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Optimizing Repayment Strategies for Egyptian Construction Companies with Multiple Loans

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[Optimizing Repayment Strategies for Egyptian Construction Companies with Multiple Loans]

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Declaration of Authorship

I, Moataz Mohamed Elkady, declare that this Thesis titled, Optimizing Repayment Strategies for Egyptian Construction Companies with Multiple Loans and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.
- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
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Signed:

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Abstract

Optimizing Repayment Strategies for Egyptian Construction Companies with Multiple Loans is of utmost importance to efficiently allocate monthly income for loan repayment. The conventional practice of paying the minimum balance due each month leads to increased interest charges and prolonged repayment periods. In response, this research proposes a model for multiple loan repayment that considers the computational complexity involved.

The model is built on the foundation of mixed integer linear programming and aims to establish an optimal repayment schedule by minimizing the total cash required to repay the loans, prioritizing the repayment of loans before personal expenses. The implementation of the model utilizes Mathematica software and the Gurobi optimizer. Through simulated cases using real data, the effectiveness of the proposed methodology is demonstrated. Debtors achieve average savings exceeding 12%, resulting in substantial reductions in payments made to banks or leasing companies. In certain scenarios, savings of up to 23.87% are attainable. The model takes into account debtors' revenue, expenses, as well as the associated interest rates, terms, and fees for each loan.

This research challenges the prevailing debt repayment practices in construction companies by introducing an efficient and practical model for addressing the multiple loan repayment problem. By adopting the proposed model, construction companies gain the ability to make informed decisions regarding their financial circumstances and repayment strategies, ultimately leading to significant cost savings and enhanced financial stability. It is recommended to utilize the model to implement an optimized payment plan that considers the unique financial situation and cash flow projections of each construction company. Regular monitoring of loan repayment performance and necessary adjustments to optimize repayment strategies are crucial for sustained effectiveness.

The proposed model presents construction companies with a powerful tool to optimize their repayment policies for multiple loans. Its adoption enables informed decision-making, resulting in notable cost savings and improved financial stability. This research makes a valuable contribution to the field of construction finance, potentially revolutionizing the approach to debt repayment within the industry.

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List of Abbreviations

LC:	**Letter of credit**
LG:	**Letters of Guarantee**
LTL:	**Long term loans**
MILP:	**Mixed-Integer Linear Programming**
MTL:	**Medium-term loans**
NP-Hard:	**Nondeterministic Polynomial-Time Hard**
RPML:	**Repayment Policy of Multiple Loans**
STL:	**Short term loans**

List of Symbols

$\boldsymbol{ai_n}$: administration fee for partial or full payment of the principle

$\boldsymbol{A_n^t}$ Is the administration fee imposed when V_l^t is more than the required installment for loan *n* at month *t*

$\boldsymbol{c_n^t}$ when there is a deficit in the payment a penalty is imposed on the deficit amount

$\boldsymbol{b_n}$: the month loan *n* begins.

$\boldsymbol{D_n^t}$: is the penalty imposed when V_l^t is less than the minimum payment for loan *n* at month *t*

$\boldsymbol{e_n}$: the month loan *n* ends

$\boldsymbol{H_n^t}$: A binary variable is used to indicate the presence (1) or absence (0) of a loan for repayment.

$\boldsymbol{i_n^t}$: monthly interest rate for loan *n* at month *t*; allowing for a variable interest rate

$\boldsymbol{M_n^t}$: monthly instalment for loan *n* at month *t*.

n : individual loan

$\boldsymbol{o_n}$: minimum required payment for overdraft.

$\boldsymbol{p_n}$: the principal of loan *n*

$\boldsymbol{Q_n^t}$ **:** A binary variable takes the value of 1 if an administration fee is applied to early repayment of the principal, and it takes the value of 0 if the fee is not applied.

$\boldsymbol{r^t}$: reserve set aside at month *t* to pay all available loans.

S={1,….,x}

S: Set of loans x

S: a set of loans x to be considered.

T: total number of months until loan is paid, where $T \geq max_l(e_l)$, T is determined by the last month the last loan is paid.

$\boldsymbol{V_n^t}$: the main decision variable, which is the total amount of money, in future value, to be paid for loan *n* at month *t*

$\boldsymbol{Y_n^t}$: remaining balance from principle of loan *n* at month *t*

Z_n^t: the amount of money remaining after each installment is paid month *t*

Chapter 1

Introduction

The Egyptian construction industry has seen significant growth in recent years, resulting in a substantial increase in loan demand. In 2022, the market size reached $70.6 billion (Global data, 2023). However, construction companies face challenges when it comes to repaying these loans, especially when dealing with multiple loans with different interest rates and terms. This complexity hinders their ability to determine an optimal repayment strategy that minimizes their cash outflow.

The construction sector relies heavily on corporate debt and loans to finance projects, considering the significant capital investments involved. The type of financing chosen depends on project size, risk profile, and financial requirements. Loans require borrowers to repay the remaining balance within a specified timeframe, typically several years, while also accounting for an annual interest rate. The interest not only incentivizes borrowers to repay the loan promptly and reduce overall interest expenses but also compensates lenders for assuming loan risks. To date, there has been a lack of research on the application of repayment strategies for multiple loans in construction companies, both in Egypt and globally. Therefore, it is crucial to explore the impact of such strategies on construction companies' operations and financial performance.

1.1 Cash flow in Construction companies

Cash flow is crucial for the success and sustainability of construction projects, but construction companies often face significant challenges in this area. Their operations involve upfront investments and substantial outflows before generating revenue, particularly in long-term projects with extended construction periods. To address these cash flow needs, proper financial planning and structuring are essential. Financing provides an effective solution by ensuring a constant stream of funds directly to the construction company, reducing reliance on fluctuating project revenue. This approach enables efficient allocation of financial resources and alleviates strain on the company's cash flow. Additionally, financing helps mitigate risks associated with construction projects, such as cost overruns, delays, and unexpected events. By securing project-specific financing, construction companies can align cash flow requirements with project revenues, ensuring smoother and more sustainable financial performance (Gatti, 2018).

1.2 Failure of Construction Companies due to debt

A study conducted on construction companies in Nigeria revealed that the debt-to-equity ratio significantly impacts their performance. Higher levels of debt relative to equity can lead to reduced profitability and financial stability. It is crucial for construction firms to maintain an optimal capital structure, where a moderate level of debt is associated with better performance. Deviating from this optimal level, whether by having excessive or insufficient debt, can negatively affect the firm's performance. Managing the cost of capital is essential in the construction industry, with careful consideration of the trade-off between the cost of debt and equity when determining the capital structure. The study also found that firm size plays a role, as larger construction firms with better leverage and access to financing options tend to achieve superior performance compared to smaller firms. Financial flexibility is crucial, enabling firms to adapt to market conditions, manage risks, and seize growth opportunities (Jayiddin et al., 2017).

Another study conducted in the USA emphasized the potential failure of construction companies due to debt-related problems. Warning signs such as excessive borrowing, high leverage ratios, and an inability to meet debt service obligations contribute to financial risk and vulnerability. Accumulating excessive debt and becoming overly leveraged can hinder a construction firm's ability to secure additional financing, fund projects, and meet operational expenses, leading to a downward spiral and potential business failure. Proactive debt management, sustainable repayment plans, and open communication with lenders are crucial in mitigating these risks and improving long-term viability (Strischek, 1998).

In the context of the construction market in Saudi Arabia, a study highlighted that excessive debt levels can strain contractors' financial resources, impacting their ability to meet financial obligations and complete projects successfully. Difficulties in paying subcontractors, procuring materials, and covering operational costs, coupled with potential delays or disruptions, can contribute to the failure of contractors in industrial projects (Assaf et al., 2015).

1.3 Financing in Egyptian Construction Companies

Exploring financing options available to construction firms, a research study emphasized the significance of effectively managing debt capacity and financial risk. While debt capital may be a cheaper form of financing, excessive debt increases the financial risk faced by companies. The ability to meet debt obligations and allocate debt capital effectively is crucial for financial stability. The capital structure analysis revealed that debt financing constitutes a significant portion, averaging around 67.8% to 68.4%, while equity finance represents the remaining percentage. The proportion of short-term debt outweighing long-term debt raises concerns regarding potential financial risks. Furthermore, financial ratios indicate potential challenges in debt repayment and capital sufficiency. Profitability measures suggest that the high debt burden may not be effectively utilized. A significant portion of the selected companies, ranging from 70% to 81.25% over the analyzed four-year period, faces a high likelihood of financial distress and debt repayment failure. These findings highlight the importance of effectively managing debt capacity and financial risk within the construction industry in Egypt (Ashraf, 2007).

1.4 Research Goal and Objectives

This section aims to delve into the research's goal and objectives, providing a clear understanding of the intended outcomes and the direction of the study.

1.4.1 Research Goal

The goal of this research is to propose and implement an efficient and practical model for optimizing repayment strategies for Egyptian construction companies with multiple loans. The model aims to establish an optimal repayment schedule by minimizing the total cash required for loan repayment, prioritizing loan repayment over personal expenses. By adopting this model, construction companies can make informed decisions regarding their financial circumstances and repayment strategies, leading to significant cost savings and enhanced financial stability. The research emphasizes the importance of regular monitoring and necessary adjustments to optimize repayment strategies, revolutionizing the approach to debt repayment in the construction industry.

This research aims to propose and apply an algorithm to determine the optimal loan repayment plan for construction companies in Egypt, considering all industry-standard loan types and incorporating specific industry policies and regulations. By addressing this prevalent complex problem in the Egyptian construction industry, the research fills a significant gap in the academic literature and provides a practical solution to optimize debt repayment strategies. The proposed algorithm has the potential to greatly contribute to the financial and treasury management field by helping construction companies effectively manage their obligations, reduce the total cash required for loan repayment, and improve their overall financial health.

1.4.2 Research Objectives

The objectives of the research are as follows:

1. Conduct a comprehensive literature review of the existing methods and algorithms to optimize repayment strategies for multiple loans across multiple industries.
2. Analyze the various forms of loans utilized in the Egyptian construction industry, such as short-term loans STL, medium-term loans MTL, and long-term loans LTL, from the bank and lease companies to identify the relevant factors influencing the repayment strategy.
3. Develop and Implement an algorithm to determine the optimal repayment plan for a construction company with multiple loans.
4. Evaluate the efficacy and efficiency of the proposed algorithm utilizing real-world data from Egyptian construction firms.
5. Determine the superiority of the proposed algorithm in comparison to standard repayment strategies in the industry using the amortization technique.
6. Provide Egyptian construction companies with recommendations and insights on optimizing their repayment strategies to reduce their debt burden and improve their financial performance.

The research addresses the complex and challenging problem of determining the optimal repayment schedule for multiple loans in the Egyptian construction industry. This study can significantly contribute to financial and treasury management by devising an algorithm and evaluating its effectiveness.

1.5 Scope

This research investigates the optimum debt repayment strategies for construction firms. The purpose of this study is to evaluate the efficacy of an optimization model in predicting and managing industry-wide debt repayment patterns. It entails the creation and validation of an optimization model employing actual data from Egyptian construction companies. The research will concentrate on identifying obstacles to debt repayment and examining methods to reduce debt loads while maintaining financial stability. In addition, the study will investigate the capability of the model to predict future repayment patterns and proactively address potential issues such as cash flow shortages and late payment fees. The research findings will enhance the comprehension of debt management in the construction industry and provide construction companies with valuable insights for optimizing their debt repayment strategies and fostering long-term success and growth.

