertainty in investing in technology-based innovations (KPMG, 2017). Fintech companies take a relatively long time to introduce a new innovative product and penetrate the market. During this period, unpredictable and predictable factors may change, adding more complexity and uncertainty related to the market and competition. Therefore, when a product or service is introduced to the market, the unforeseen risk challenges the accurate projection of revenues, costs, and growth rates (De Meyer et al., 2002). Furthermore, many fintech companies have limited historical data, which makes it difficult to depend on historical data to forecast future revenues, costs, and growth rates. Thus, the high uncertainty and complexity involved in technology-based innovations make it challenging to use the traditional valuation methods, which require an adequate projection of revenues, costs, and expected growth rates.

Another challenge faced when using traditional methods to value fintech is the nature of fintech investment. Fintech investments require great flexibility in the decision-making process to fast act to market dynamics as fintech innovations pass through multiple stages of the product life cycle before showing potential success and marketability (Kang, 2009). Thus, in each stage, there is an option to change the investment decision based on the result of the previous stage (Olsson, 2006); for instance, the investor may decide to expand investment or exit based on the potential of the developed product and the market conditions. Therefore, a valuation methodology that ignores managerial flexibility, particularly when a high degree of uncertainty is involved in the business, may lead to understated business value and inappropriate decisions (Block, 2007). Therefore, the methodology used to assess the value of fintech investments should reflect this managerial flexibility in the value of fintech firms.

The large upfront capital required for investment in technology-based innovations is another critical challenge in valuing fintech companies (Demers and Lev, 2001, as cited in Klobucnik & Sievers, 2013). This upfront capital is necessary before introducing the product to the market, resulting in negative cash flow for several years until customers successfully adopt the product or service and gain widespread use, then be able to generate revenues to pay back for the large upfront capital paid in developing and introducing the product (Bartov et al., 2002 as cited in Klobucnik & Sievers, 2013). Furthermore, the high risk associated with fintech investments increases the probability of business failure before reaching the stage of generating positive cash flow. If the business fails, investors will incur significant losses resulting from the large upfront invested capital. So, the negative cash flow generated from operations for several years combined with the risky nature of fintech presents a challenge for financial experts when applying traditional valuation methodologies, such as NPV or discounted cash flow, to assess the value of the business.

In sum, fintech companies are characterized by their risky nature as technology-based companies, which typically pass through multiple stages of the product life cycle that need great flexibility in decision-making. Fintech firms often require large upfront capital investments before the product/service grows widespread and generate revenues. Thus, it may take several years to pay back the large upfront capital paid in the development stage and generate positive cash flow. These unique characteristics represent challenges in using traditional valuation methodologies and should be addressed in the dynamics of the methodologies applied to valuing fintech companies.

Previous researchers have suggested several methodologies to value technology-based companies. Smith and Parr (2000) use the cost approach and assume that the value of the technology can be assessed by estimating the reproduction cost or the substitution cost of acquiring similar technology and then reflecting depreciation. This method is proper when valuing intangible assets such as software. In the case of valuing a technology-based project or business, an equal amount of investment does not necessarily result in the same technology, as risks and benefits related to each technology are different. Relative valuation, which allows valuing an asset by comparing it to another similar asset, is also used to determine the value of a technology-based business. Comparable company valuation is a relative valuation methodology widely used in valuing technology-based investment opportunities. It is a market-based valuation technique, arguing that similar investments should trade at the same price (Meitner, 2006). When using a comparable company valuation, it is essential to determine a set of peer firms that share similar characteristics and then compute the average and median of some ratios for those firms to conduct the valuation. The commonly used multiples are price per earnings (P/E) and price per sales (P/sales). In practice, the most used multiples are enterprise value multiples of earnings and sales, such as EV/EBITDA and EV/sales. Practitioners widely use these metrics as they are independent of capital structure. The critical step when using the comparable company valuation is identifying suitable peer firms. Due to the substantial differences among companies in size, business model, financial conditions, capital structure, and risk exposures, finding suitable peer firms can be challenging. This difficulty in finding peer companies is considered a criticism of using this approach for valuing high-growth technology companies, particularly fintech companies. Such companies' unique characteristics and rapid growth rate can make finding peer groups with similar innovative technology, business model, and financial position difficult, leading to inappropriate valuations. Considering this limitation, comparable valuations may not be reliable when valuing fintech companies. It is more reliable to use a more reliable valuation method that focuses on the intrinsic value of the fintech companies.

Of the most popular methods in the valuation of firms is the Discounted Cash Flow method (DCF). Practitioners extensively apply the DCF model due to its straightforward application that relies mainly on accounting information. This method forecasts the future cash flows and discounts them back at a proper discount rate that reflects the risk involved in the cash flow. The DCF model is applicable when the firm's future cash flows are predictable. When the firm's future cash flow projection is based on uncertain or subjective assumptions, the DCF model can lead to unreliable results (MacMillan & Van Putten, 2004). In the case of valuing fintech companies where a new technology-based product is introduced, it becomes difficult to reliably predict market information and forecast revenues and costs without uncertainty (O'Connor et al., 2008; Salerno et al., 2015 as cited in Brasil et al., 2018). Meanwhile, the DCF model may not be a suitable valuation method for companies experiencing negative cash flows, as a large part of the firm's value is generated from the terminal value (Amram & Kulatilaka, 2000), which is typically the situation for fintech firms that often make significant upfront investments and take several years to generate positive cash flows.

While the DCf method "can be used to value any type of asset – physical, financial or intangible," as Sudarsanam et al. (2006) said, Myers (1984) questioned the application of the DCF in valuing a business that requires managerial flexibility in decision making by saying that "Discounted cash flow analysis may fail in strategic applications even if it is properly applied." Myers (1984) suggests using modern techniques such as real options valuation to capture the value options embedded in managerial and strategic flexibility in decisions instead of the traditional ones that miss the value of these options. For Smit and Trigeorgis (2004), the traditional DCF model hardly incorporated managerial flexibility in capital investment, such as the decision to expand or defer an investment upon the emergence of new information. Consequently, using DCF or any similar traditional methodology has many limitations when valuing fast-growing fintech companies.

Responding to these challenges in valuing innovative technology-based investments, the real option technique is proposed for valuing such risky investments. According to Copeland and Antikarov (2001), the real options method gives the investor the right but not the obligation to correct their decision according to future changes. Using this method in a technology investment decision gives flexibility against future uncertainty and recognizes the uncertainty as an opportunity. Schwartz and Moon (2000,2001) introduced a model based on real options theory and capital budgeting techniques for pricing internet companies. The model aims to capture these firms' high growth potentials that traditional methodologies fail to capture. An essential assumption in the model is that the high growth rates of these firms tend to converge to the industry's long-term growth rate due to competition. The revisited Schwartz-Moon model (2001) incorporates three sources of uncertainty, including the growth in revenues R_t, the expected growth rate in revenues μ_t , and variable costs as a fraction of revenues γ_t. The model has six state variables, three of which are deterministic and the other three are stochastic, and all stochastic variables follow a mean reverting process. The stochastic variables are revenues, growth rates in revenues, and variable costs, while the deterministic path-dependent variables are the amount of cash available, the loss-carry-forward, and the accumulated Property, Plant, and Equipment. Monte Carlo simulation is used to deal with these variables and the complex path dependencies of the problem. Unlike the discounted cash flow (DCF) model, the Schwartz-Moon model relies not on subjective estimates but on firm parameters and long-term industry averages. The model introduces a bankruptcy option, enabling the company to pursue future financing. This option addresses a significant challenge faced in valuing technology firms with projected negative cash flows while the firm still has growth potentials that enable management to get funding.

In this thesis, we will use Schwartz and Moon's (2001) model to value fintech companies as technology-based companies. While Schwartz and Moon (2001) model introduced only the option to default (bankruptcy option) when the company has negative cash flow, the thesis extends the model by introducing more options for the companies with growth potential to expand and grow while preserving the right (option) exit the investments. The thesis attempts to value the real options by applying the Least Squares Monte Carlo (LSM) approach introduced by Longstaff and Schwartz (2001), which is a numerical methodology based on the Monte Carlo simulation optimized using least square linear regression to determine the optimal time to exercise the options, given that real options embedded in the capital budgeting problem are usually recognized as American-type options or claims. The remainder of the thesis is structured as follows: Section 2 provides a literature review.

Section 3 describes the methodology used, while Section 4 presents a numerical investigation. In Section 5, an empirical study is applied. Finally, Section 6 draws conclusions based on the findings.

I. Literature Review

Real Options valuation is a relatively new methodology used to value investment opportunities involving high uncertainty. According to Trigeorgis and Reuer (2017), an option is "the right, but not the obligation, to take some future specified action at a specified cost." Unlike a financial option, a real option is a decision opportunity that provides the right to make a particular business decision in the future, contingent on the price of the underlying real asset (Alexander & Chen, 2021). Therefore, the option derives its value from a real asset rather than a financial asset. The real options approach assumes that managers can make managerial decisions in the future, depending on the new information that emerges, and this is referred to as managerial flexibility (Wang & Yang, 2012).

Real options, similar to financial options, can be either call options or put options. A call option is the right to purchase an asset, whereas a put option is the right to sell an asset in the future. These options are typically classified into two major styles: European-style and American-style, which differ in terms of the flexibility given to the holder for exercising the options. European options can only be exercised on their expiration date, while American options can be exercised at any time before their expiration date. Real options are generally considered American-style options because the holder can exercise the option at any time before expiry. Real options are distinguished from financial options by the nature of the underlying asset. Financial options are typically written on financial securities, such as stocks or bonds, representing claims to income generated by real assets. Conversely, real options derive their value from real assets that have the ability to generate cash flow, such as investments in new projects, businesses, or technologies. Therefore, while financial assets only define distribution rights of a "given value," real assets can create "economic value." Another distinction between financial and real options lies in the future action embedded in the option. Financial options require the holder to decide whether to exercise the option based on the future price of the financial security, which is often observable in the financial security market. In contrast, real options are based on a set of flexible managerial decisions, making the decision-making process more complex. For example, the holder of a real option can decide whether to exercise the option and which real option to exercise (expand, defer, or abandon a business) based on the changes in the future value of the real asset, which is uncertain. Moreover, in the case of real options, managers' decisions throughout the life cycle of the underlying real asset can significantly impact the asset's value and/or riskiness. On the contrary, the value of financial options is mainly influenced by market factors. For instance, managers can take corrective actions in their business that have the capacity to change the value of the business, subsequently affecting the value of the real option. Moreover, the decisions made by managers on the level of risk they are willing to undertake in their business will impact the risk embedded in the real options.

In this thesis, we use the real options valuation approach as it addresses the limitations of traditional valuation methodologies. Compared to traditional valuation approaches, real options valuation methods consider two critical limitations: the flexibility in managerial decisions and the risk involved in operations. The literature review section aims to provide an overview of real options applications, previous research in this area, and real option types. It also presents fundamental real options valuation methodologies.

I.1 Real Options Applications

The concept of the real option was first introduced by Myers (1977). The article aims to elucidate why firms do not maximize their debt financing to take advantage of the tax shield it creates. Mayers (1977) considers future investment opportunities that depend on corporate growth as an option. He argues that a portion of the firm’s value is derived from the option to make future investments. Therefore, the firm's total value comprises its intrinsic value and the net present value of its option for future investments. Myers (1977) highlights that the traditional valuation methods understate the firm’s value as these methods fail to consider the value of future growth. Myers (1977) also emphasizes the importance of growth options in determining the value of firms, as traditional valuation methods ignore the value of these options. Furthermore, the application of contingent claims analysis to value corporate securities was discussed by Mason and Merton (1985). The article states that corporate securities can be considered a contingent claim on the firm's assets. They shed light on the option feature inherent in corporate securities; hence, the equity and debt of the firm can imitate the option's payoff. Mason and Merton (1985) further emphasize the suitability of contingent claim analysis to value managerial flexibility embedded in firms. Since its introduction, the real option valuation approach has been addressed in pioneers' research works. Dixit and Pindyck (1994) explore the decision-making process of firms when facing future uncertainty. They argue that the potential future uncertainty and the flexibility in managerial decisions to react to future uncertainties should be considered when valuing investment opportunities. Dixit and Pindyck (1994) highlight the inadequacy of conventional valuation methodologies in valuing projects in the presence of uncertainty and discuss using real options to value these projects. They emphasize the importance of real options in investment decisions, wherein managers can exercise the option by making decisions such as delaying an investment or abandoning a project.

In his work, Trigeorgis (1996) explores the advantages of using real options in valuing projects in the presence of high risk and uncertainty. His article explains applying real options in various contexts, including capital budgeting, project valuation, and risk management. Additionally, he emphasizes utilizing real options in valuing investment opportunities where the timing and flexibility of investment decisions are important. Furthermore, Amram and Kulatilaka (1999) highlight the significance of incorporating managerial flexibility into decision-making processes when valuing investment opportunities. They also shed light on the constraints of traditional discounted cash flow methods in capturing the value associated with managerial flexibility. Amram and Kulatilaka (1999) present the use of real options valuation methods for valuing investments in different industries, such as pharmaceuticals, oil and gas, and high-tech, and provide insights into how these methods can be integrated into strategic decisions.

Moreover, utilizing real option valuation methods can enhance the value of the firm. Unlike traditional valuation methods and capital budgeting approaches, real options effectively capture the value of managerial flexibility in decision-making and consider the potential upside in the case of uncertainty. When the real option valuation method is used, changing volatility can be used to capture the riskiness of cash flow generated over the project lifespan (˘Culík, 2016). Consequently, uncertainty can lead to a higher valuation of an investment, in contrast to the discounted cash flow method, where the uncertainty decreases its value by applying a higher discount rate to account for the risk (Schwartz & Trigeorgis, 2001; Copeland & Antikarov, 2001).

Real options are considered capital budgeting decisions contingent on future information, which will impact the value of the investment. The work of early scholars discusses the application of real options valuation to value projects involving a single option. Majd and Myers (1984) demonstrate the application of real options theory to value a petroleum exploration project. Their model incorporates the uncertainty of future cash flows and the flexibility to abandon the project at any time. Additionally, McDonald & Siegel (1985) apply the real options valuation method to value the managerial flexibility inherent in the option of temporarily shutting down if the firm's variable costs exceed its revenue. McDonald & Siegel (1986) numerically examine the use of real options theory by valuing an option to defer investment, a "wait and see" option. The option is used to value investment in a new plant where the firm has the option to wait for one year before investing in the plant until it receives new information about the demand for its product, allowing for updating its cash flow estimation based on the revealed information. They conclude that the firm's value in waiting to invest can be greater than in the case of immediate investment when applying the real option pricing method. In real-life, investments often involve multiple, rather than single, real options. This idea was first introduced by Brennan and Schwartz (1985). Brennan and Schwartz (1985) apply the real options method in valuing mining projects by combining the value of the options to shut down and the option to abandon at a salvage value. Later, Triantis & Hodder (1990) value a flexible production system in case of multiple products and capacity constraints from the multiple real options theory perspective. They conclude that the real options approach proves to be an effective tool for capturing managerial flexibility when valuing investment in production systems. They highlight that the result supports “the descriptive notions regarding flexibility which are often cited.”

A portfolio of interacting real options is introduced in Trigeorgis (1991). The paper tackles the problem of valuing complex options in the context of a generic project. It uses a log-transformed binomial option pricing method to value options to defer, abandon, expand, and switch the use of investment. Then, the portfolio of more complex and interacting real options is investigated in Trigeorgis (1993b). The article is concerned with quantifying the interactions among a collection of real options to value an investment. It compares the value of independent options with the value of a collection of interacting options. It sheds light on the fact that "the combined value of a collection of options may differ from the sum of separate option values," as the interaction among options not only can change the value of each option (compared to the value of independent options) but also may change the value of the underlying asset through the sequence of exercised options. Considering real options' interaction in valuing an investment opportunity is handled later in several academic works such as Trigeorgis (1993a) and Trigeorgis & Reuer (2017).

Real options theory's capacity to address uncertainty and benefit from managerial flexibility has motivated scholars to discuss real options methods in valuing various technology-based investments. Boer (2000) and McGrath & MacMillan (2000) emphasize the use of real options in valuing technology investments and challenge traditional methods in capturing uncertainty associated with technology when valuing an investment opportunity. Additionally, real options methods are used in the air freighter industry to value a risky new technology-based project introduced by Boeing (Mathews, 2009). Furthermore, the appropriateness of using real options theory in valuing investments in multi-stage high-tech projects is tested by Song et al. (2017). Moreover, Brasil et al. (2018) explain why using real options suits valuing innovation projects. They emphasize that considering the options contingent on managerial decisions is the rationale behind adopting real options methods. Recently, the real options approach has been adopted in valuing agricultural technology. Wilson et al. (2022) apply real options to value an agricultural technology startup. In conclusion, real options theory has evidenced its appropriateness to value technology-based investments in various fields.

In the context of valuing internet-based companies, Schwartz and Moon (2000; 2001) apply capital budgeting techniques and real options theory to value Amazon and eBay. They justify the apparent high valuations of internet firms at the time of the dot-com bubble with high growth in revenues in combination with high volatility in key variables. The Schwartz-Moon (2000; 2001) model captures the uncertainty and high growth for internet-based companies as it uses Monte Carlo simulations to forecast stochastic variables that can handle complex processes and path dependency. Klobucnik and Sievers (2013) study the application of the Schwartz and Moon (2001) model on a large-scale dataset of around 30 thousand technology firms in the US. The study reveals the feasibility of the Schwartz-Moon model in valuing technology firms with key advantages in valuing small and non-listed firms. The Schwartz and Moon (2001) model is also illustrated by Doffou (2015) in the valuation of Google, Amazon, eBay, Facebook, and Yahoo.

In conclusion, real options theory has gained significant attention in recent years due to its ability to capture managerial flexibility and uncertainty embedded in investment decisions. The real options approach enhances the value of investments compared to traditional methodologies, particularly in industries with high levels of uncertainty and risk that require managerial flexibility in decisions. The literature review has shown that real options theory has been applied in various fields, including capital budgeting, valuation, risk management, and strategic decision-making. It is also used in various industries, including but not limited to air freighters, agriculture, pharmaceuticals, oil and gas, and high-tech.

I.2 Real Option Types

Copeland and Antikarov (2001) and Trigeorgis (1993a) identify common types of real options. They explore the following types of real options:

1- ***Option to defer***: The investment decision can be postponed, making it possible to benefit from the new information revealed during the option's lifetime. This option can be presented as a call option with the right to delay investment, where the exercise price is the investment cost at the start of the project. This real option type is proper to value natural resources, real estate development, extraction industries, and farming. The option to defer has been analyzed by Tourinho (1979), Titman (1985), McDonald and Siegel (1976), Paddock, Siegel, and Smith (1988), and Ingersoll and Ross (1992).

2- ***Time to build (staged investment)***: Involve staging capital expenditures in an investment. Each stage can be viewed as an option on the value of subsequent stages, creating a compound option. This option is often proper to value research and development (R&D) intensive industries, such as pharmaceuticals and long-term development capital-intensive projects. This concept has been studied by several researchers, including Majd & Pindyck (1987), Carr (1988), and Trigeorgis (1993).

3- ***Option to alter operating scale***: The option to modify the operating scale enables the firm to adjust production levels by expanding, contracting, or accelerating resource utilization based on market conditions. If market conditions are more favorable than expected, the firm can increase its production level while reducing it in the opposite situation. Industries such as natural resources, mine operations, facilities planning, construction in cyclical industries, fashion apparel, consumer goods, and commercial real estate can benefit from this option, as analyzed by Brennan and Schwartz (1985), McDonald and Siegel (1985), Pindyck (1988), and Trigeorgis and Mason (1987).

4- ***Option to abandon***: If market conditions decline severely, management may decide to abandon current operations and sell capital equipment and other assets and realize their resale value. This option can be presented as a put option with the right to exit a project where the exercise price is the liquidation value of the project. The option is useful in capital-intensive industries like airlines, railroads, and financial services in uncertain markets, as discussed in Myers and Majd (1990).

5- ***Option to switch***: The option provides flexibility to respond to changes in prices or demand. If prices or demand change, management can change the output (product flexibility), or the outputs can be produced using different types of inputs (process flexibility). This option is similar to a portfolio call or put options that allow the holder to switch among different operating strategies at a fixed cost. This approach is particularly useful in industries subject to volatile demand, such as consumer electronics, toys, specialty paper, machine parts, and autos. The flexibility to change inputs is especially important for feedstock-dependent facilities, such as those that use oil, electric power, chemicals, or crops. These options are discussed in works by Margrabe (1978), Kensinger (1987), and Kulatilaka and Trigeorgis (1993).

6- ***Option to expand***: Expand investments if conditions are favorable. Certain investments provide the opportunity to increase their scale and generate more profits if they yield favorable returns during their initial stage. In such cases, the initial investment gives the investor an option to expand. This option can be presented as a call option with the right to expand investment at a fixed value. The expansion option is useful in a range of investments, including a lease on undeveloped land or oil reserves, strategic acquisitions, or the development of a new generation product or process as explored by Kester (1993), Trigeorgis (1990), Pindyck (1988), Chung & Charoenwong (1991).

7- ***Option to scale back***: The option to reduce the size of a project's operation. This option can be presented as a put option with the right to sell a part of a project at a fixed value.

In addition to the above types of options, Trigeorgis (2005) identifies one more type of real option: corporate growth. The corporate growth option is a type of option to expand. Unlike the expansion option, the growth option concerns strategic expansions such as penetrating new markets or acquiring a partner. In other words, growth options set the path for future opportunities.

I.3 Real Options Valuation methodologies.

The option valuation methods can be classified into two main approaches, as illustrated (Baecker et al., 2003). The first approach is the analytical approach which depends on closed-form solutions or approximation models to value for simple option problems. However, they can be challenging to apply to more complex options. The Black and Scholes option-pricing method is one of the pioneer models based on closed-for solutions to price options. The second approach is the numerical approach which can be divided into two subgroups: approximation of the stochastic processes and approximation of partial differential equations. The first group includes the Monte Carlo simulation method and the binomial tree approach. The Monte Carlo simulation method developed by Boyle (1976) determines the option's price by simulating returns on an underlying asset. The binomial tree methods developed by Cox, Ross, and Rubenstein (1979) and the log-transformed binomial method introduced by Trigeorgis (1991) allow for the modeling of possible future paths of projects and can take contingent decisions into account. Finite differences and finite elements are examples of the second group, which is the approximation of partial differential equations approach. In addition to these methods, numerical integration is one of the approaches used to value options which is a method for approximating integrals numerically.

As financial and real options are closely related, most of the presented methods apply to both options. However, throughout the section, we focus on exploring the fundamental methods for valuing real options. We will investigate the following methods: the Black and Scholes option-pricing method, the Binomial method, and the Monte Carlo simulation method. Given that real options embedded in investment decisions are American-style options, the dynamic programming technique often used in real options valuation to find an optimal exercise policy is also investigated.

I.3.1 The Black-Scholes Model

The Black-Scholes financial option pricing model was first proposed by Black and Scholes in 1973. Black and Scholes (1973) model is one of the most widely used closed-form solution models in pricing options. The model applies stochastic differential equations to derive an analytical solution for the option price. The formula of Black and Scholes (1973) model for a European call option is：

$$\mathrm{C} = S_0\, N(d_1) - K\, e^{-rt}\, N\,(d_2),$$

where N(▪) is the cumulative normal distribution function and:

$$d_1 = \frac{\ln\frac{S_0}{K} + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}},$$

$$d_2 = d_1 - \sigma\sqrt{T},$$

where C is the price of the call option, S is the price of the underlying asset, K is the exercise price, r is the risk-free interest rate, and T is the time to maturity. The Black and Scholes model works under the following assumptions: no transaction costs or differential taxes, a constant short-term risk-free rate of interest, borrowing and lending are allowed at the same rate of interest, continuous trading over time, and the underlying stock price is described by Geometric Brownian Motion.