1.6 Methodology

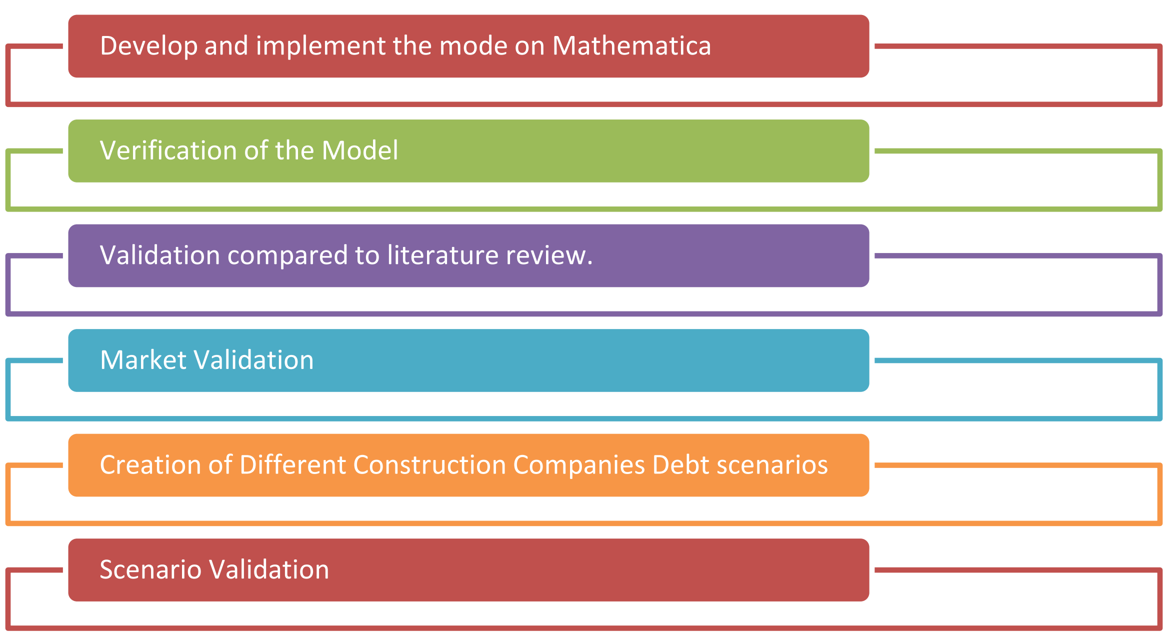
The research methodology involved multiple steps to verify and validate the proposed model for optimizing debt repayment strategies in Egyptian construction companies with multiple loans, Figure 1 illustrates the research methodology. Firstly, the model was developed using Mathematica and the Gurobi optimizer to minimize the total loan repayment amount. Verification of the model's performance was conducted through an analysis of extreme scenarios.

To validate the model, a comparison was made with a set of test cases obtained from a literature review on personal loans. These test cases evaluated the model's performance across various scenarios, ensuring accurate results. Market validation was achieved through interviews with financial and treasury administrators from Egyptian construction companies, gathering real-world information on debt management practices, repayment strategies, and industry challenges. The feedback received confirmed the need for an optimization tool and expressed interest in utilizing the proposed model.

In addition, realistic debt scenarios in Egyptian construction firms were generated based on the interview data. These scenarios served to evaluate the model's effectiveness in solving real-world problems, producing results for different situations. The obtained results were analyzed to assess the precision and effectiveness of the model. A sensitivity analysis was conducted to examine the model's performance under various constraints and identify influential factors. Based on these findings, the model was refined, and recommendations for future research were formulated.

Overall, the proposed methodology successfully verified and validated the model for optimizing debt repayment strategies in Egyptian construction companies with multiple loans. The verification process ensured accurate implementation, while the validation process demonstrated the model's capability to address real-world challenges. The insights gained from the sensitivity analysis provide valuable contributions to future research in this field.

Figure 1. Research Methodology Overview



1.7 Thesis Organization

The thesis is organized into several chapters to provide a clear structure and systematic presentation of the research.

i. Introduction
- An overview of the research topic
- The objectives of the study
- The significance of the study

ii. Literature Review
- A review of the relevant literature on loan repayment optimization models
- A discussion of the challenges faced by construction companies in managing their loans and debt repayments.
- An overview of the existing models and approaches used to optimize loan repayments.

iii. Model Description
- A description of the loan repayment optimization model proposed in the study.
- An explanation of the model components, including the financial metrics and calculations used to optimize loan repayments.
- A discussion of the potential benefits and limitations of using the proposed model

iv. Model Verification

Evaluate the model's performance using extreme scenarios.

v. Literature review Validation
- A demonstration of the model's effectiveness in optimizing loan repayments through a comparison with results in the Literature Review.
- An evaluation of the model's performance in optimizing loan repayments for the selected construction company

vi. Interviews and Market Validation
- An analysis of the interviews conducted with financial executives in construction companies to validate the model's effectiveness and feasibility.
- Discuss the feedback received and its implications for the model's implementation and use in the construction industry.

vii. Model Validation

- Validation of the proposed model through a comparison with the existing loan repayment method
- An explanation of the validation methodology and criteria used to evaluate the model's performance.
- A discussion of the results and implications for the model's implementation and use in the construction industry.

viii. Conclusion and Recommendation

- A summary of the study's main findings and contributions
- A discussion of the implications of the study for the construction industry
- A recommendation for future research in the area of loan repayment optimization models
- A conclusion highlighting the significance of the study and its potential impact on the construction industry.

Chapter 2

Literature review

Loan repayment plays a vital role in the financial management of construction companies within the industry. These companies strive to effectively allocate their resources by carefully considering their monthly cash flows and loan portfolio. However, the variability of monthly receivables among organizations is a common occurrence, often leading to unpredictable changes from one month to the next. This fluctuation can create challenges, as insufficient funds may be available to cover all loan installments, thereby jeopardizing the financial stability of the debtor.

2.1 Corporate Debt

Corporate debt is a prevalent aspect of the construction sector, known for its capital-intensive nature and substantial financial requirements. In order to undertake and complete projects, construction companies often rely on loans and other forms of corporate debt as a means of financing their operations.

2.1.2 Types of Corporate Debt and Loans:

Commercial loans are available through various financial institutions such as commercial banks, credit unions, and others. These loans are commonly utilized in the construction industry to finance the acquisition of building supplies, equipment, and labor costs. They are typically secured by assets or collateral and feature variable interest rates and repayment terms based on the creditworthiness of the lender and borrower (US Small Business Administration, 2022).

Construction loans, on the other hand, provide funding for new construction projects or the renovation of existing. These loans are short-term and can be renewed for up to a year. They are usually secured by the receivables of the property being built or restored, and the loan is disbursed in stages as the development progresses. Mezzanine financing is a hybrid financing option that combines both debt and equity. It is often utilized when borrowers require additional funding on top of senior debt financing. This type of financing involves taking on additional risk but offers higher returns to the lender (Hunt, 2022).

Leasing, which is a newly introduced financial instrument in Egypt, is commonly employed in the construction industry to acquire equipment and machinery. In leasing, a lessor (the equipment owner) and a lessee (the equipment user) are involved (Gatti, 2018). The lessor owns the equipment, and the lessee makes recurring lease payments to utilize it for a specific duration. At the end of the lease, the lessee can choose to purchase the equipment, return it, or negotiate a new lease. Leasing offers several advantages for construction companies. It allows them to access equipment without a substantial upfront expenditure, thus preserving cash flow for other expenses. The terms of the lease, including the lease period, payment schedule, and buyout options, can be customized to meet the specific needs of the construction firm. Leasing also enables companies to leverage cutting-edge equipment and technology, enhancing project efficiency and competitiveness. Additionally, leasing equipment may provide tax benefits, as lease payments can be deducted as operational expenditures (Gatti, 2018).

2.1.3 Special terms used in corporate loans.

Working capital loans are an essential source of funding for businesses, providing financial support for day-to-day expenses such as payroll, rent, inventory, and operational costs. These loans come in various forms, including overdraft facilities, lines of credit, or cash credits, tailored to meet the specific needs of companies. Businesses can choose between demand loans, suitable for short-term financial needs, or term loans, which offer longer-term funding alternatives. Access to working capital is crucial for sustaining daily operations and enabling businesses to seize growth prospects. Insufficient working capital can lead to difficulties in meeting financial obligations, maintaining inventory levels, and paying bills, potentially resulting in insolvency (Kagan, 2020).

A line of credit is a form of borrowing extended by banks to customers with good credit. With a line of credit, borrowers have access to a predetermined maximum loan amount that can be utilized as needed. One notable advantage of a line of credit is that interest is charged only on the amount withdrawn, not the entire maximum limit, making it an attractive financing option for businesses requiring regular access to funds without paying interest on unused portions. Lines of credit are commonly used to manage cash flow, address short-term needs, and capitalize on unexpected opportunities, providing businesses with a safety net of accessible funds (Hayes, 2022).

Asset-backed loans are a financing solution that allows companies to use existing assets as collateral to obtain a loan. These loans are commonly utilized to fund initiatives related to short-term working capital, capacity expansion, modernization, or the implementation of advanced technical procedures. Tangible and intangible assets such as real estate, merchandise, equipment, and accounts receivable can be used as collateral for asset-backed loans. By pledging these assets, businesses can secure financing at a reduced interest rate, as the lender has the security of the collateral. One of the primary advantages of asset-backed loans is that they allow firms to leverage the value of their existing assets without having to sell them. This is particularly beneficial for businesses that require funding to support their growth plans while maintaining ownership and control without involving outside investors. Asset-backed loans are prevalent in industries such as manufacturing, construction, and transportation, where enterprises possess substantial assets that can serve as collateral. They can be utilized for various purposes, including financing the acquisition of new equipment, facility refurbishment, and increasing production capacity (Hayes, 2021; Corporate Finance Institute,2023).

Finance against future receivables is a valuable financing solution for construction enterprises. This type of loan is secured by the borrower's assured future receivables, primarily consisting of customer payments for services or items supplied. Construction companies can benefit from this form of financing to manage their cash flow or obtain funding for working capital. By using their future receivables as collateral based on current contract projects, construction enterprises can access finance at a cheaper interest rate compared to unsecured loans. However, it is essential for a construction company to have a track record of steady revenue and a robust client base with predictable payment histories to qualify for funding against future receivables. Once accepted, the borrower can receive a loan based on a proportion of their expected future receivables (Supply Chain Finance Forum, 2023; Universal Funding, 2023).

Short-term loans are a sort of finance offering firms quick cash to meet their immediate financial needs. These loans are usually structured with shorter repayment terms and lower loan amounts than regular corporate loans. Short-term loans might benefit firms that must bridge a temporary cash-flow shortfall or cover unexpected expenses. These loans can assist businesses in maintaining operations while they wait for a more substantial and consistent source of cash to become available. Short-term loans can be an effective financing alternative for organizations with immediate financial needs. While they have higher interest rates, they can offer businesses the cash flow they need to keep operations running and bridge a short financial gap until a more substantial and dependable source of financing becomes available (Corporate Finance Institute,2023; Bankrate,2023; Wall Street Mojo,2023).