The Black and Scholes (1973) model was used to value real options that only include one investment option, such as an option to defer an investment, as examined by McDonald & Siegel (1986), and an option to abandon a project as illustrated by Majd and Myers (1984). One of the limitations of using Black and Scholes (1973) in pricing real options is that the model works for simple options (Arnold, 2014), while real options in practice are more complex options. Another shortcoming of the use of this model in real option pricing is that it used to value European options, which is not the case in real options where the holder has the right to exercise the option by taking an investment decision at any time before the option is expired at its maturity date. Although the Black-Scholes model has some limitations, it remains a valuable tool for having important insights and path-breaking findings that are essential when pricing real options (Borison & Triantis, 2001).

I.3.2 Binomial Tree

The binomial tree model was first introduced by Cox, Ross, and Rubinstein in 1979. According to Borison and Triantis (2001), the binomial tree model provides a higher degree of flexibility by enabling the optimal timing of the exercise decision and the distribution of the underlying asset's value at different time points. The binomial tree is a graphical presentation that depicts the possible future asset price movements during the lifespan of an option, as depicted in figure.2. In each time step, the asset price has a probability of moving up with probability p by a certain factor u and it at the same time has a probability of moving down with probability $(1 - p)$ by a certain factor d. Let S_0*, r, and* p be the current price of the underlying asset, the risk-free rate of interest, *and* the risk-neutral probability, which could be derived as in (Cox, Ross, and Rubinstein, 1979):

$$p = \frac{e^{r\Delta t} - d}{u - d}.$$

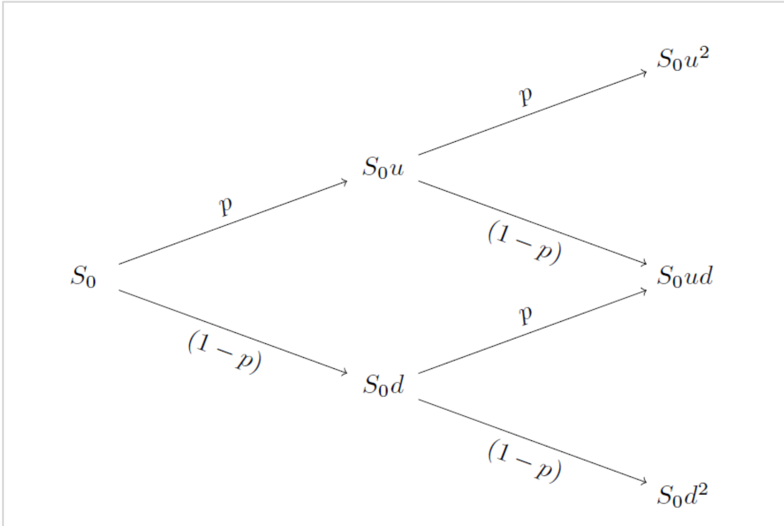

Figure 2: Binomial tree Illustration

To solve the binomial tree model, a risk-neutral portfolio must be set up under the assumption of an arbitrage-free market. The option values are calculated backward in time, starting at the maturity of the option. For an American option with multiple exercise opportunities, one must compare the exercise value to the holding value at each decision date. Let f be the value of the option at the inception date, f_u and f_d the overall value of the option for each branch. The following equation must hold:

$$S_0\, u\, \Delta - f_u = S_0\, d\, \Delta - f_d,$$

$$\Delta Su - \Delta Sd = f_u - f_d,$$

$$\Delta = \frac{f_u - f_d}{Su - Sd}.$$

At time T, the value of the real option in each state is known, so equation (5) can be solved for Δ, which is the hedge ratio as in equation (7). It means that a portfolio is riskless if you possess Δ shares in the asset and short one option. Because a riskless portfolio, in the absence of arbitrage opportunities, must earn the risk-free rate of interest, both sides of the equation can be discounted at the continuously compounded risk-free rate of interest to derive the option value at the previous state.

So, we have:

$$S_0\, \Delta - f = (S_0\, u\, \Delta - f_u)\, e^{-r\Delta t}.$$

Now we could solve for *f*, the initial price of the real option

$$f = \Delta S - (\Delta S u - f_u)\, e^{-r\Delta t}.$$

Trigeorgis (1991) develops the log-transformed binomial tree for valuing complex real options. He argues that this approach improves the traditional binomial tree method by reflecting the actual distribution of asset values more accurately. At the same time, it allows for pricing multiple options, specifically American-style options. Using the log-transformed binomial tree approach, the model becomes more efficient and accurate while maintaining the binomial approach's flexibility. Trigeorgis (1991) demonstrated the effectiveness of this method through several examples, including valuing a multi-option investment such as abandonment and switching options. The binomial tree model was also discussed by Brennan and Schwartz, 1985 in the valuation of natural resource investments, such as mines, forests, and oil fields. They propose a binomial tree model for valuing natural resource investments as a series of options, including the option to delay development and the option to abandon the project. The binomial tree model was also examined in valuing complex real options embedded in an investment opportunity in the energy sector by Song et al. (2017). The strength of the binomial tree model is its simple intuition and flexibility to handle various stochastic processes and multiple options. However, a limitation of these models is that they require more time when dealing with multiple underlying assets.

I.3.3 Monte Carlo simulation

Monte Carlo is a numerical method that approximates the option value by simulating the underlying stochastic process under the risk-neutral probability measure. Boyle (1977) used the Monte Carlo simulation to price European options. Longstaff and Schwartz (2001) proposed a least square Monte Carlo simulation method suitable to value American options. We present the stochastic differential equation that describes the path of the underlying asset S_t for time t ≥ 0:

$$dS_t = \mu S_t dt + \sigma S_t dz_t,$$

where μ is the instantaneous return, σ the standard deviation, and z_t is a Standard Brownian Motion.

The goal of the Monte Carlo simulation is to create sample paths for the underlying asset for each point in time t for a specific time interval up to time T with a time step dt. For example, a European call with maturity T and exercise price K uses the pricing function: $P_j = max\,(S_T - K, 0)$, where index j reflects the price of the j^{th} simulated path. The mean of the prices constitutes the option valuation at time $t = 0$:

$$V_0 = e^{-rT} \frac{1}{N} \sum_{j=1}^{N} P_T^j.$$

Monte Carlo presents the advantage of handling different complex processes of the underlying asset, including for example jump processes and stochastic volatility. In addition, it can accommodate the pricing of exotic options with more complex payoff functions. Finally, numerous articles have demonstrated the feasibility of using Monte Carlo simulation to value real options, including works by Broadie and Glasserman (1997), Smith and Nau (1995), Schwartz and Moon (2001), Mathews (2009), Klobucnik and Sievers (2013), and Doffou (2015).

I.3.4 Dynamic Programming

Dynamic programming (DP) is a technique used to solve complex optimization problems that involve sequential decision-making over time. In the context of real options, dynamic programming can be used to model the decision-making process involved in determining the optimal time to exercise an option at each point in time, like the optimal stopping problem of American options. DP starts the resolution at a future date when the value function is known. For example, the value of an American option at maturity is its exercise value. Using backwardation, at each decision date, we compute the value function as the maximum between the exercise value and the holding value. The holding value at a certain date is the expected future value function. This expectation is calculated by taking into account all the possible scenarios at the next date given the current position. In order to calculate this expectation, an interpolation for the value function is used.

DP is used by many researchers in various contexts to solve complex optimization problems. It is used to value American options (Ben-Ameur, Breton, & L'Ecuyer, 2002; Ben-Ameur, Chérif, & Rémillard, 2016; Ben-Ameur, Chérif, & Rémillard, 2020), derivative contracts (Ben-Ameur et al., 2013), installment option (Ben-Ameur, Breton, & François, 2006), and corporate securities (Ayadi, Ben-Ameur, and Fakhfakh (2016) ; Ben Ameur, Chérif, & Remillard, 2022; Ben-Ameur, Fakhfakh, & Roch, 2022).

II. Methodology

The methodology applied in this thesis extends Schwartz and Moon (2001) model to incorporate various types of real options, thereby accounting for the value of managerial flexibility embedded in these options that are not considered in Schwartz and Moon (2001). Our valuation approach consists of two components. The first component aims to determine the main firm value by adopting the capital budgeting technique that Schwartz and Moon (2001) introduced, which captures the high-growth rate potential of fintech companies. The second component aims to assess the value of the managerial flexibility embedded in the fintech investment decision using real option valuation. To value the real options, we apply the dynamic programming method proposed by Longstaff and Schwartz (2001) to value American-style options. This approach is appropriate as real options embedded in the capital budgeting problem are typically considered as American-type options. In this section, we discuss the Schwartz and Moon (2001) model for valuing high-growth companies, as well as the dynamic programming model introduced by Longstaff and Schwartz (2001).

II.1 The Schwartz-Moon Model

For Schwartz and Moon (2001) the dynamic of firms' revenues follows the following stochastic differential equation:

$$\frac{dR_t}{R_t} = \mu_t \, dt + \sigma_t \, dz_1, \qquad (1)$$

where μ_t, the drift, is the expected growth rate in revenues following a mean reverting process with a long-term average of $\bar{\mu}$. The initial high growth rates are assumed to converge stochastically to a long-term growth rate of the industry. The expected growth rate of revenue follows the following stochastic differential equation:

$$d\mu_t = k\,(\bar{\mu} - \mu_t)\,dt + \eta_t \, dz_2,$$

where η_t is the volatility of the expected growth in revenues and k is the mean reversion coefficient of all the stochastic processes. Schwartz and Moon (2001) stated that the mean-reversion coefficient k affects the speed at which the growth rate converges to its long-term average. The unanticipated changes in revenues σ_t are also assumed to deterministically converge to the normal level $\bar{\sigma}$, while its drift converges deterministically to zero.

$$d\sigma_t = k_1\,(\bar{\sigma} - \sigma_t)dt, \qquad (2)$$

$$d\eta_t = -\,k_2 \eta_t dt. \qquad (3)$$

Further, they assumed that total costs include variable and fixed costs. The variable costs are assumed to be a fraction of revenues as in equation (1):

$$Cost_t = \gamma_t \, R_t + F,$$

where F is the fixed costs and γ_t is the variable costs parameter in the cost function. Unlike Schwartz and Moon (2000) model, the revised Schwartz and Moon (2001) allows for the variable costs to follow a stochastic differential equation:

$$d\gamma_t = k_3(\bar{\gamma} - \gamma_t)dt + \varphi_t dz_3,$$

where k_3 is the mean-reversion coefficient which reflects the speed at which the variable costs are anticipated to approach their long-term average $\bar{\gamma}$, and φ_t is the volatility of variable costs. Additionally, the unanticipated changes in variable costs are assumed to deterministically converge to the long-term level, $\bar{\varphi}$, by the following equation:

$$d\varphi_t = k_4 (\bar{\varphi} - \varphi_t)dt. \tag{4}$$

Schwartz and Moon (2001) model allows for correlation between the Brownian motions z_1, z_2, z_3,

$$dz_1 dz_2 = \rho_{12} dt,$$
$$dz_1 dz_3 = \rho_{13} dt,$$
$$dz_2 dz_3 = \rho_{23} dt.$$

The net income after tax Y_t is given by the following equation:

$$Y_t = (R_t - Cost_t - Dep_t)(1 - \tau_c),$$

where τ_c is the tax rate of the firm and Dep_t is the depreciation. Taxes are only paid if the firm has net profit or if the firm has no accumulated loss-carry-forward. The loss-carry forward dynamics are given by

$$dL_t = \begin{cases} -Y_t\, dt & if\ L_t > 0, \\ Max\,[-Y_t\, dt, 0] & if\ L_t = 0\,. \end{cases} \tag{5}$$

Equation (5) shows that the loss-carry forward increases by the firm's net loss. These dynamics enable the model to account for taxes and accumulated loss-carry-forward in the valuation model. Moreover, the accumulated Property, Plant and Equipment, P_t, depends on capital expenditures, $Capex_t$, and the depreciation, Dep_t, as demonstrated in the following equation:

$$dP_t = [Capex_t - Dep_t]dt.$$

The following equations illustrate the calculations of capital expenditures and depreciation where CX_t, is the initial capital expenditures which after that are assumed to be a fraction, CR, of revenues. Depreciation is assumed to be a fraction DR of the accumulated Property, Plant and Equipment:

$$Capex_t = CX_t \qquad for\ t \ \leq \ \bar{t},$$

$$Dep_t = DR \times P_t.$$

Then, the amount of cash available to the firm, given by X_t, evolves according to:

$$dX_t = [r\ X_t + Y_t + Dep_t - Capex_t]dt.$$

The interest on the available cash balance is included in the dynamics of the cash available to ensure that the valuation results are not affected by the timing of cash flow distribution to shareholders. Schwartz and Moon (2001) model works under the risk-neutral framework enabling the risk-adjusted cash flow to be discounted at the risk-free interest rate. To avoid the need to define a dividend policy, the authors assume that the cash generated by the firm's operations remains within the firm, earning a risk-free rate of interest. This accumulated cash is available for distribution to shareholders at an arbitrary long-term horizon T , by this time the firm is expected to revert to a normal firm. Additionally, the model considers the possibility that the firm may reach a negative cash level without going bankrupt, by getting new financing opportunities. To incorporate this, they assume that the firm goes bankrupt when its available cash reaches a predetermined negative threshold, X^*.

Finally, Schwartz and Moon (2001) state that the value of the firm at the horizon T has two components: the outstanding cash balance and the terminal value of the firm, which can be determined as a multiple M of the $EBITDA$ at time T:

$$V_0 = E_Q[X_T + M\,(R_T - Cost_T)]\, e^{-rT},$$

where E_Q is the equivalent martingale measure, indicating that the model uses the risk-neutral measure to discount the expected value of the firm.

Moreover, the model considers the discrete nature of the input data used in the model such as quarterly and annual financial reports by proposing a discrete-time approximation of the model. In their implementation, they also assume that all the mean reversion coefficients, k, are equal. Then, they apply the following discrete version of the risk-adjusted processes:

$$R_{t+\Delta t} = R_t e^{\left\{\left[\mu_t - \bar{\lambda}\,\sigma_t - \frac{\sigma_t{}^2}{2}\right]\Delta t + \sigma_t \sqrt{\Delta t}\,\varepsilon_1\right\}},$$

$$\mu_{t+\Delta t} = e^{-k\Delta t}\,\mu_t + (1 - e^{-k\Delta t})\bar{\mu} + \sqrt{\frac{1-e^{-2k\Delta t}}{2k}}\,\eta_t\,\varepsilon_2,$$

$$\gamma_{t+\Delta t} = e^{-k\Delta t}\,\gamma_t + (1 - e^{-k\Delta t})\bar{\gamma} + \sqrt{\frac{1-e^{-2k\Delta t}}{2k}}\,\varphi_t\varepsilon_2,$$

where:

$$\sigma_t = \sigma_0\, e^{-kt} + \bar{\sigma}\,(1 - e^{-kt}), \tag{6}$$

$$\eta_t = \eta_0 e^{-kt}, \tag{7}$$

$$\varphi_t = \varphi_0\, e^{-kt} + \bar{\varphi}\,(1 - e^{-kt}). \tag{8}$$

Schwartz and Moon (2001) state that equations (6) to (8) are obtained by integrating equations (2), (3) and (4) with initial values σ_0, η_0 and φ_0. The variables ε_1, ε_2, and ε_3 are standard correlated normal variates.

II.2 The backward dynamic programing of Longstaff and Schwartz (2001)

As mentioned earlier, real options are mostly American options as there is an opportunity to exercise the options at any point until the investment opportunity disappears. The problem with pricing American options is that they can be exercised at any time before the expiration date, unlike a European option that can only be exercised at the expiration time. So, to price an American option, we need to find the optimal stopping time τ, and then estimate the expected value of the option. Hence, the methodology used in this thesis to value American-style real options is The LSM method, developed by Longstaff and Schwartz. It is a widely used numerical technique in evaluating investment opportunities under uncertainty. The LSM method uses adynamic programming which is a widely used approach for solving optimization problems to find the optimal stopping time. Meanwhile, the expected value can be approximated through Monte Carlo simulation. The LSM algorithm solves the model backward in time, evaluating at each node whether it is optimal to exercise the option or hold it for at least one more period.

Longstaff and Schwartz (2001) propose a random visit via Monte Carlo simulation. The variable $X_{n,t}$ represents the level of the underlying process X at the evaluation node (n,t) where n represents a simulated path and t an evaluation date, for $n = 1, \dots, N$ and $t = 1, \dots, T$. The exercise value at node (n,t) is indicated by $v_t^e\ (X_{n,t})$. For example, upon exercise at node (n,t), a call option on an underlying asset pays:

$$v_t^e(X_{n,t}) = Max\ (\ X_{n,t} - K, 0).$$

The holding function at note (n,t), under the risk-neutral probability measure, is given by

$$v_t^h\left(X_{n,t}\right) = E_Q\left[e^{-r\tau_n}\ v_{\tau_n}^e\left(X_n\right)\right].$$

The numerical procedure of Longstaff and Schwartz (2001) is referred to as least-squares Monte Carlo (LSMC) since it alternates between Monte Carlo estimations and linear least square approximation to solve the model backward in time, from the option maturity to the origin. In sum, LSMC combines Monte Carlo simulation, dynamic programming, and multilinear regressions, and consists of a backward construction of the $N \times T$ exercise table $\boldsymbol{E}$, where:

$$\boldsymbol{E}_{\boldsymbol{n},\boldsymbol{t}} = \begin{cases} 1 & \text{if the option is exercised at node } (n,t), \\ 0 & \text{if it is held at node } (n,t) \text{for at least another period.} \end{cases}$$

Longstaff and Schwartz (2001) start the evaluation procedures at the option maturity and set:

$$v_T(X_{n,T}) \ = \ v_T^e\left(X_{n,T}\right).$$

Thus, they set 1 at the last column of $\boldsymbol{E}$, i.e., at all nodes (n,T) for $n = 1,\dots,N$, with the convention that $v_T = v_T^e = 0$ if the option expires out of the money. Now, assume that the model has been solved and that the exercise table $\boldsymbol{E}$ has been filled from T to $t+1$. Longstaff and Schwartz (2001) use a two-step approach to approximate the holding values $v_t^h\left(X_{n,t}\right)$, for $n = 1,\dots,N$ and fill in column t of table $\boldsymbol{E}$. The algorithm works as follows:

1- Move forward from node (n,t) along row n of table $\boldsymbol{E}$, stop at the first time $\tau_n \in (t;\ T]$ at which it is "optimal" to exercise the option, discount the associated option cash flow, and set:

$$\tilde{v}_{\mathrm{t}}^h\left(X_{n,t}\right)= e^{-r(\tau_n-t)}\, v_{\tau_n}^e\left(X_{n,t}\right),$$

where $\tilde{v}_{\mathrm{t}}^h\left(X_{n,t}\right)$ is a Monte Carlo estimation of the option holding value $v_t^h\left(X_{n,t}\right)$ at node (n,t) based on only one path (path number n). This poor estimation is dictated by the fact that the N simulated paths never intersect.

2- To address this shortcoming, the Monte Carlo estimations $\tilde{\upsilon}_{\mathrm{t}}^{h}\ (X_{n,t})$ of $\upsilon_t^h\ (X_{n,t})$ for $n\ =\ 1, \dots, N$, are regressed. This is motivated by the fact that $\upsilon_t^h\ (X)$ is a function of X, as it is a conditional expectation of $X_t\ =\ X$. This second step results in an adjusted approximations $\hat{\upsilon}_{\mathrm{t}}^{h}\ (X_{n,t})$ of $\upsilon_t^h\ (X_{n,t})$ for $n\ =\ 1, \dots, N$, as if the first step were made of Monte Carlo estimations of size N. Thus, this step of LSMC consists of regressing $\tilde{\upsilon}_{\mathrm{t}}^{h}$ $(X_{n,t})$ on 1, $(X_{n,t})$ and $\left(X_{n,t}\right)^2$, for $n\ =\ 1, \dots, N$, solve the least square optimization problem:

$$min_{(\alpha,\beta,\gamma)}\ \textstyle\sum_{n=1}^{N}[\ \tilde{\upsilon}_{\mathrm{t}}^{h}\ \left(X_{n,t}\right) - \left(\ \alpha +\ \beta\ X_{n,t} +\ \gamma\ X_{n,t}^2\ \right)],$$

which results in the LSMC approximation of $\upsilon_t^h\ (X)$

$$\hat{\upsilon}_t^h\ (X_{n,t}) = \hat{\alpha} + \hat{\beta} X_{n,t} + \hat{\gamma} X_{n,t}^2\ for\ n =\ 1, \dots, N.$$

It is worth noting that $\hat{\upsilon}_t^h\ (X_{n,t})$ is now dependent on all paths, but not only on one path, and that $\hat{\upsilon}_t^h$ is now defined on the overall state space, but not only on the evaluation nodes.

Column t of $\boldsymbol{E}$ is filled as follows:

$$\mathbf{E}_{n,t} = \begin{cases} 1 & if\ \upsilon_t^e\left(S_{n,t}\right) >\ \hat{\upsilon}_{\mathrm{t}}^{h}\ \left(S_{n,t}\right), \\ 0 & \qquad elsewhere. \end{cases}$$

By backward induction, the $N\ \times\ T$ exercise table $\boldsymbol{E}$ is now assumed to be fully filled. Finally, Longstaff and Schwartz (2001) use table $\boldsymbol{E}$ and propose a Monte Carlo estimation of $\upsilon_0^h\ (X_0)$ of size N as follows:

$$\hat{\upsilon}_0^h(X_0) =\ \tfrac{1}{N}\textstyle\sum_{n=1}^{N}\ \upsilon_{\tau_n}^e\left(X_{\tau_n}\right),$$

where $\tau_n \in (0;\ T]$ is the first time (column number) 1 is encountered along row n of table $\boldsymbol{E}$. The overall option value at $(0, X_0)$ is

$$v_0(X_0) = \max\left(\hat{v}_0^h\,(X_0)\,, v_0^e(X_0)\right).$$

Shortly after its publication, LSMC was proven to be convergent when the number of random paths and the number of basis functions tend to infinity.

III. Numerical Investigation

In our implementation, we use the parameters summarized in Table (1) to test the reliability of our model. We use the MATLAB program to implement the methodology discussed earlier provided in the appendix. To conduct the numerical investigation, we run the code using 100,000 simulations with steps of one year over a 6-year forecasting period. Additionally, we incorporate an EV/EBITDA multiple of $1/r$ times in the calculations. We execute the model to get NPV of EGP 17.058 per share.

Table 1: Numerical investigation parameters

Valuation horizon	T	6 years
Initial revenue	R_0	500 million
Initial volatility of revenues	σ_0	15%
Long-term volatility of the rate of growth in revenues	$\bar{\sigma}$	8%
Mean-reversion coefficient	k	0.2
Initial expected rate of growth in revenues	μ_0	40%
Long-term growth rate in revenues	$\bar{\mu}$	5%
Initial volatility of expected growth rate in revenues	η_0	30%
Correlation	ρ	0
Initial variable cost as fraction of revenues	γ_0	70%
Fixed cost	F	35 million

Market price of risk in the revenue factor	λ	1.25
Risk-free interest rate	r	5%
Corporate tax rate	τ	35%
Number of shares	n	100 million

Our extended model includes the following real options:

1. Option to expand with an expansion rate, e, investment cost, K, and maturity T_1.
2. Option to abandon with an abandon value, A, and abandon rate $a = e/2$.

To evaluate these real options, we adopt both American and European option valuation approaches. The parameters for the real option valuation are presented in Table (2).