2.2 Debt Repayment Plan

A debt repayment plan is an effective financial strategy for businesses to handle their outstanding debts in a controlled and organized manner. It entails making a plan to pay off debts over a set period of time, usually with defined payments and deadlines. The principal purpose of a debt repayment plan is to assist businesses in regaining control of their finances, minimizing interest payments, and eventually becoming debt-free. The repayment plan can be tailored to the company's specific financial constraints, considering elements such as income, expenses, and debt balances. Implementing a debt payback plan can help businesses successfully manage their debts, increase financial stability, and accomplish long-term financial goals (Ramsey, 2021). There are mainly four different strategies used to repay debt:

2.2.1 Snowball Method

The debt snowball technique is a debt reduction strategy that involves repaying debts in ascending order of size, regardless of interest rates (Ramsey, 2021). The goal is to build momentum and a sense of accomplishment by paying off smaller debts first. Once the smallest debt is paid off, the debtor moves on to the next smallest debt, and so on, until all debts are paid (Ramsey, 2021). The money used to pay off each debt is then rolled into the next debt, creating a snowball effect. The debt snowball strategy offers psychological benefits that go beyond financial gains. Seeing progress helps keep people motivated, and paying off a small debt quickly can provide the encouragement needed to tackle larger debts.

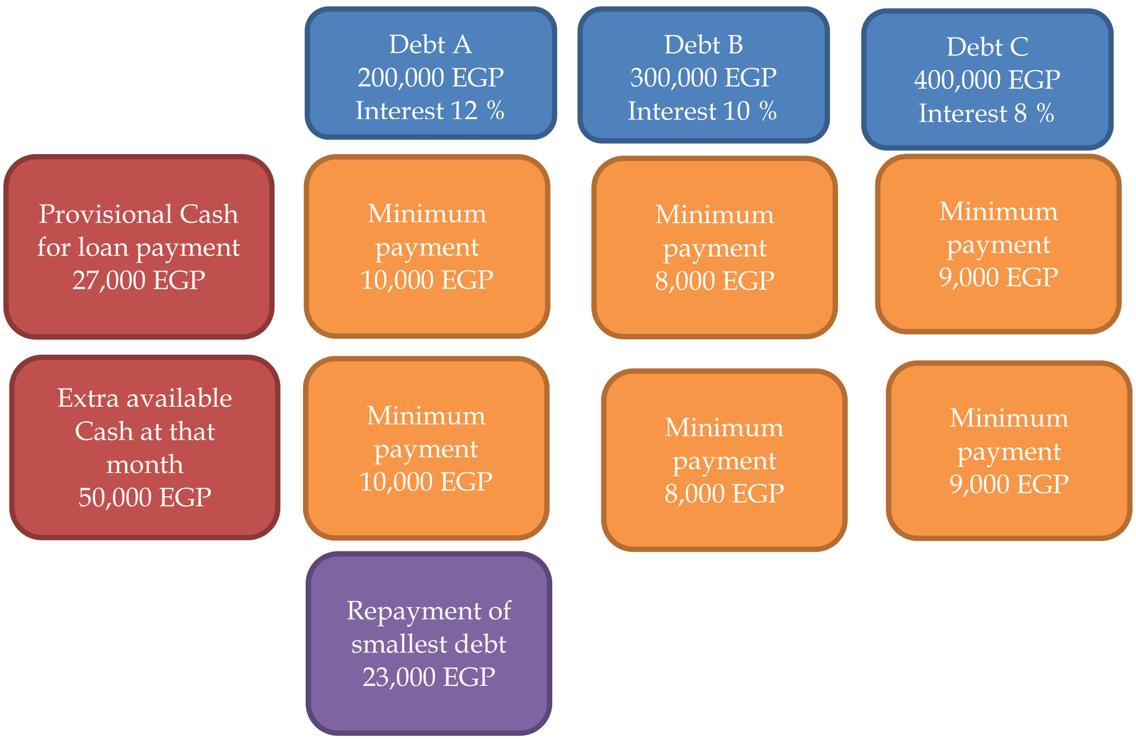
While some financial analysts argue that the debt snowball method may not be the most cost-effective option, many debtors and experts continue to support it. Ramsey believes that debt repayment is 20% head knowledge and 80% behavior (Ramsey, 2021). In other words, while the debt snowball method may not be the most mathematically ideal strategy, it can be helpful for those who require motivation to stay on track with debt repayment.

The debt snowball method can be particularly beneficial for individuals or businesses with multiple obligations, such as a construction company with long-term loans, short-term loans, and equipment installments. By listing the debts in descending order according to their balance or the amount still owed, the debtor can focus on paying off the lowest debts first and then move on to higher obligations. It simplifies the debt repayment process and provides a sense of progress and achievement.

Despite potential disadvantages, such as not prioritizing higher-interest debts, many people utilize the debt snowballing heuristic debt management strategy. This involves paying off the debt with the smallest balance first, followed by the next smallest balance, regardless of the interest rate. While it may not be the most cost-effective approach, it fosters a sense of success and motivates individuals to continue reducing their debt . The debt snowball method has been shown to effectively reduce debt by starting with the smallest debt and gradually moving on to larger ones (Ramsey, 2021).

Figure 2 illustrates an example of how the snowball method operates. In this scenario, there are three debts: Debt A, Debt B, and Debt C. Debt A has the smallest principal amount, and therefore, when there is extra cash available, it is the first debt to be paid off. Once Debt A is fully paid, the repayment focus shifts to Debt B, and finally, Debt C.

Figure 2.   Illustration of the Debt Snowball Method



<u>Psychological Aspects</u>

The psychological aspects of this strategy in further detail, as well as why some payment systems are favored over others. According to the experts, the debt snowball strategy may be successful because it delivers a sense of accomplishment and progress toward a greater goal {Brown et al, 2014).

<u>Advantage</u>

Paying off smaller debts first can provide a psychological boost and generate momentum that can be applied to tackling larger obligations. This approach, known as the debt snowball method, is often preferred over other payment strategies due to its simplicity and ease of implementation (Ramsey, 2021). By focusing on eliminating smaller bills initially, individuals can experience a sense of accomplishment and motivation, which can then be channeled towards addressing larger debts.

Disadvantages

While the debt snowball strategy may bring motivational benefits, it may not always be the best financial decision. Individuals should carefully assess their financial status and ambitions before deciding on a debt repayment option. The researchers also point out that many experimental setups used to test the efficacy of various debt repayment approaches fail to account for the motivating benefits that may be acquired in real-life scenarios (Brown et al, 2014).

2.2.2 Debt avalanche

The debt avalanche method is an effective approach for businesses dealing with multiple interest-bearing debts, including long-term loans, short-term loans, and equipment installments. This method involves prioritizing the repayment of debts with the highest interest rates first, and then proceeding to debts with lower interest rates by arranging them in a non-increasing order based on their interest rate. By adopting this strategy, companies can minimize the total amount of interest paid overtime and expedite the repayment process. The debt avalanche method may be particularly beneficial for debtors seeking to save money on interest payments. Orman emphasizes that prioritizing high-interest debt can result in significant long-term savings. Financial analysts also endorse the debt avalanche strategy, as it provides the most optimal interest plan for managing multiple loans (Omran, 2018).

To implement the debt avalanche strategy, debtors need to create a comprehensive list of their loans and prioritize paying off the debt with the highest interest rate first, while making minimum payments on all other obligations. Once the highest-interest debt is fully paid, they can proceed to the next debt with the highest interest rate and continue the process until all debts are successfully settled. Unlike the debt snowball method, the debt avalanche approach may not offer the same psychological benefits, such as a sense of accomplishment and momentum gained from paying off smaller debts first. However, it can be an effective strategy for corporate debt reduction. The debt avalanche method provides the most optimal interest plan for managing multiple loans and can help debtors save money in the long run. It is recommended as a strategy that prioritizes paying off high-interest debt to achieve greater savings. By systematically addressing debts based on interest rates, the debt avalanche strategy enables companies to minimize interest payments and work towards becoming debt-free (Omran, 2018).

Advantages

The debt avalanche method offers a number of benefits to individuals and enterprises seeking effective debt management and reduction. Here are some of the most significant benefits, with references and citations:

1. The debt avalanche strategy prioritizes paying off debts with the highest interest rates first in order to save money. Individuals and businesses can save money on interest payments over time by focusing on high-interest obligations. Focusing on high-interest debt can result in substantial savings, (Orman, 2018).

2. Efficient Debt Reduction: The debt avalanche method enables debtors to resolve their debts systematically based on interest rates. By focusing on debts with the highest rates of interest, individuals and businesses can expedite the repayment process and reduce the total time required to become debt-free.

3. Optimal Interest Plan Financial analysts advocate the debt avalanche strategy because it offers the most effective interest plan for managing multiple loans. By addressing debts with the highest rates of interest first, debtors can reduce the total amount of interest paid over the course of repayment.

Long-Term Financial Benefits Prioritizing high-interest debts using the debt avalanche method can result in long-term financial advantages. Individuals and businesses can improve their financial health and establish a solid foundation for future financial objectives by minimizing interest payments and reducing debt balances (Orman, 2018).

Disadvantages

Despite the fact that the debt avalanche method has advantages, it is essential to consider its potential drawbacks. The following are some disadvantages of the debt deluge strategy, supported by in-text citations:

1. One disadvantage of the debt avalanche method is that it may not provide the same immediate sense of accomplishment and motivation as the debt snowball method. Smaller debts with lesser interest rates may not be prioritized initially under the debt avalanche method, resulting in slower progress on these smaller obligations (Ramsey, 2021).

2. Impact on Psychology: The debt avalanche strategy may not offer the same psychological advantages as the debt snowball method. Individuals and enterprises may not experience the same momentum and motivation that comes from paying off smaller debts first if the focus is on interest rates rather than debt balances (Amar et al., 2011).