Table 2: Real option parameters in the numerical investigation

T	4 years
e	50%
K	1000 million
A	1000 million

To evaluate the real options, we consider the firm value, V_0, obtained from the Schwartz and Moon model, as the underlying asset for these options. The numerical results are presented in Table (3). The table displays the values of each option calculated using both the American and European approaches, along with their respective standard errors. It demonstrates that the adoption of a real option valuation leads to a higher per share value compared to the traditional NPV. This difference reflects the captured value of managerial flexibility embedded in the real options. Furthermore, it shows that the value of the American option exceeds that of the European option. This difference highlights the additional value derived from the flexibility to exercise the option before its maturity date, which is uniquely captured by the American option framework.

Table 3: Numerical investigation real option results

	American		European	
	Value	SE	Value	SE
Expansion	19.042	0.0027	18.923	0.0035
Abandon	22.773	0.0002	21.138	0.0019

IV. Empirical Study

We present in this section an application of the Real Options Valuation (ROV) theory to value a fintech company listed on the Egyptian Exchange: Fawry for Banking Technology and Electronic Payment (S.A.E). To achieve this objective, we will first introduce Fawry and its financial overview. Then proceed to apply the ROV model to estimate the value of the company and perform a sensitivity analysis.

IV.1 Introduction to Fawry

Fawry for Banking Technology and Electronic Payment (S.A.E) is a premier fintech company in Egypt that provides electronic payments and digital finance solutions. Fawry is listed on the Egyptian Exchange (EGX) since 2019. The company provides a wide range of electronic payment solutions, including bill payment, mobile top-up, e-commerce payments, and digital wallets. With its innovative and user-friendly platform, Fawry serves around 45 million users in its network, offering a convenient and secure way to financial services. In 2008, Fawry was established by a group of visionary entrepreneurs with the aim of introducing electronic payment systems in Egypt. The idea was to address the challenges faced by consumers who encountered difficulties and complex procedures when attempting to pay their bills. The company's primary objective was to provide simple and convenient payment alternatives to the people (Fawry, 2023).

By 2010, Fawry had deployed 5,000 Points of Sale (POS) machines, enabling it to make its services available for bank payments. This development facilitated the processing of bill payments through conventional banking channels. Since then, Fawry has grown substantially, expanding its network to include over 295,000 POS terminals and over 200 branches located throughout the country. The company has also formed partnerships with over 90% of the Egyptian banks. Fawry has a network of entities that integrates to constitute its ecosystem as illustrated in the following figure (Fawry, 2023):

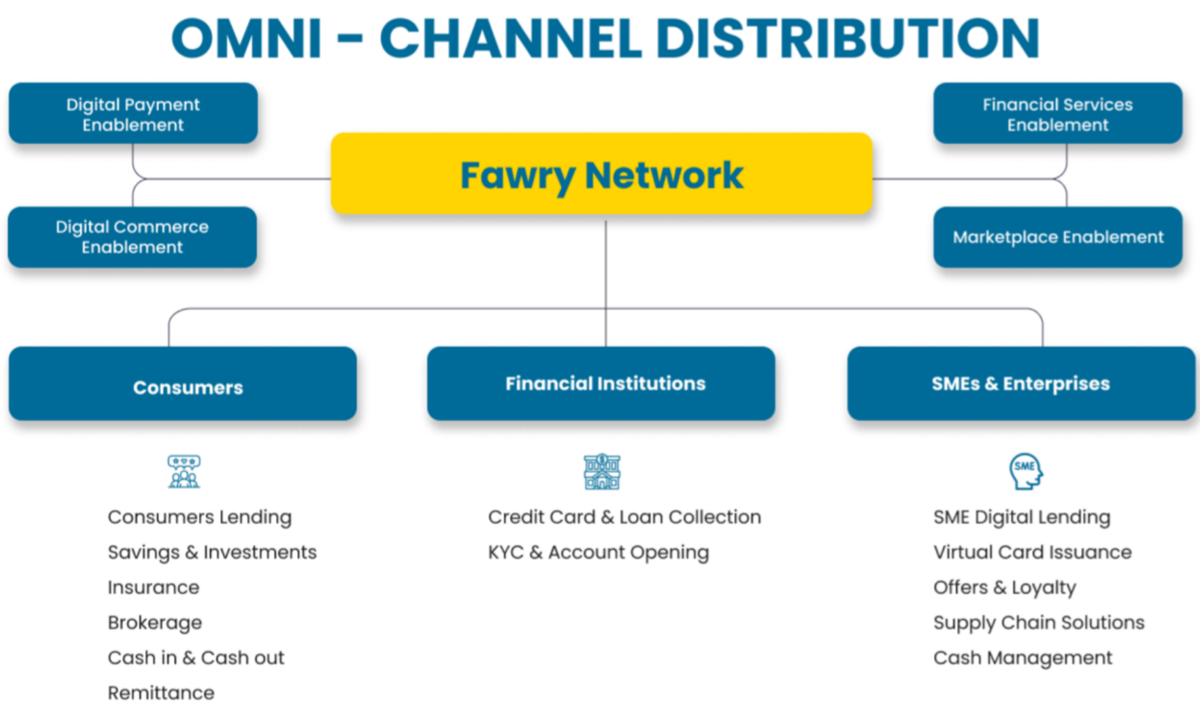

Figure 3: Fawry Ecosystem (about Fawry,2023)

In addition to its payment solutions, Fawry has been at the forefront of digital transformation in Egypt, providing technology-based services to businesses and government entities. The company's services include e-commerce solutions, digital marketing, and data analytics, among others. Fawry has also established several subsidiaries, as depicted in the figure below, to expand its range of services and offer innovative products to its customers. To further strengthen its ecosystem, Fawry has invested in many start-ups and joint ventures, including Bosta, Tazcara, Roaderz, and Waffarha.com. By raising strategic partnerships and investing in start-ups, Fawry has positioned itself at front of the fintech companies in Egypt.

Figure 4: Fawry Subsidiaries (about Fawry, 2023)

Fawry has also extended its reach beyond Egypt, with operations in the United Arab Emirates and Saudi Arabia. In 2023, Fawry has signed a memorandum of understanding (MoU) with the Emirati calling app BOTIM to offer residents in the UAE and Gulf Cooperation Council (GCC) countries money transfer and invoice payment services through the BOTIM app. According to the MoU, individuals in Egypt will be able to withdraw funds transferred from users in the UAE through the BOTIM app, using payment channels facilitated by Payby and Fawry across the country ("Fawry, BOTIM cooperate to enable international money transfers for Egyptian expats in UAE," 2023, February 28).

IV.2 Financial Overview

Fawry experiences high growth in revenues, as the figure illustrates. In 2022, Fawry's revenue amounts to EGP 2,279 million, reflecting a 38% revenue growth from 2021. The compounded annual growth rate between 2018 and 2022 is 40%. The net income in 2022 is EGP 327 million.

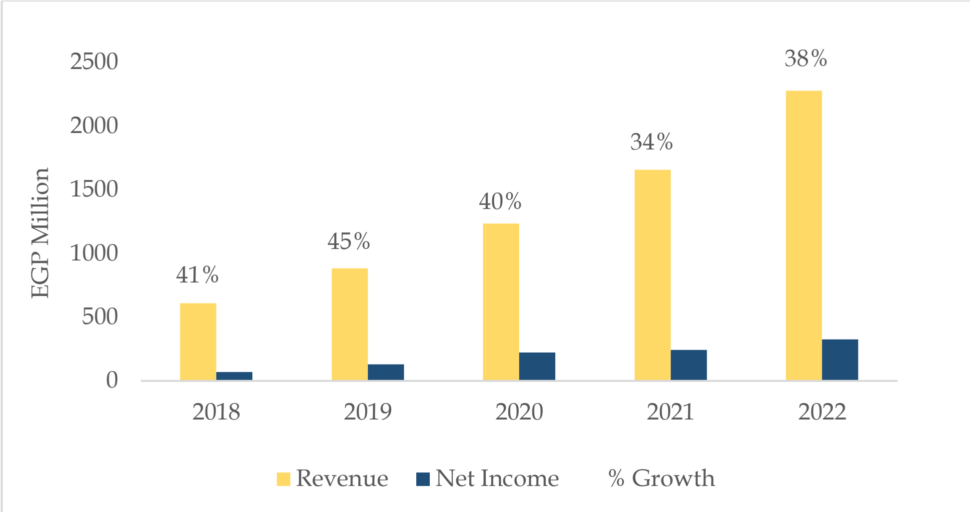

Figure 5: Revenue, Growth rates, and Net Income from 2018 to 2022

Fawry's COGS as illustrated in the figure below that shows the how the COGS develops in relation to revenues, amounting to EGP 948 million reflecting 58% gross profit margin in 2022. Its compounded annual growth rate between 2018 and 2022 is 33%.

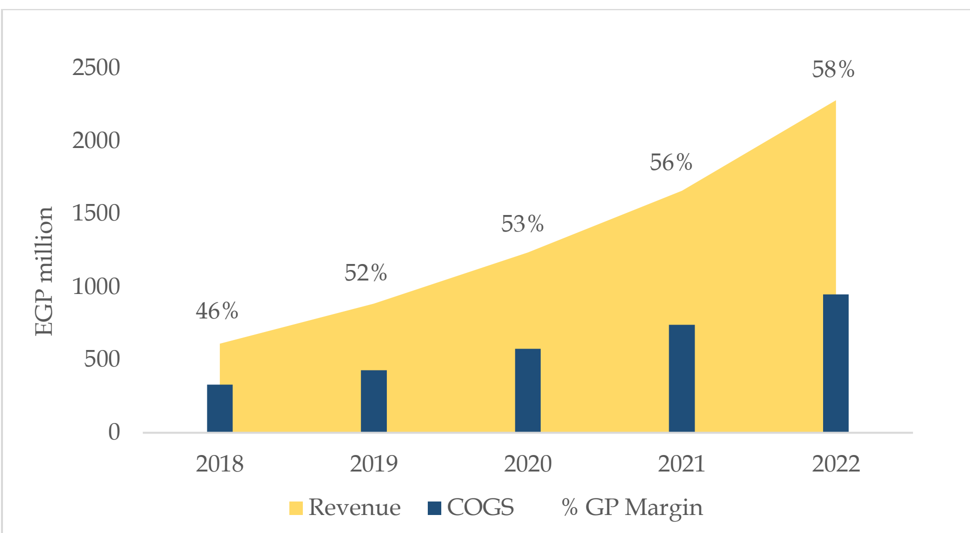

Figure 6: COGS in relation to Revenue and GP margin

Furthermore, as displayed in the figure showing the relation between revenue and total operating costs which consists of COGS, R&D expenses, Selling and Marketing expenses, General & Administrative expenses. The operating costs amount to EGP 2,016 million make up 88% of revenues in 2022.

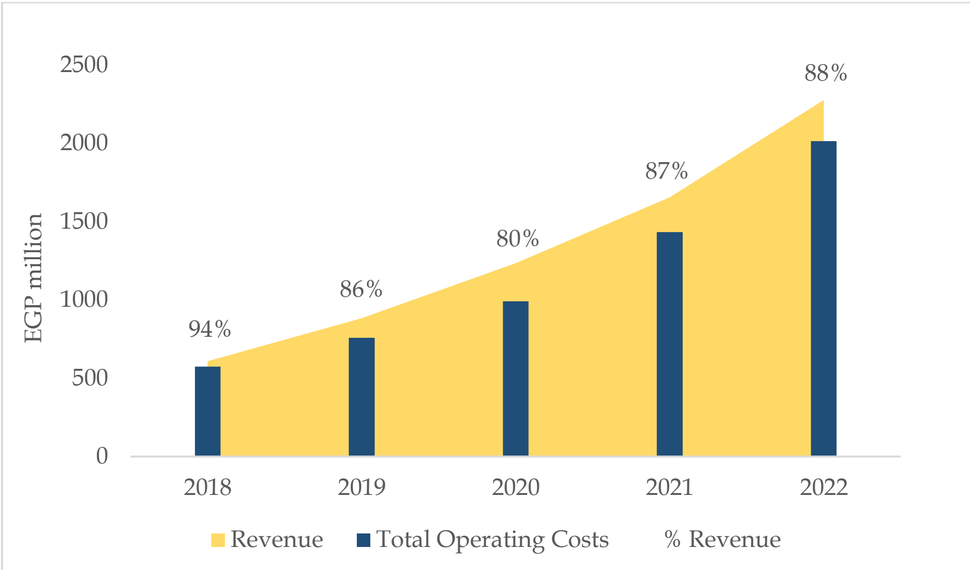

Figure 7: Total Operating Costs COGS in relation to Revenue

IV.3 Real option valuation of Fawry

Fawry was listed on the Egyptian Exchange (EGX) in 2019. Since then, it has released annual reports for the years 2019 to 2022 that are publicly available. The data from these reports, as well as the report from 2018 which is also publicly available, were used as a basis for estimating input parameters (https://www.fawry.com/financials-and-earning-releases/).