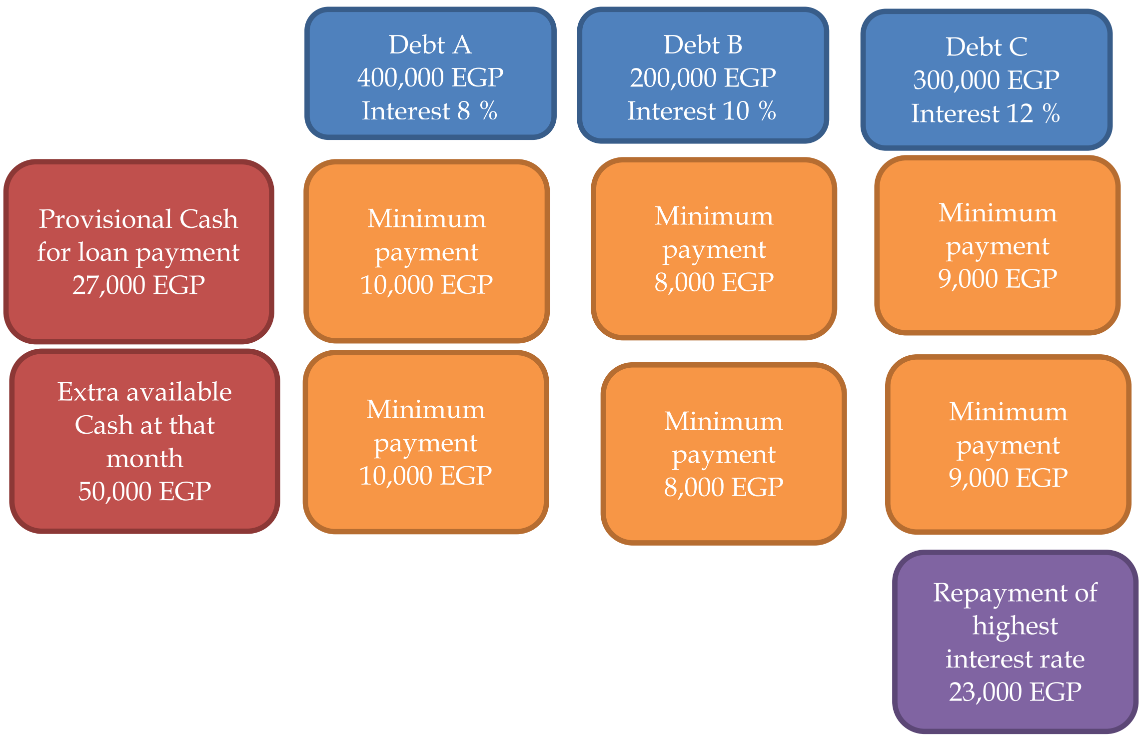
3. Some individuals and businesses may experience a lack of motivation as a result of the debt avalanche method's requirement to pay off debts with higher interest rates over a prolonged period of time. On lesser debts, the absence of immediate rewards or visible progress can make it difficult to maintain motivation throughout the repayment process (Brown & Lahey, 2014).

4. Implementing the debt deluge strategy may necessitate additional effort and organization. Individuals and businesses must manage multiple debt accounts, prioritize debts based on interest rates, and monitor progress over time, which can be time-consuming and complex (Gupta et al., 1987).

When deciding which debt repayment strategy, such as the debt avalanche method, is most suitable for their particular needs and circumstances, it is essential for individuals and businesses to thoroughly evaluate their financial situation and consider these drawbacks.

Figure 3 illustrates an example of loan repayment using the avalanche method. The scenario involves three debts: debt A, debt B, and debt C. Each month, after all the regular installments are paid, the debt with the highest interest rate, which in this case is debt C, is given priority for additional funds. Once debt C is fully paid off, the focus shifts to debt B, followed by debt A, until all the debts have been completely repaid.

Figure 3. *Illustration of the debt avalanche method*



2.2.3 Debt Consolidation

Debt consolidation is a frequent corporate technique for managing numerous existing debts by taking out a new loan with a lower interest rate. By merging several debts into a single monthly payment, firms can save money on interest payments in the long run and make debt repayment more manageable (Kagan, 2023). According to financial experts, debt consolidation can be an efficient option for businesses with several high-interest obligations. According to the experts, "debt consolidation can help simplify monthly payments, potentially save money on interest, and help pay off debts faster." Debt consolidation is a realistic alternative for businesses with high-interest loans or credit card debt, making managing multiple monthly debt payments challenging. Corporations can cut their monthly payments and save money on interest payments by consolidating their debt, freeing up resources to invest in other areas of their organization.

Advantages:

1. Debt consolidation enables individuals to consolidate multiple obligations into a single loan or payment. This simplifies the repayment process by eliminating the need to manage numerous creditors and due dates, thereby making it easier to track payments and avoid late fees (Kagan, 2023).

2. Lower Interest Rates: Obtaining a new loan with a lower interest rate than the individual's existing debts is typically required for debt consolidation. This can result in reduced overall interest costs and save money in the long run (Kagan, 2023).

3. Individuals who consolidate their debts may be able to negotiate lengthier repayment terms, which could result in lower monthly payments. This can provide immediate cash flow alleviation and make it simpler to manage financial obligations (Kagan, 2023).

4. Credit Score Improvement Debt consolidation can have a positive influence on credit scores. By paying off existing debts and making consistent payments on the new consolidated loan, individuals demonstrate responsible financial behavior, thereby enhancing their creditworthiness over time (Kagan, 2023).

Disadvantages:

1. While debt consolidation can result in reduced interest rates, it is essential to consider the total cost of the new loan. Even with a lower interest rate, extending the repayment term may result in higher long-term interest payments (Kagan, 2023).

2. Depending on the type of debt consolidation loan, collateral may be required to secure the loan, or a high credit score may be required to qualify for a loan with favorable terms. Individuals with poor credit or few assets may have difficulty gaining access to debt consolidation options (Kagan, 2023).

3. Risk of Re-Debt: Debt consolidation does not address the underlying causes of financial difficulties. Without addressing spending habits and implementing proper budgeting and financial management strategies, there is a risk of incurring new debts and plunging back into financial difficulties (Kagan, 2023).

4. Debt consolidation may incur a variety of fees, such as loan origination fees or balance transfer fees. It is essential to thoroughly evaluate the costs associated with the consolidation procedure to ensure that the potential savings outweigh these costs (Kagan, 2023).

2.2.4 Balance Transfer

Balance transfers might be a feasible alternative for borrowers trying to save money on high-interest credit debt when managing credit card debt. Transferring existing debt to a new credit card with a lower interest rate is what a balance transfer entails. In the short run, this can save corporations money on interest payments, but it may come with balance transfer fees and other expenses (Barone, 2022).

<u>Advantages:</u>

1. The ability to transfer high-interest debt to a new credit card or loan with a reduced interest rate is one of the primary benefits of balance transfer. This can help individuals save money on interest fees and potentially pay off their debts more quickly (Federal Reserve Bank of New York, 2017).

2. Consolidation of Debt: Balance transfer enables individuals to consolidate multiple debts into a single account, making it simpler to track and manage their payments. This simplifies the repayment procedure and can assist borrowers in staying organized (Federal Reserve Bank of New York, 2017).

3. Many balance transfer credit cards provide attractive introductory offers, such as 0% interest for a limited time. This enables individuals to make interest-free payments on their transferred balance, providing temporary respite and the chance to pay down debt

more efficiently (Federal Reserve Bank of New York, 2017).

Disadvantages:

1. Balance Transfer Fees There may be balance transfer fees associated with transferring a balance to a new credit card. These fees are typically a percentage of the transferred balance and can nullify any savings from a reduced interest rate (Federal Reserve Bank of New York, 2017).

2. Limited Promotional Period: The attractive introductory offers of balance transfer cards are transient and typically last for a limited time, such as six or twelve months. After the promotional period concludes, the interest rate may increase substantially, potentially nullifying the balance transfer's benefits (Federal Reserve Bank of New York, 2017).

3. The application for a new credit card and the transfer of a balance can have an effect on a person's credit score. Credit utilization and length of credit history are factors that influence credit scores (Federal Reserve Bank of New York, 2017). Opening new accounts and closing old accounts can have an effect on these factors.

4. Balance transfers can provide temporary relief from high-interest debt, but they can also create the temptation to incur additional debt on the newly opened credit card. Individuals may find themselves in a worse financial situation if they continue to rely on credit without addressing underlying spending patterns (Federal Reserve Bank of New York, 2017).

2.3 Debt snowball vs. debt avalanche

Interest rates and fees boost the debt snowball method's chances of success. In most circumstances, the debt avalanche method outperforms the debt snowball method quantitatively, indicating that paying off loans with the highest interest rates first can result in lower overall interest payments and faster debt repayment. The primary benefit of the debt snowball strategy is simply psychological, as it can create a sense of accomplishment and incentive to continue debt payments (Gal et al, 2012).

The two most common debt-reduction strategies, the debt snowball and the debt avalanche. The debt avalanche involves paying off bills from greatest to lowest interest rate, whereas the debt snowball involves paying off debts from lowest to highest balance (McAllister, 2018). Numerous research comparing the potency of these two approaches was included in McAllister's literature assessment. He pointed out that advocates of the debt snowball frequently contend that the psychological boost gained from eliminating smaller obligations first might aid people in maintaining their motivation to pay off their debts. On the other hand, advocates of the debt avalanche argue that paying off loans with higher interest rates initially can help people save more money over time.

The debt snowball strategy was linked to increased rates of debt payback among participants, according to one study. The debt snowball method users had better credit scores and lower debt-to-income ratios than the debt avalanche method users. The study did not, however, investigate whether using the debt snowball strategy truly resulted in longer-term financial savings for people (McAllister, 2018). According to a different study, the debt avalanche approach was ultimately less expensive than the debt snowball approach. According to the results of this study, individuals who utilized the debt avalanche approach paid off their debts more quickly and with lower interest rates than those who employed the debt snowball method. However, this study did not examine how the two approaches affected people psychologically (McAllister, 2018).

In conclusion, using either the debt snowball or debt avalanche strategies to pay off debt can be successful, depending on the situation. When selecting a debt repayment strategy, he advised people to consider variables including their motivation level, debt levels and interest rates, and overall financial goals (McAllister, 2018).

2.4 Prioritizing Debt Repayment

A study explored how individuals prioritize and approach debt repayment. Their findings revealed that people prioritize paying off smaller debts first, even if they have higher interest rates than more significant debts. According to the researchers, the interest-ordered payment approach prioritizes paying off debts with the highest interest rates and is the most cost-effective. According to the researchers, this behavior could be attributable to a phenomenon known as "debt account aversion," in which debtors avoid dealing with their debt accounts because they find it stressful or anxiety-inducing. A previous study has demonstrated that borrowers tend to underestimate the pace at which debt accrues interest over time and may be unaware of the procedures involved in debt repayment (Amar et al, 2011).

2.5 RPML an NP-Hard Problem

The repayment policy for multiple loans (RPML) problem is a well-recognized financial problem. It entails handling several debts acquired by a debtor and identifying the most efficient repayment method. The problem is classified as NP-hard, making it computationally difficult to solve using existing algorithms as the problem grows (Gupta et al., 1987). The RPML problem focuses on reducing the total amount of money required to repay all obligations. It is not used to plan new loans or determine how to schedule them. Instead, it deals with a pre-existing set of debts and attempts to discover the most economical way to repay them.

NP-hard problems are computing problems that are at least as difficult as the most complex problems in the NP (nondeterministic polynomial time) class. In other words, these are the issues for which no polynomial-time algorithm exists. An NP-hard problem is at least as complex as the most complicated NP problems. It is thought that no polynomial-time algorithm exists to solve these problems. (Cook, 1971).

NP-hard issues have critical applications in many domains, including computer science, operations research, and engineering. Many major optimization tasks are known to be NP-hard, including the traveling salesman problem, the knapsack problem, and the graph coloring problem (Garey et al, 1979). As a result, developing effective algorithms to tackle these challenges is critical.

Several ways to deal with NP-hard problems have been presented, including approximation algorithms, heuristics, and exact algorithms. Approximation algorithms provide a substandard solution to an optimization problem while guaranteeing that it is within a specified factor of the optimal answer (Papadimitriou et al, 1998). These methods are commonly employed to solve NP-hard problems, particularly when finding an exact solution is impossible. Greedy algorithms, local search algorithms, and randomized algorithms are some of the most commonly used approximation algorithms (Papadimitriou et al, 1998). The success of these algorithms is determined by the nature of the problem and the algorithm employed.

Heuristics are algorithms that direct the search for a solution using domain-specific knowledge. They are intended to identify a suitable solution quickly but do not guarantee that the solution is optimal. In practice, heuristics are commonly utilized to solve many NP-hard problems, mainly when the problem is vast or complex (Papadimitriou et al, 1998).

Exact algorithms are type algorithms that solve NP-hard problems exactly; they discover the best answer. Algorithms like this are frequently based on mathematical concepts like linear programming, integer programming, and dynamic programming. Exact algorithms are computationally expensive and may be impractical for large or complex situations (Garey et al, 1979). This type of algorithm is similar to that proposed for the research problem.

NP-hard problems are a type of computer problem that has wide-ranging practical ramifications. It is critical to develop efficient methods to address these challenges. To address these issues, approaches such as approximation algorithms, heuristics, and exact algorithms have been presented. The success of these approaches is determined by the nature of the problem and the algorithm used (Garey et al, 1979).

2.6 Optimization Algorithms for Debt Repayment

In a study that expanded the use of scheduling theory to handle debt-related financial problems. They created a branch and bound algorithm to discover the best repayment plans for various loans, considering each loan's due date and the possibility of discounts or fees for early or late payments. The algorithm aims to minimize the current value of all future financial outflows, emphasizing the necessity of considering psychological and financial aspects when devising debt repayment plans (Gupta et al, 1987).

While the debt snowball method may have psychological benefits, individuals should carefully consider the potential impact on overall interest payments and consider alternative repayment methods, such as the debt avalanche method or optimization algorithms, based on their financial situation and goals. A balanced approach that considers both financial and psychological aspects is essential when devising debt repayment plans (Amar et al., 2011; Brown & Lahey, 2014).

Prioritizing loans is critical because the debtor must pick which loans to make installments on and in what order. When monthly receivables are high, the debtor must decide how to divide the excess monies. This is known as the Repayment Policy of Multiple Loans (RPML) dilemmas in academic circles. Previous research, titled "Optimal repayment strategies for numerous loans," provided a strategy to handle this issue (Gupta et al., 1987). The RPML was designed as a single-machine scheduling problem to minimize the maximum debt completion time. Debt principles were supposed to rise as a function of their starting time, and debts were non-preemptive, meaning borrowers were obligated to continue making payments until the loan was completely paid off (Gupta et al., 1987).

Another study investigated a simplified version of the Repayment Policy of Multiple Loans (RPML) problems to develop a solution further. The work, dubbed "Preemptive Repayment Policy," reformulated the RPML problem as a scheduling problem with preemptive processing times. This allowed the debtor to suspend and resume loan repayment as needed. The author used a nonlinear optimization model to approximate the scheduling problem. Although this study was a simplified version of the RPML problem, it provided valuable insights into prospective remedies. Both previous models, including the one above, are of polynomial complexity. This means that the algorithm needed to solve the problem is already available, allowing for the development of more efficient solutions (Ng et al, 2012).

In a recent study, researchers developed the most successful approach for solving the RPML problem. The mixed-integer linear programming (MILP) method was used in this work to develop an optimal payback scheduling model that minimizes the overall amount of cash necessary to repay the loans. This method could represent a significant improvement in the field of construction industry finance, and it has the potential to change how construction businesses handle loan repayment (Rios-Solis et al, 2017).

2.7 Scheduling Concepts in Financial Management

The scheduling concepts that have been developed in research have primarily focused on the manufacturing industry and its applications. However, other functional areas of an organization, such as financial management, have yet to benefit from such advances. This section discusses the challenges of scheduling cash in the financial sector, which differs significantly from scheduling machines in the manufacturing industry.

<u>Cash Scheduling Challenges</u>

Unlike machines, cash can be carried forward and potentially invested, adding another layer to the scheduling problem. Additionally, monetary resources are used in chunks rather than continuously, further complicating the scheduling process. This section delves into the specific challenges of scheduling cash and the importance of considering cash's unique qualities as a resource when developing optimal payment rules for credit purchases or loans (Gupta et al, 1987).

Resource Utilization in Financial Management

This section highlights the significant differences in resource utilization patterns between the financial and manufacturing industries. Unlike machines, monetary resources are only used when a loan is repaid, leading to a need for customized scheduling solutions for various industries and functional areas of an organization. It emphasizes the importance of developing customized scheduling solutions that consider each industry's unique characteristics and functional area (Gupta et al, 1987).

Their research primarily focused on identifying the most effective repayment plan for various loans. This work required considering various elements, including loan amounts, payment constraints, and any cash borrowed or purchases made on credit and included in the loan. To accomplish their goal, the researchers devised a strategy that would base the efficacy of the payback schedule on the present value of all incoming financial outflows. This present value was then decreased by an ideal timetable that accounted for the loan constraints (Gupta et al, 1987).

The researchers employed scheduling theory concepts to discover the optimum practices for repaying several loans. While the finance literature has historically considered cash management a short-term financing issue, the researchers recognized that borrowing money to pay off debts could be a realistic option. The researchers developed an algorithm that made specific assumptions to determine optimal payback options. The first was that daily cash generation was constant. This could be the case if the borrower set aside a percentage of their daily cash receipts to repay the loans, including sales revenues and other cash receipts. However, the researchers recognized that the amount of money created daily in real life could vary or be represented by a probabilistic distribution. Despite this uncertainty, their algorithm was intended to determine the most effective payback plans for many loans while considering various variables and assumptions (Ng et al,2012).

In addition to the unpredictability of daily cash generation, it is possible that other variables, such as the interest rate, may alter over time and not be known with absolute certainty. This can make it challenging to develop an optimal long-term repayment strategy. To address this difficulty, researchers have investigated various multiple-loan repayment strategies. For example, preemptive and time-dependent processing times were employed in a research study by (Ng et al,2012) to express the problem as a single-machine scheduling problem. The objective was to reduce the make span, which is the time required to complete all duties.

The researchers transformed the scheduling issue into a continuous nonlinear optimization problem that could be solved by solving a succession of associated linear programming problems. Using this method, they were able to arrive at a preliminary solution that had practical applications. Nonetheless, the problem's computational complexity remained challenging, and the researchers investigated permissibly constraints for its solution. Despite these obstacles, the research provided valuable insights into the problem of multiple-loan repayment and proposed novel solutions (Gupta et al, 1987).

Preemptive scheduling is utilized in various real-world contexts, such as financial management, industrial settings, and commercial sectors. In financial management, preemptive scheduling methods can assist in optimizing the repayment strategy for multiple loans, enabling more efficient cash flow management. In their study, the authors determined the optimal proactive repayment strategy for a collection of loans. The loan principals were exogenous and deterministic, meaning they were fixed quantities that did not fluctuate over time. For each loan, the lending terms were also specified, including the loan release and due dates and any additional fees, such as interest and late payment penalties (Ng et al,2012).

The debtor business was required to collect cash from its daily income during a collection period, analogous to the task processing time in a manufacturing environment to repay the loans. Payments in advance were permitted without incurring additional fees. The ultimate objective of the study was to reduce the total quantity of money required to repay the loans or, more precisely, the entire collecting period. Using preemptive scheduling techniques, the researchers optimized the loan repayment strategy and reduced the total loan repayment cost. This study has significant implications for financial management in a variety of industries. By utilizing preemptive scheduling methods to optimize the repayment strategy for multiple loans, businesses can improve their cash flow management and reduce their total cost of debt repayment (Gupta et al, 1987).

The researchers formulated the scheduling problem in their study as a continuous quadratic program and used an algebraic technique to solve it. In addition, they investigated the computational complexity of the problem. They identified acceptable constraints for addressing issues involving savings accounts, variable interest rates, fluctuating currency flows, and situations where idle time in repayment is allowed. This approach allowed them to comprehensively comprehend the problem and devise more effective solutions for determining the optimal repayment schedule for multiple loans (Ng et al,2012).

2.8 Repayment policy for multiple loans

In a recent study that sought to determine the optimal model for determining the optimal repayment schedule for multiple loans, researchers employed mixed-integer linear programming (MILP). They created an optimal repayment scheduling model to minimize the total cash needed to repay the loans. Using this methodology, the researchers devised a highly efficient and effective solution to the problem. Despite the effectiveness of these approaches, determining the optimal repayment strategy for multiple loans remains a complex problem. In fact, according to the repayment policy for multiple loans study, the RPML problem complexity is classified as NP-hard, meaning that the time required to solve such a problem using known algorithms would increase exponentially with the problem's size. This emphasizes the significance of continued research in this area and the need for novel and creative approaches to addressing this intricate problem (Rios-Solis et al., 2017).

The RPML problem is predominantly concerned with determining the optimal repayment strategy for a given set of existing debts to minimize the total amount of cash required for repayment. However, it is essential to note that the RPML problem is not intended to plan the loans a debtor can obtain or how those loans should be scheduled (Rios-Solis et al., 2017).

2.9 Summary of the literature review

The literature review offers a thorough summary of the research conducted on loan repayment optimization and scheduling strategies in financial management. It begins by emphasizing the significance of effective debt repayment strategies and the potential repercussions of ineffective approaches. The review emphasizes the significance of incorporating both financial and psychological factors into repayment programs.

The debt snowball and debt avalanche methods are discussed as popular approaches, with the debt snowball method emphasizing psychological benefits and the debt avalanche method emphasizing minimizing interest payments. Literature suggests, however, that alternative methods, such as optimization algorithms, can provide more efficient solutions based on an individual's financial situation and objectives.

The review examines the concepts of scheduling utilized in financial management, with a focus on the challenges unique to currency scheduling, in which monetary resources are utilized in discrete chunks rather than continuously. Utilizing scheduling theory, researchers have developed algorithms and optimization models for determining optimal repayment schedules for multiple loans. In these models, various factors such as loan quantities, payment constraints, and interest rates are considered.

The effectiveness of preemptive scheduling techniques as means to optimize loan repayment strategies is investigated. These methods permit the suspension and resumption of loan repayment as required, thereby reducing the overall cost of repayment. Researchers have used mixed-integer linear programming and other techniques to tackle the difficult problem of determining the optimal repayment schedule for multiple loans. The problem's computational complexity is acknowledged, emphasizing the need for ongoing research and novel approaches.

Overall, the literature review highlights the significance of customized scheduling solutions that take into account the distinct characteristics and needs of various industries and functions. It highlights the need for ongoing research to develop more efficient and effective strategies for optimizing loan repayment, taking into consideration variables such as savings accounts, variable interest rates, and fluctuating cash flows. The conclusion of the review emphasizes the importance of addressing the RPML problem and devising innovative techniques to optimize loan repayment schedules in financial management.

2.10 Gap in the Research

The existing literature primarily focuses on the management of personal loans and the repayment of multiple personal debts, including car loans, credit card loans, mortgages, and personal debts. However, there is a significant gap in research when it comes to applying these repayment strategies to corporate loans and corporate debt. This area remains largely unexplored, offering an opportunity to extend the existing knowledge and strategies to the realm of corporate debt management. Furthermore, within the scope of corporate debt, there is a specific need to examine the debt management practices of construction companies. Construction companies often have unique types of debt that require tailored repayment strategies. However, limited research has been conducted in this specific context, leaving construction companies without adequate guidance for effectively managing their debts.