- **Descriptive statistics**

Table (4) shows the key descriptive statistics for Fawry's Stock prices since its listing in the Egyptian Exchange (EGX) in 2019 till 21st May 2023 to give insights about the firm stock prices.

Table 4: Fawry's stock prices descriptive statistics

Mean	5.56
Standard Error	9%
Median	5.27
Standard Deviation	2.87
Kurtosis	-0.73
Skewness	0.48

IV.3.1 Schwartz-Moon Model Implementation

- **Revenue and growth rate dynamics**

The initial revenue for Fawry is EGP 2,279 million. As proposed in Schwartz and Moon (2001), the initial volatility of revenue σ_0 is calculated using the standard deviation of revenue changes over the preceding sixteen quarters from 2018 to 2022 and annualized to get 14%. Also, σ_0 converges to the long-term volatility $\bar{\sigma}$ = 7%, which is half the initial volatility consistent with Schwartz and Moon (2001). Further, the initial growth rate of revenues μ_0 = 39.5% is calculated as the mean of change in revenues from 2018 – 2022. On the long run, μ_0 converges to the growth rate of a normal firm $\bar{\mu}$ = 10% which is the long-term average annual inflation rate as suggested by Klobucnik and Sievers (2013). Meanwhile, the initial volatility of expected growth rates in revenues η_0 is estimated by the standard deviation of the residuals from an $AR(1)$ regression on the growth rates like the estimation method used in Klobucnik and Sievers (2013) and can be expressed by the following equation:

$$\boldsymbol{\eta_0 = \sqrt{\frac{1}{n-1} \sum_{j=0}^{t} (\hat{\varepsilon}_{t-j} - \bar{\varepsilon})^2}}$$

where $\hat{\varepsilon}_j$ are the estimated residuals of an $AR(1)$ process: $\mu_t = \alpha + \beta\mu_{t-1} + \varepsilon_t$. Figure (8) presents the historical revenues generated from 2019 to 2022 and the simulated revenue progression throughout the valuation horizon.

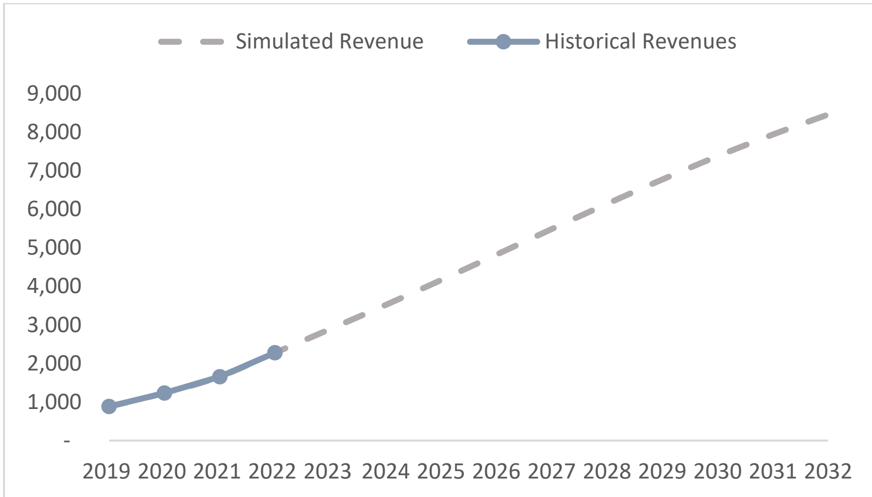

Figure 8: Fawry's Historical & Simulated Revenues

Further, figure (9) depicts the frequency distribution of revenues in year 4, where the initial growth rate of revenues begins to converge towards the long-term growth rate of the industry.

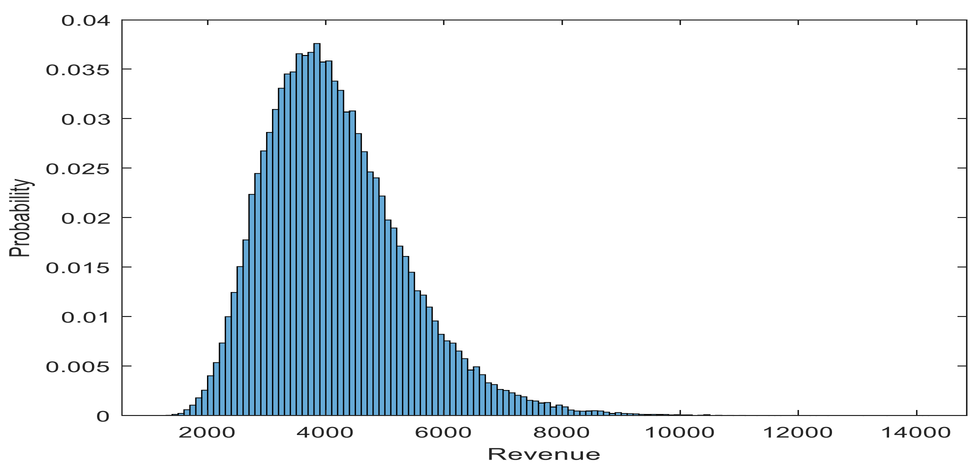

Figure 9: Revenue distribution in year 4

Figure (10) depicts the mean-reverting process of Fawry's growth rate in revenues, showing its convergence to the long-term growth rate $\bar{\mu}$ = 10%.

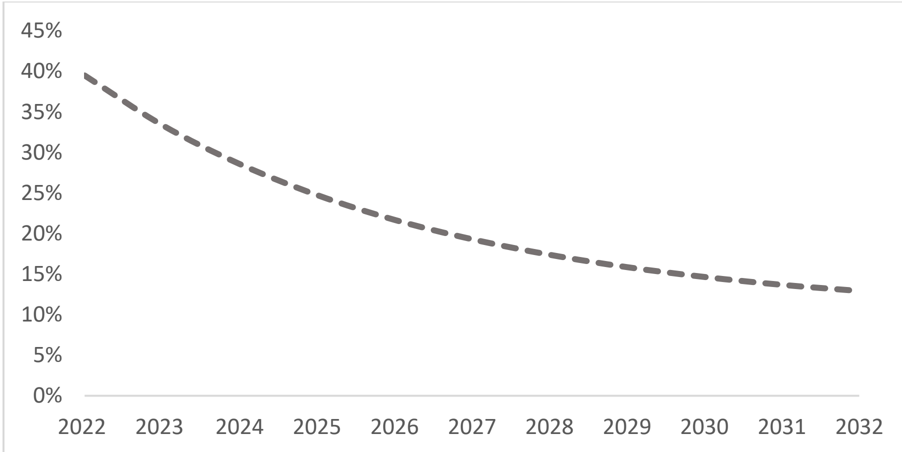

Figure 10: Fawry's growth rate in revenues

- **Variable cost dynamics**

The variable cost is calculated as described by Klobucnik and Sievers (2013), where the initial variable cost γ_0 is calculated using the average of total costs (variable cost + fixed cost) divided by revenues over the period from 2018 to 2022, considering fixed costs to be zero. This calculation yields in γ_0 = 0.871, which will converge to the long-run variable cost $\bar{\gamma}$ = 0.65 as a fraction of revenue. The initial volatility of costs φ_0 is obtained using the same method applied by Klobucnik and Sievers (2013). This method proposes running $AR(1)$ regression on the firm's cost ratios and calculating the standard deviation of the residuals. This calculation yields in φ_0 = 11%, which will converge to the long-term volatility in variable cost $\bar{\varphi}$ = 5.5%, which is half the initial volatility consistent with Schwartz and Moon (2001). Moreover, the mean-reverting process of Fawry's variable costs as a fraction of revenues is illustrated in Figure (11) where it convergences to the long-term fraction of revenues of $\bar{\gamma}$ = 65%.

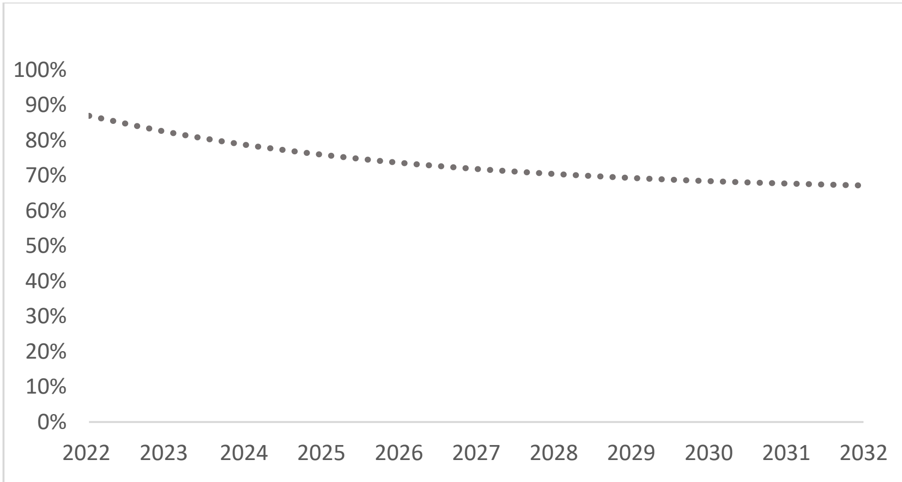

Figure 11: Fawry's Variable Costs as a Fraction of Revenues

- **Half-life of deviations and correlations**

In line with Schwartz and Moon (2001), we assume that all the mean reversion processes have the same speed adjusting coefficient, *k,* and are estimated using the half-life deviation. Schwartz and Moon (2001) use a half-life of 2.8 years. Nevertheless, considering Fawry's strong competitive position and well-diversified portfolio of products with strong growth potential, we use a half-life of 3 years and calculate the mean-reverting coefficient, $k = 0.231$. The mean-reverting coefficient has a significant effect on the valuation as it reflects the speed at which the initial high growth rate of revenues, which is 39.5% in the case of Fawry, converges to the long-term growth rate of the industry which is 10%. Therefore, a sensitivity analysis to the critical parameters is discussed later in the thesis.

- **Data from annual financial reports**

The cash and cash equivalents available at the end of the year 2022 amount to EGP 2,279 million, and Fawry has no loss-carry-forward as of 2022. The property, plant, and equipment reported in the 2022 balance sheet is EGP 749 million. Additionally, analysts expect the expected capital expenditure to be 20%, as indicated in Fawry's 2022 earnings release, where management plans to maintain capital expenditure at a level of 20% of revenues. The annual depreciation rate is determined by calculating the average past depreciation rate as a percentage of the property, plant, and equipment. The outstanding debt as of 2022 amounted to EGP 937 million, and the company has other liabilities of EGP 118 million. Furthermore, the company has 3,307 million shares outstanding. Finally, the corporate tax rate is 22.5%.

- **Risk parameters and simulations**

The risk-free interest rate is determined at 15% which is estimated from the average yield to maturity of the EGP 10-year treasury bond (https://www.cbe.org.eg/en/auctions/egp-t-bonds-fixed-coupon/historical-data). Additionally, a market risk premium of 15.4% is adopted. To conduct our analysis, we use 100,000 simulations with steps of one year over a 10-year forecasting period. As for the terminal value, we use an EV/EBITDA multiple of 15 times as used by analysts. Finally, Table (5) provides a summary of the parameters used in Schwartz and Moon model.

Table 5: Summary of Schwartz & Moon model parameters

Parameter	Notation	Value
Initial revenue (in EGP million)	R_0	2,279
Initial loss carry-forward (in EGP million)	L_0	0
Initial cash and cash equivalents (in EGP million)	X_0	2,228
Initial property, plant and equipment (in EGP million)	P_0	749
Initial volatility of revenues	σ_0	14%
Long-term volatility of revenues	$\bar{\sigma}$	7%
Initial expected growth in revenues	μ_0	39.5%
Long-term rate of growth in revenues	$\bar{\mu}$	10%
Initial volatility of expected growth rates in revenues	η_0	14%
Initial volatility of variable costs	φ_0	11%
Long-term volatility of variable costs	$\bar{\varphi}$	5.5%
Initial variable cost as fraction of revenues	γ_0	87%

Long-term variable cost fraction of revenues	$\bar{\gamma}$	65%
Fixed cost (in EGP million)	F	0
Correlation between change in revenues, change in expected growth rate and variable cost fraction	ρ	0
Market price of risk in the revenue factor	λ	15.4%
Mean-reversion coefficient	κ	0.231
Depreciation and Amortization rate	DR	24%
Capital Expenditures rate	CR	20%
Risk-free interest rate	r	15%
Corporate tax rate	τ	22.5%
Horizon for forecast period	T	10
Time increment	Δt	1
Terminal value multiple	M	15

IV.3.2 Real Option Implementation

In this section, we aim to extend the Schwartz and Moon (2001) model by examining two types of real options: the option to expand and the option to abandon. As of 2022, we assume that Fawry made an investment in ABC company (a hypothetical company), that facilitates the connection between small merchants and delivery agents, enabling them to deliver goods and receive payments from clients. Also, we assume that Fawry currently holds a 30% ownership stake in ABC, which has significant growth potential as it targets the market gap in providing last-mile digital delivery services for SMEs. Given the good prospects associated with ABC, we aim to capture the value of managerial flexibility embedded in this investment. To conduct a real option valuation, we assume that Fawry has the option to increase its share in ABC by acquiring an additional 22% of the company's ownership through an investment of EGP 500 million within the next six years. Additionally, Fawry has the option to abandon half of its share, valued at EGP 500 million, in ABC over the same period. Table (6) provides a summary of the parameters used in real option valuation.

Table 6: Real Option Parameters

Time to maturity, T	6 years
Expansion rate, e	22%
Investment, K	EGP 500 million
Abandon rate, a	50%
Abandon value, A	EGP 500 million

IV.3.3 Results

By applying the proposed methodology discussed earlier, the estimated enterprise value (EV) as well as the per share value of Fawry is presented in table (7).

Table 7: Fawry estimated enterprise value and per share value.

	Enterprise Value	Per Share Value
Expansion Option	EGP 40,346 million	EGP 11.90
Abandon Option	EGP 26,744 million	EGP 7.78
NPV	EGP 22,796 million	EGP 7.20

As of May 21st, 2023, the closing stock market price for Fawry is EGP 5.5 per share. However, the estimated value per share based on our analysis is higher than the current market price. The NPV per share is higher by 31%, indicating that the stock is undervalued in the market. Additionally, the value of the abandon option per share exceeds the market price by 42%, while the expansion option per share is higher by 116%.

IV.3.4 Sensitivity analysis

The valuation results are sensitive to the estimated parameters involved in the model. Therefore, it is crucial to examine the impact of changing critical parameters. First, we examine the effect of Schwartz and Moon's (2001) critical parameters on the enterprise value independent of real options. Then, we assess the effect of the expansion and abandonment costs on the enterprise value for each of the options.

Schwartz and Moon (2001) highlight the significant impact of the following parameters on the valuation results: initial growth rate of revenue, μ_0, mean-reverting coefficient, k, and the long-term variable cost as a fraction of revenues, $\bar{\gamma}$. Table (8) highlights the effect of 5% increase or decrease in each of these parameters on the enterprise value. The sensitivity analysis is conducted in two scenarios: the first involves applying the Schwartz and Moon model without the extended real option valuation, while the second involves using the extended model that incorporates the real option valuation approach. The table shows that the mean-reverting coefficient has the most significant effect on Fawry's enterprise value as well as long-term variable cost. The initial growth rate of revenues also impacted the value Fawry's enterprise value but not to the extent of variable cost long-term rate or the mean reverting coefficient.

Table 8: Sensitivity analysis of Schwartz and Moon critical parameters

Parameter		EV without Option	% Change	EV with Options	% Change
Initial growth rate of revenues (μ_0)					
+ 5%	41.5%	28,724	7.4%	43,545	7.9%
Base	39.5%	26,743	-	40,346	-
- 5%	37.5%	24,814	-7.2%	37,590	-6.8%
Mean-reverting coefficient (k)					
3.5 years	0.198	30,283	13.2%	45,955	13.9%
3 years (base)	0.231	26,743	-	40,346	-
2.5 years	0.277	23,451	-12.3%	35,374	-12.3%
Long-term variable cost ($\bar{\gamma}$)					
+ 5%	68.3%	24,194	-9.5%	36,542	-9.4%
Base	65.0%	26,743	-	40,346	-
- 5%	61.8%	29,257	9.4%	44,279	9.7%

Regarding the real options valuation, a sensitivity analysis was conducted to examine the impact of changes in the expansion cost and expansion rate on the enterprise value. Table (9) clearly demonstrates that the effect of changing the expansion cost is marginal when compared to the substantial effect observed when changing the expansion rate. On the other side, the examination of changes in the abandon value and abandon rate revealed minimal sensitivity in the enterprise value as the abandon option becomes more valuable in situations involving financial distress. This finding is consistent with the previous analysis, which highlights the significant strategic value of the expansion option for Fawry, given its strong growth prospects.

Table 9: Sensitivity analysis of real option valuation

Parameter		EV	% Change
Expansion cost (K)			
+ 50%	750	39,943	-1%
Base	500	40,346	-
- 50%	250	40,917	1%
Expansion rate (e)			
32%		47,172	16.9%
22% (base)		40,346	-
12%		33,930	-15.9%

V. Conclusion

The aim of our thesis is to propose a model for valuing fintech companies, which are recognized as technology-based firms. Valuing these firms using the conventional valuation methodologies is challenging because of their unique characteristics. fintech companies are characterized by their risky nature as technology-based companies, which typically pass through multiple stages of the product life cycle that need great flexibility in decision-making. Fintech firms often require large upfront capital investments before the product/service grows widespread and generate revenues. Thus, it may take several years to pay back the large upfront capital paid in the development stage and generate positive cash flow. These unique characteristics represent challenges in using traditional valuation methodologies and should be addressed in the dynamics of the methodologies applied to valuing fintech companies.

Our proposed model extends the Schwartz-Moon (2001) model by incorporating real options to value fintech companies. This extension allows us to address the limitations of traditional methods in valuing high-growth technology companies, characterized by their risky nature and their need for flexibility in decision-making. Therefore, we add a real option valuation to the Schwartz and Moon (2001) model to capture the value of managerial flexibility in investment decisions, which was not considered in their model. Our valuation approach consists of two components. The first component aims to determine the main firm value by adopting the capital budgeting technique that Schwartz and Moon (2001) introduced, which captures the high-growth rate potential of fintech companies. The second component aims to assess the value of the managerial flexibility embedded in the fintech investment decision using real option valuation.

To value the real options, we apply the dynamic programming method proposed by Longstaff and Schwartz (2001) to value American-style options. This approach is appropriate as real options embedded in the capital budgeting problem are typically considered as American-type options. Our extended model includes two options: the option to expand and the option to abandon. However, the setting of the model is flexible to incorporate multiple and complex options. This contributes to the valuation of fintech companies by offering an enhanced valuation methodology that enables investors, analysts, and managers to make more informed decisions when valuing such companies.

The valuation of Fawry reveals that the real option valuation enhances the value of the firm through quantifying the value of the managerial flexibility embedded in the fintech investments that conventional techniques fail to capture. Further, the value of the expansion option is higher than the abandon option highlighting the strategic value of the expansion plan to value. The sensitivity analysis highlights that the mean-reverting coefficient has the most significant effect on Fawry's enterprise value as well as long-term variable cost.

Although real option theory is suitable for valuing fintech companies, it does have certain limitations. Firstly, it relies on specific assumptions that may not capture all the risks associated with a business or project. Secondly, it places more emphasis on risks associated with revenue generation while giving less consideration to cost-related risks. Despite these limitations, real option valuation theory remains a valuable tool for assessing investment opportunities that involve high risk and uncertainty and requires managerial flexibility in decision-making.