Another gap in the literature pertains to the specific lending and banking policies prevalent in the Egyptian market. Each country has its own distinct lending policies and regulations, and the Egyptian market is no exception. However, there is a lack of research that explores the repayment of multiple loans within the context of the Egyptian market, considering the local lending practices and policies. This creates a significant knowledge gap, leaving borrowers and lenders in Egypt without comprehensive insights and strategies for efficient debt repayment.

To address these gaps, further research is needed to investigate the repayment of multiple loans in the context of corporate debt, particularly focusing on construction companies. Additionally, studies that delve into the lending and banking policies in the Egyptian market would provide valuable insights for borrowers and financial institutions operating in Egypt. By exploring these uncharted territories, researchers can contribute to the development of effective debt management strategies and fill the existing gaps in the literature.

Chapter 3

Model Development

In this chapter of the research, the model used to optimize debt repayment strategies for Egyptian construction companies with multiple loans will be illustrated and explained in detail. The concentration will be on the model's operation, including the specific equations, symbols, and variables employed. The primary objective is to clarify the mathematical method utilized to address the problem of debt repayment optimization.

Mathematica is used to create the model, and the Gurobi optimizer is used to generate solutions. The model's equations were derived from a literature review of personal loans and interviews with financial and treasury executives from various Egyptian construction companies. Multiple loans, interest rates, repayment periods, and constraints such as cash flow and minimum payment requirements are accounted for in the equations.

Each equation will be thoroughly defined and explained to construct the model in Mathematica. This will include a description of each variable, its definition, and its function in the overall process of optimization. Additionally, the equation symbols will be explicitly defined to avoid confusion.

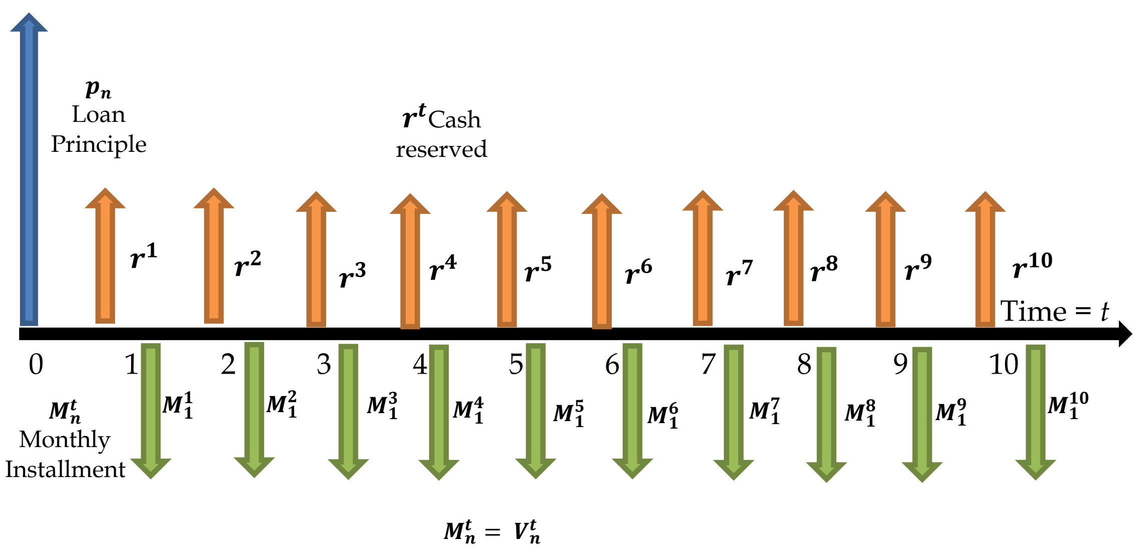
3.1 Single Loan Repayment

When a company acquires a loan, it engages in an agreement with the lender on the loan principle and negotiates the payment terms, including the repayment duration and interest rate. Based on these terms, a monthly installment is calculated. The agreement also covers aspects such as penalties for late or incomplete payments and administrative fees for voluntary payments that reduce the loan balance. In this section, we will present illustrations of three distinct cases to demonstrate the mechanics of the payment plan. Cash flow diagrams will be drawn with the three scenarios.

3.1.1 Standard Payment

The depicted cash flow diagram in Figure 4 illustrates a scenario where a company obtains a loan and proceeds to repay it through 10 installments over a 10-month duration. In this particular scenario, the company effectively allocates enough cash to meet the installment requirements, ensuring consistent and timely payments throughout the loan period. The repayment process continues until the loan reaches its maturity, with the final installment concluding the entire process. Importantly, the total amount paid to the lender V_n^t matches the agreed-upon monthly installments M_n^t in this scenario.

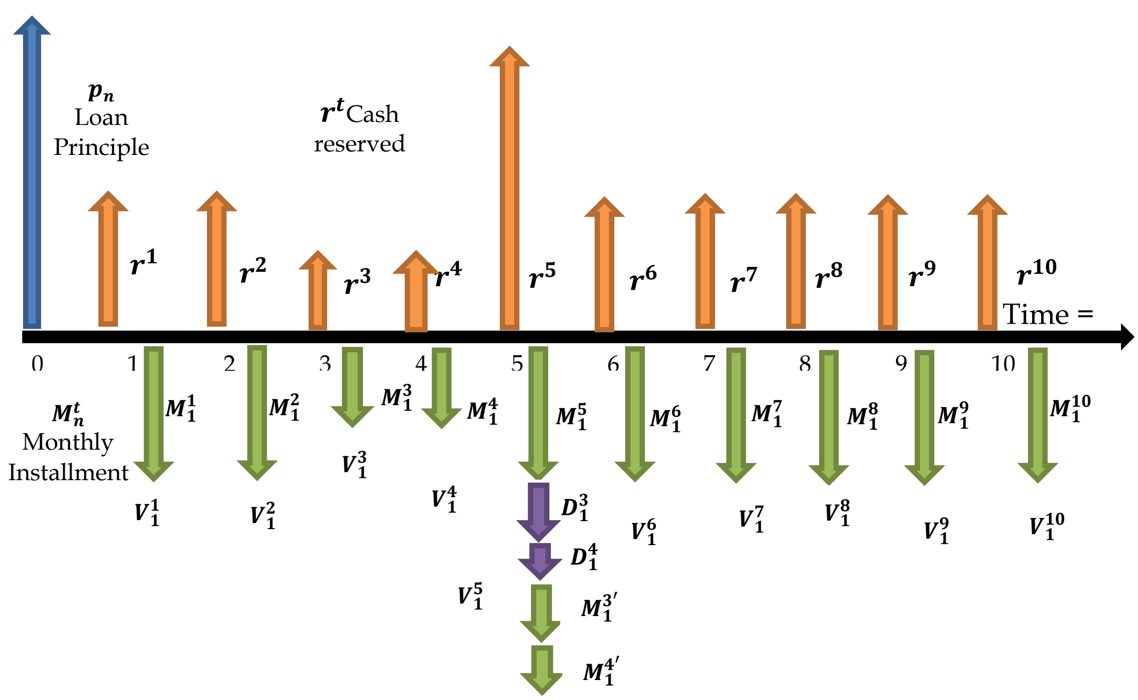
Figure 4 Planned Payment Schedule for a Single Loan



3.1.2 Deficit Payment

In the next cash flow diagram figure 5 illustrates the situation where the company after taking the loan and receiving the principle they plan the repayment of the installments but along the way they face shortages in cash and thus they have a deficit in the reserved cash so they cannot pay the full amount that results in incurring penalties until the amount is repaid. That shows changes in the cash flow diagram and an increase in the amounts to be paid to the lender. During the repayment period, a shortage in reserve cash arose in month 3, causing the company to make a partial payment for that month's installment and incur a penalty. The same situation occurred in month 4, resulting in another partial payment and penalty. By month 5, the company had to secure enough cash to cover the installment for that month, as well as the remaining installments from months 3 and 4, and the doubled penalty for month 3 and the penalty for month 4. Subsequently, the payment process continued smoothly until the end of the loan term. It's worth noting that the total amount paid to the lender V_1^3 and V_1^4 was less than the agreed-upon monthly installment M_1^3 **and** M_1^4 in months 3 and 4. However, in month 5, the total amount paid to the lender V_1^5 was the sum of the installment for that month, the remaining installments $M_1^{3'}$ **and** $M_1^{4'}$ from months 3 and 4, and the penalties for those two months D_1^3 **and** D_1^4 **and so** $V_1^5 = M_1^{3'} + M_1^{4'} + D_1^3 + D_1^4$.

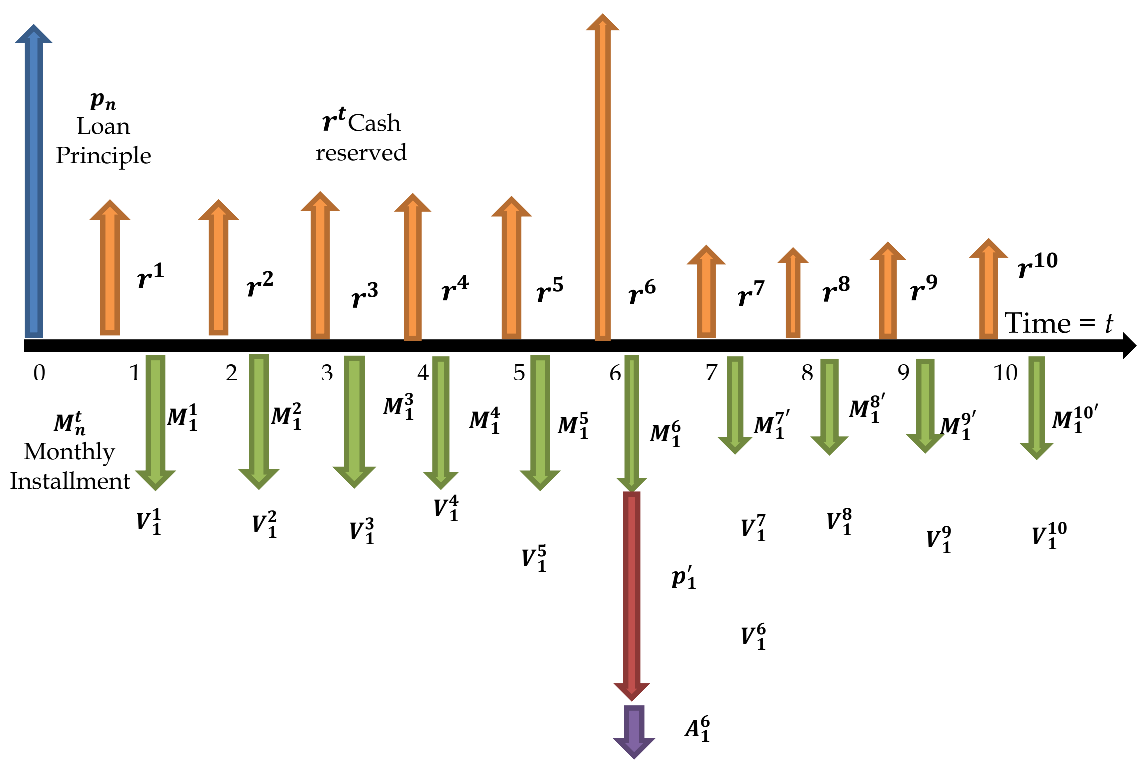
Figure 5 Loan Repayment with Payment Deficit for a Single Loan



3.1.3 Principal Payment

The next scenario involves a company experiencing a higher cash influx, prompting them to make additional payments beyond the agreed-upon installments. This partial repayment of the principal amount results in a reduction of the subsequent installments. Figure 6 illustrates this scenario, specifically in month 6, where there is an increase in the cash reserve r_1^6 , prompting the company to pay more than the required installment M_1^6. As a result, both the principle and the installments for the upcoming months are reduced. In month 6, the company paid an amount p_1' more than the installment, along with an administrative fee A_1^6 previously agreed upon to be paid in case of a reduction in the loan principal and so paying a total of V_1^6 , where $V_1^6 = M_1^6 + p_1' + A_1^6$.