VI. Appendix

MATLAB Code:

```
clear;
clc;
% Set seed
rng('default');

% Reading the inputs file
file = readtable('data.csv');

T = file.Var2(1);          % valuation horizon (years)
R0 = file.Var2(2);         % initial value of revenue (in mln)
L0 = file.Var2(3);         % initial loss carry-forward
X0 = file.Var2(4);         % initial cash & cash equivalent
Capex0 = file.Var2(5);     % initial PPE
sigma0 = file.Var2(6);     % initial volatility of revenue
sigma_bar = file.Var2(7); % long-term volatility of revenue
k = file.Var2(8);          % mean-reversion rate
mu0 = file.Var2(9);        % initial growth rate
mu_bar =file.Var2(10);     % long-term growth rate
eta0 = file.Var2(11);      % initial volatility of growth rate
rho = file.Var2(12);       % correlation
F = file.Var2(13);         % annual fixed income (in mln)
gamma0 = file.Var2(14);    % Initial variable costs (% of revenues)
gamma_bar = file.Var2(15);% Long term  variable costs (% of revenues)
psi0 = file.Var2(16);      % initial of volatility of variable costs
psi_bar = file.Var2(17);  % long term of volatility of variable costs
lambda = file.Var2(18);    % market unit risk premium
Dep_rate = file.Var2(19); % Depretaition and amortization rate per annum
Capex_rate = file.Var2(20);% Capital expenditure per annum (% of revenue)
tax = file.Var2(21);        % tax rate
r = file.Var2(22);          % risk-free rate
WACC = file.Var2(23);       % weighted average cost of capital
N_shares = file.Var2(24);  % number of outstanding shares (in mln)
n = file.Var2(25);          % time steps
Multiple = file.Var2(26);  % EV/EBITDA exit multiple to get terminal value
N = file.Var2(27);          % Simulation steps
NPV = file.Var2(28);
EV = file.Var2(29);

dt = T / n;                 % Time increment

if NPV == 1
  r = WACC;
end

factor = 1;
if EV == 1
    factor = 0;
end
```

```
% Parameters of real option valuation
file_RO = readtable('data_RO.csv');

sigma = file_RO.Var2(1);
alpha = file_RO.Var2(2);

% Expand option (Call)
T1 = file_RO.Var2(3);          % option maturity
K = file_RO.Var2(4);           % investment cost
e =file_RO.Var2(5);            % expansion rate
n1 = 12 * T1;                  % time steps
dt1 = T1 / n1;                 % time increment

% Abandon option (Put)
T2 = file_RO.Var2(6);          % option maturity
A = file_RO.Var2(7);           % abandon strike price
a = file_RO.Var2(8);           % 0.3*0.5
n2 = 12 * T2;                  % time steps
dt2 = T2 / n2;                 % time increment

% 1- Simulate Revenues

% calculate the volatility in expected growth rate eta(t)
eta_t = zeros(N, n);
eta_t(:, 1) = eta0;
for j = 1:N
    for i = 1:n
        eta_t(j, i) = eta0 * exp(-k * i * dt);
    end
end

% simulate mu(t); expected growth rate on revenues
zm = randn(N, n) * factor;   % generate random N(0,1)
mu_t = zeros(N, n);
mu_t(:, 1) = mu0;
for i = 1:n
    mu_t(:, i+1) = mu_t(:, i) .* exp(-k * dt) + (1 - exp(-k * dt)) .* mu_bar +
sqrt((1 - exp(-2 * k * dt)) / (2 * k)) .* eta_t(:, i) .* zm(:, i);
end

% calculate sigma(t); Volatility in the revenue
sigma_t = zeros(N, n);
sigma_t(:, 1) = sigma0;
for j = 1:N
    for i = 1:n
        sigma_t(j, i) = sigma0 * exp(-k * i * dt) + (1 - exp(-k * i * dt)) *
sigma_bar;
    end
end

% simulate revenue
zr = randn(N, n) * factor;   % generate random N(0,1)
Rt = zeros(N, n);
Rt(:, 1) = R0;
```

```
for i = 1:n
    Rt(:, i+1) = Rt(:, i) .* exp((mu_t(:, i) - lambda .* sigma_t(:, i) - (sigma_t(:, i) .^ 2) / 2) .* dt + sigma_t(:, i) .* sqrt(dt) .* zr(:, i));
end

means = mean(Rt);

% 2- Calculate Total Cost

% Calculate the volatility in variable cost
psi_t = zeros(N, n);
psi_t(:, 1) = psi0;
for j = 1:N
    for i = 1:n
        psi_t(j, i) = psi0 * exp(-k * i * dt) + (1 - exp(-k * i * dt)) * psi_bar;
    end
end

% Simulate the fraction of variable cost
zc = randn(N, n)  * factor;   % generate random N(0,1)
gamma_t = zeros(N, n);
gamma_t(:, 1) = gamma0;
for i = 1:n
    gamma_t(:, i+1) = gamma_t(:, i) .* exp(-k * dt) + (1 - exp(-k * dt)) .* gamma_bar + sqrt((1 - exp(-2 * k * dt)) / (2 * k)) .* psi_t(:, i) .* zc(:, i);
end

% Calculate the Total cost
TC = gamma_t .* Rt + F;

% 3-Calculate Capital Expenditure, Depreciation & Earnings before Tax (EBT)

% Calculate Capital Expenditure
capex_t = zeros(N, n);
capex_t(:, 1) = Capex0;

for i = 1:n
    capex_t(:, i) = Rt(:, i) .* Capex_rate;
end

% Calculate Ending Balance of PPE and Depreciation
Gross_PPE = zeros(N, n);
Gross_PPE(:, 1) = Capex0;
Dep_t = zeros(N, n);
PPE_t = zeros(N, n);
PPE_t(:, 1) = Capex0;
for i = 1:n
    Gross_PPE(:, i+1) = Gross_PPE(:, i) + capex_t(:, i);
    Dep_t(:, i+1) = Gross_PPE(:, i) .* Dep_rate;
    PPE_t(:, i+1) = Gross_PPE(:, i+1) - Gross_PPE(:, i) .* Dep_rate;
end

% Calculate Earnings before Tax (EBT)
EBT = Rt - TC - Dep_t;
```

```
% 4- Calculate Loss Carry-Forward, Tax Expense & Earnings after Tax

% Calculate Loss Carry-Forward
Lt = zeros(N, n);
Lt(:, 1) = L0;
for j = 1:N
    for i = 1:n
        if Lt(j, i) > 0
            Lt(j, i+1) = Lt(j, i) - EBT(j, i+1);
        elseif Lt(j, i) == 0
            Lt(j, i+1) = max(-EBT(j, i+1), 0);
        else
            Lt(j, i+1) = -EBT(j, i+1);
        end
    end
end

% Calculate Tax Expense
Tax_Expense = zeros(N, n+1);
for j = 1:N
    for i = 1:n
        if Lt(j, i+1) < 0
            Tax_Expense(j, i+1) = -Lt(j, i+1) * tax;
        elseif Lt(j, i+1) == 0 && EBT(j, i+1) > 0
            Tax_Expense(j, i+1) = EBT(j, i+1) * tax;
        end
    end
end

% Calculate Net Income(Loss) (Earning after Tax)
Yt = EBT - Tax_Expense;


% 5- Calculate Ending cash balance (FCFF) & Cumulative Cash at T.

% Calculate Ending cash balance (FCFF)
Cash_t = zeros(N, n);
Cash_t(:, 1) = X0;
for i = 1:n
    Cash_t(:, i+1) = Cash_t(:, i) .* exp(r * dt) + Yt(:, i+1) + Dep_t(:, i+1) -
capex_t(:, i);
end

% 6- Calculate Terminal Value (TV)

% Calculate Earnings before Interest, Tax, Depreciation & Amortization (EBITDA)
EBITDA_T = Rt(:, n) - TC(:, n);

% Calculate Terminal Value (TV)
TV_T = Multiple * EBITDA_T;

% 7- Calculate Firm Value (V0)

Vl = zeros(N, n);
Vl(:, n) = Cash_t(:, n) + TV_T;
```

```
VT = sum(Vl, 2);

% Firm value under risk-neutral
V0 = mean(VT) * exp(-r * T);

% Firm value using
price_share = V0 / N_shares;

% Plotting Revenue Distribution in Year 3
figure;
histogram(Rt(:,4), 'Normalization', 'probability');

xlabel('Revenue');
ylabel('Probability');

%% EUROPEAN OPTION TO EXPAND (call)
S0= price_share;
call_closed = blsprice(e * price_share, K, r, T1, sigma);
% Generate stock price paths
S = zeros(N, n1+1);
S(:, 1) = S0;
for i = 2:n1+1
    S(:, i) = S(:, i-1) .* exp((r - 0.5 * sigma^2) * dt1 + sigma * sqrt(dt1) *
randn(N, 1));
end

% Calculate the payoff of the call option at maturity
Payoff_C = max(e * S(:, end) - K, 0);
% Calculate the call option price using standard Monte Carlo simulation
Call = exp(-r * T1) * mean(Payoff_C);
% Calculate the total real option value as the sum of the project value and the call
option value
V_C_eur = price_share + Call;
% Calculate the per-share value for the expansion option (European)
%VC_eur_per_share = V_C_eur / N_shares;
disp(V_C_eur);

%% AMERICAN OPTION TO EXPAND (call)
% Calculate the call option value at time 0
call_Amr = LSMC_put_call(e * price_share, K, T1, sigma, r, N, n1, alpha, 1);
% Calculate the total real option value as the sum of the project value and the call
option value
V_C_Amr = price_share + call_Amr;
% Calculate the per-share value for the expansion option (American)
%VC_Amr_per_share = V_C_Amr / N_shares;
disp(V_C_Amr);

%% EUROPEAN OPTION TO ABANDON (Put)
[call,put]=blsprice (a * price_share,K,r,T2,sigma);
% Generate stock price paths
S_P = zeros(N,n2+1);
S_P(:,1) = S0;
for i = 2:n2+1
    S_P(:,i) = S_P(:,i-1).*exp((r-0.5*sigma^2)*dt2+sigma*sqrt(dt2)*randn(N,1));
end
```

```
% % Calculate the payoff of the put option at maturity
Payoff_P = max(A -a.*S_P(:,end), 0);
% Calculate the put option price using standard Monte Carlo simulation
Put = exp(-r*T2) * mean(Payoff_P);
% Calculate the total real option value as the sum of the project value and the put
option value
V_P_eur = price_share + Put;
% Calculate the per-share value of the abandon option (European)
%VP_eur_per_share = V_P_eur / N_shares;
% Calculate the confidence interval of the put option price estimate
P_std = std(exp(-r*T)*Payoff_P)/sqrt(n); % standard error
CI_P = [Put - norminv(1-alpha/2)*P_std ; Put + norminv(1-alpha/2)*P_std ]; %
Confidence interval
se_p = P_std/sqrt(n);
disp(V_P_eur)

%% AMERICAN OPTION TO ABANDON (Put)
% Calculate the Put option value at time 0
put_Amr = LSMC_put_call(a * price_share, K, T2, sigma, r, N, n2, alpha, 0);
% Calculate the total real option value as the sum of the project value and the put
option value
V_P_Amr = price_share + put_Amr;
% Calculate the per-share value of the abandon option (American)
%VP_Amr_per_share = V_P_Amr / N_shares;
disp(V_P_Amr);
```

Longstaff and Schwartz function to value call/put option

```
function [option_price] = LSMC_put_call(S0, K, T, sigm, r, N, n, alpha, index)

dt= T/n;        % Duration between exercise times

% SDE Simulation Process
U= rand(N,n);       % Random uniform sample
Z1= norminv(U);     % Deriving random sample for the Browning Motion
Z2= norminv(1-U);   % Deriving the antithetic sample to double the simulation size
S1= zeros(N,n+1);   % Constructing matrix to simulate "original" share price paths
S2= zeros(N,n+1);   % Constructing matrix to simulate "antithetic" share price paths
S1(:,1)= S0;        % Setting initial price in original paths
S2(:,1)= S0;        % Setting initial price in antithetic paths

for i= 1:n
    S1(:,i+1)= S1(:,i) + S1(:,i)*r*dt + S1(:,i)*sigm*sqrt(dt).*Z1(:,i);
    S2(:,i+1)= S2(:,i) + S2(:,i)*r*dt + S2(:,i)*sigm*sqrt(dt).*Z2(:,i);
end
S=[S1;S2];          % Combining Simulation Paths (50K original + 50K antithetic)


% Simulating Optimal Exercise Values (American LSM)
if index == 1
    cf_m= max(0, S(:,2:n+1) - K);  % Creating cash flow matrix of (call payoff) each
node.
elseif index == 0
```

```
    cf_m= max(0, K - S(:,2:n+1));  % Creating cash flow matrix of (put payoff) each
node.
end

E_m= zeros(N,n);            % Creating matrix to mark optimal exercise value (vt).
E_i= zeros(N,1);            % Vector to index the optimal exercise date (t).

for i=1:N                   % Starting with the Terminal Payoff Setting
    if cf_m(i,n)>0
        E_i(i)=n;
        E_m(i,n)=cf_m(i,n);
    end
end

for i= n-1:-1:1             % Starting Backward induction (Longstaff & Shwartz
Process)

    Xn=zeros(N,1);          % Prepare the first predictor of the regression (St).
    for i_2= 1:N
        if  cf_m(i_2,i) > 0
            Xn(i_2)=S(i_2,i+1);
        else
            Xn(i_2)= nan;
        end
    end

    Xs=[ones(N,1),Xn,Xn.^2];    % Predictor matrix including alpha, St, and St-squred

    Y=zeros(N,1);               % Prepare the predicted value (discounted future
payoff)
    for i_3=1:N
        if E_i(i_3)>0
            Y(i_3)= cf_m(i_3,E_i(i_3)) * exp(-r * dt * (E_i(i_3)-i));
        end
        if  cf_m(i_3,i) < 0
            Y(i_3)= nan;
        end
    end

    beta= mvregress(Xs,Y);    % Calculating regression coeffeficients

    v_h= Xs*beta;             % Predicting (approximating) holding value

    for i_4=1:N
        if cf_m(i_4,i)>v_h(i_4)     % Comparing exercise to holding values
            E_i(i_4)=i;
            E_m(i_4,:)= 0;
            E_m(i_4,i)= cf_m(i_4,i);
        end
    end

end

putvector_lsm= zeros(N,1);          % Calculate v0 as the holding values of paths
for i_5=1:N
```

```
    if E_i(i_5)>0
        putvector_lsm(i_5) = exp(-r * dt * E_i(i_5)) * E_m(i_5,E_i(i_5));
    end
end



put_lsm = mean(putvector_lsm);           % Calculate the put price as a mean of
holding values
put_lsm_s = std(putvector_lsm)/sqrt(N);   % Standard Erorr
put_lsm_ci = [put_lsm-norminv(1-alpha/2)*(put_lsm_s/sqrt(N)); ...
    put_lsm + norminv(1-alpha/2)*(put_lsm_s/sqrt(N))]; % Confidence interval
option_price = put_lsm;
%standar_error = put_lsm_s;
%CI = put_lsm_ci;
%disp(option_price);
%disp(standar_error);
%disp(CI);
end
```

VII. References

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