Figure 6 Early Repayment of a Single Loan



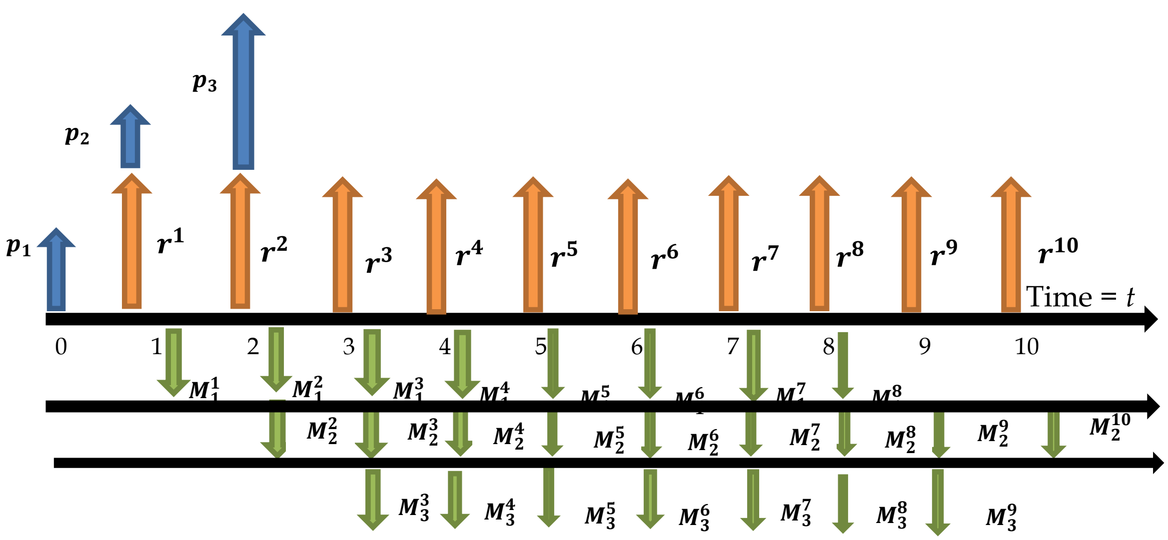
3.2 Multiple Loan Repayment

This section presents a complex scenario involving three loans, each with its own unique characteristics such as starting date, amortization period, loan principle, interest rate, penalties for late payment, and administration fees for exceeding the agreed-upon installment. These loans may come from different lenders. A cash flow diagram will be used to illustrate the repayment scenario for these three loans.

3.2.1 Standard Payment Multiple Loans

Figure 7 displays a cash flow diagram where a company obtains three loans. The first loan (t=0) is repaid over eight installments in eight months. The second loan (t=1) is repaid over nine installments in nine months. Lastly, the third loan (t=2) is repaid over seven installments in seven months. In this scenario, the company manages its cash flow effectively to meet the installment obligations, ensuring consistent and timely payments throughout each loan's duration. The repayment process continues until all loans reach maturity, with the final installment marking the completion. The total amount paid to the lender $\boldsymbol{V_n^t}$ aligns with the agreed-upon monthly installments $\boldsymbol{M_n^t}$ for each loan in this scenario.

Figure 7 Planned Payment Schedule for a Multiple Loans



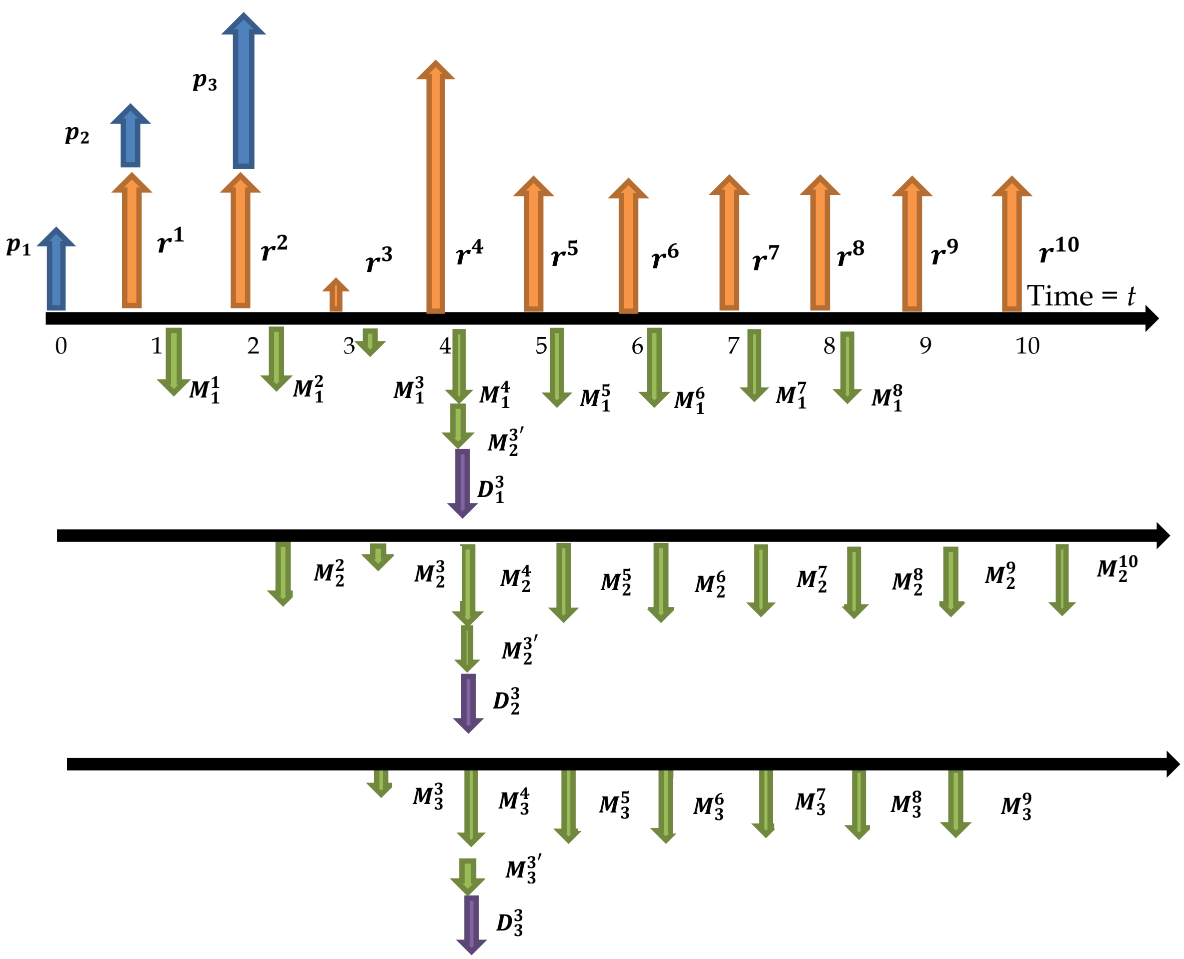
$$\sum V^3 = V_1^3 + V_2^3 + V_3^3 = M_1^3 + M_2^3 + M_3^3$$

3.2.2 Deficit Payment Multiple Loans

Figure 8 presents a cash flow diagram depicting a situation where a company, after obtaining multiple loans and receiving the loan principal, plans the repayment of installments. However, during the repayment period, they encounter cash shortages, leading to deficits in reserved funds. As a result, the company is unable to pay the full installment amounts, resulting in the incurrence of penalties until the outstanding amounts are repaid. At t=3, a shortage of funds leads to payment deficits for all loans. However, at t=4, the company has a more substantial influx of funds, enabling them to fully repay all loans, including the accrued penalties.

$V_1^5 = M_1^{3'} + M_1^{4'} + D_1^3 + D_1^4$.

Figure 8 Loan Repayment with Payment Deficit for a Multiple Loans

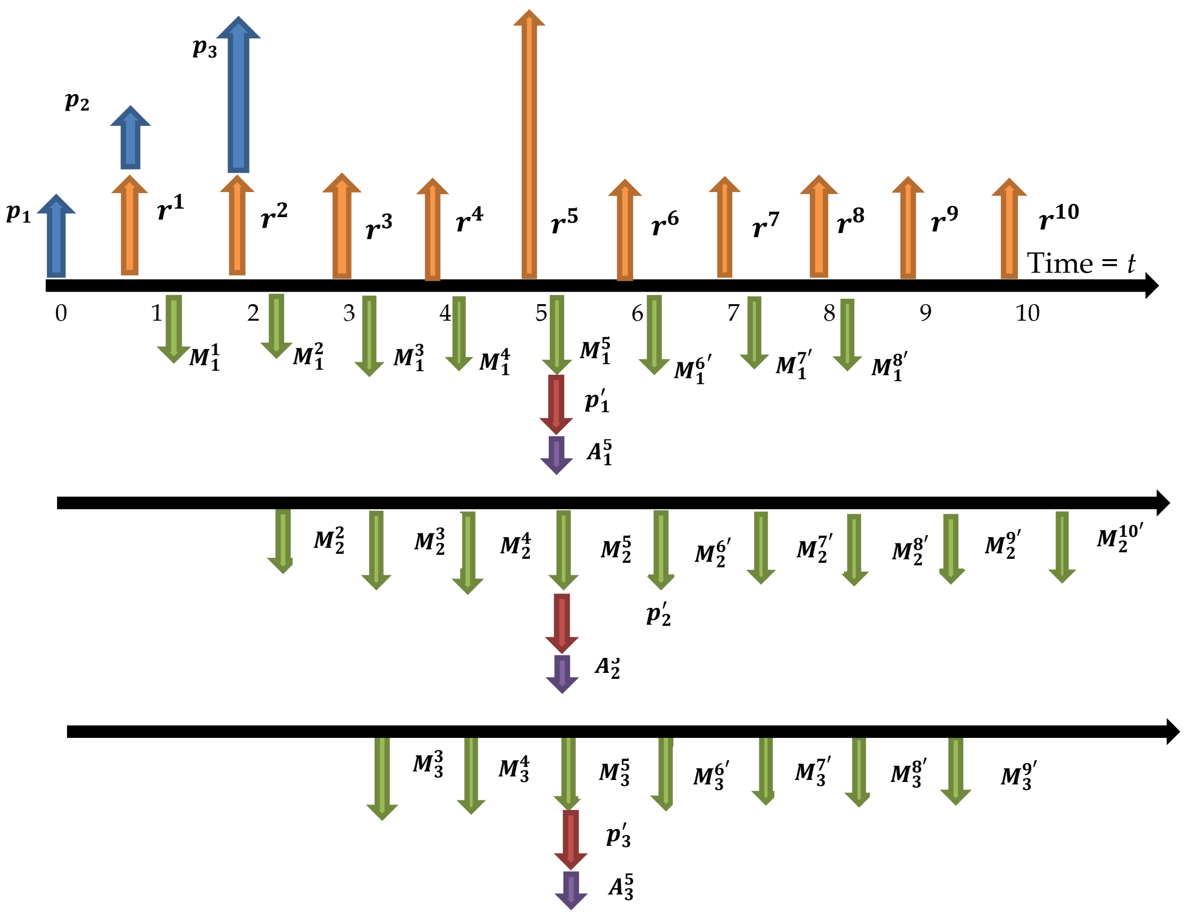


$$\sum V^4 = V_1^4 + V_2^4 + V_3^4 = \left(M_1^4 + M_2^{3'} + D_1^3\right) + \left(M_2^4 + M_2^{3'} + D_2^3\right) + \left(M_3^4 + M_3^{3'} + D_3^3\right)$$

3.2.3 Principal Payment Multiple Loans

Figure 9 showcases a scenario where a company encounters a significant increase in cash flow, leading them to make additional payments exceeding the agreed-upon installments. As a result, the outstanding principal amount for each loan is reduced, leading to a corresponding decrease in the subsequent installments. Specifically, in month 4, the company's cash reserve experiences a notable rise, prompting them to repay a portion of the principal amount for all three loans. For a detailed representation of this scenario, please refer to Figure 9.

Figure 9 Early Repayment of a Multiple Loans



$$\sum V^5 = V_1^4 + V_2^4 + V_3^4 = \left(\boldsymbol{M_1^5} + p_1' + A_1^5\right) + \left(\boldsymbol{M_2^5} + \boldsymbol{p_2'} + A_2^5\right) + \left(\boldsymbol{M_3^5} + p_3' + A_3^5\right)$$

3.3 Example to explain the model.

Consider a hypothetical situation where a construction company has secured two loans to finance its operations. The first loan, Loan one, has an eight-month repayment period, while the second loan, Loan two, has a twelve-month repayment period. Loan one has a principal of EGP 7,000, while Loan two has a principal of EGP 3,000. Table 1 provides a comprehensive breakdown of the data used in this example. This example illustrates how the proposed model can be utilized to optimize the repayment of multiple loans for Egyptian construction firms. By employing the methodology detailed in this research , businesses can reduce the total amount of money required to repay multiple loans and avoid incurring penalties for late payments.

Table 1. Illustrative Data Example for Demonstrating the Model's Functionality

	Loan 1	Loan 2
Principle p_n	7,000	3,000
Month (t)	8	12
Interest rate i_n^t	1.67%	1.67%
Total debt x	7,535.12	3334.842
Established installments M_n^t	941.89	277.90
Penalty not paying installment	2 %	2 %
Admin fees for paying principle	3 %	3 %

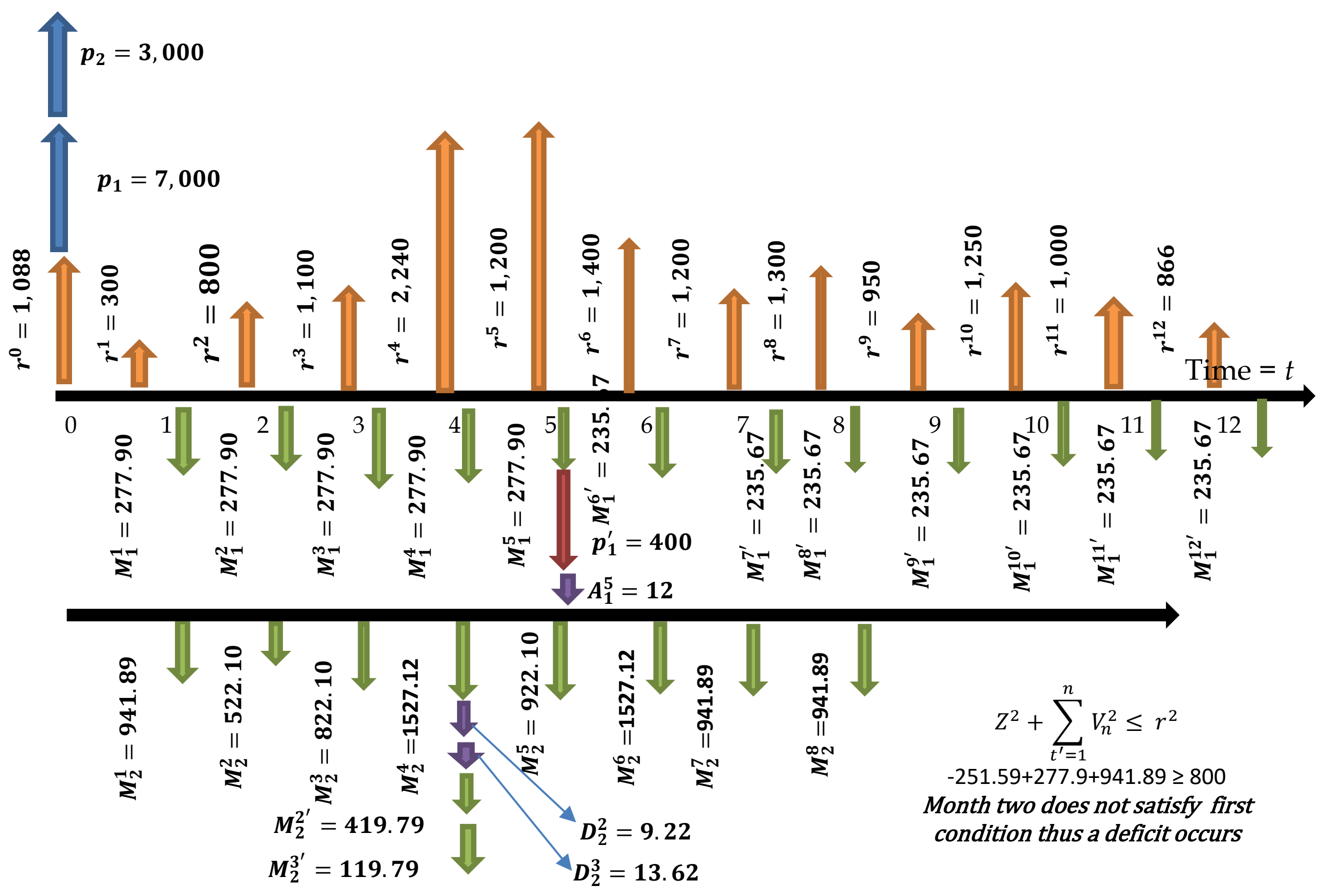
Table 2 portrays the monthly fluctuating available cash flow r^t, which is dependent on the company's expenses and revenues. The installment distribution is based on an amortization method that considers the principal, interest rate, and the number of months of repayment. The first loan has a 12-month repayment period, so its repayment can be tracked throughout the entire period. In contrast, the second loan is repaid in eight months, and its repayment is tracked from the date of release until the date of full repayment.

Table 2 reveals that there was a shortfall in the repayment for months 2 and 3, which resulted in the imposition of a penalty D_n^t , which intensified the shortfall until all payments were made in month 4. In month 5, however, savings Z^t accumulated, and a portion of it was used to pay off a portion of the principal for loan one, incurring a penalty for early payment A_n^t. This decreased the principal and, as a result, the monthly payment.

Table 2. Performance Comparison of the Model using Data from Table 1

T	available cash r^t	Loan 1	Loan 2	Saving Z^t	Penalty D_n^t	Payment + Administrative fee A_n^t
0	1088			1088.00		
1	300	277.90	941.89	168.21	0.00	
2	800	277.90	522.10	-251.59	9.22	
3	1100	277.90	822.10	-371.38	13.62	
4	2240	277.90	1527.12	434.98	0.00	
5	1200	277.90	922.10	415.18	0.00	412.00
		0.00	0.00	3.18	0.00	
6	1400	235.67	941.89	225.62	0.00	
7	1200	235.67	941.89	248.06	0.00	
8	1300	235.67	941.89	370.50	0.00	
9	950	235.67	0.00	1084.83	0.00	
10	1250	235.67	0.00	2099.15	0.00	
11	1000	235.67	0.00	2863.48	0.00	
12	866	235.67	0.00	3493.81	0.00	

Figure 10 illustrates the cash flow diagram corresponding to the example presented in Table 2.



All loans were repaid by the conclusion of the repayment period, as seen in the 12th month, which marks the end of the model. The penalty incurred during the operation of the model was added to the corresponding installment to reflect the actual cost of the loan. It was based on the outstanding principle at the time. This example illustrates the proposed model's usefulness for managing multiple loan repayment, minimizing penalties, and maximizing savings.

3.4 Equations of the model

This section will present a comprehensive list of equations that will serve as the foundation of the model. Additionally, the corresponding Mathematica code for each equation will be provided, ensuring clarity and practical implementation.

3.3.1 The Objective Function

The objective function of the model, as shown below in equation 1, is to minimize the total amount of money needed to pay the entire loan amount. The objective function is to minimize the entire amount paid for all loans V_n^t from loan 1 to loan x, and from installment at month 1 to installment at month T

$$\boldsymbol{min} \sum_{l=1}^{x} \sum_{t=1}^{T} V_n^t$$

Equation 1

Mathematica Code

```
obj = Total[x, 2];
```

3.3.2 Restriction for the availability of funds

A constraint is indicated, as seen in equation 2, every month t, where the debtor has a fixed amount of money $\boldsymbol{r^{t'}}$, that is available for the payment of loans in addition to any other cash provisions from the previous month. The total loans (*1 to x*) to be paid each ($\boldsymbol{V_n^t}$) for the first month, added to saving for the month Z^1, should be less than or equal to the amount of money available to repay the debt:

$$Z^1 + \sum_{t'=1}^{n} V_1^{b_l} \leq r^{t'}$$

Equation 2

<u>Mathematica code</u>

$$\text{con2} = \sum_{t'=1}^{x} \text{Indexed}[v, \{t', \text{Sort}[bn][[1]]\}] + \text{Indexed}[z, 1] \leq rt[[1]];$$

And so each month, as can be seen in equation 3, the total loans (1 to x) to be paid each $V_{[\mathrm{bn}]+(n-1)}^{t'}$, added to the saving for that month Z_n^t Should be less than or equal to the amount available to repay debt and savings from the previous month.

$$\sum_{t'=1}^{x} V_{[\mathrm{bn}]+(n-1)}^{t'} + Z_n^t \leq r_n^t + Z_{n-1}^{t-1} \quad , \qquad n = 2, \dots \dots . . -\mathrm{bn} + t$$

Equation 3

<u>Mathematica code</u>

$$\text{con3} = \text{Table}\left[\sum_{t'=1}^{x} \text{Indexed}[v, \{t', \text{Sort}[bn][[1]] + n - 1\}] + \text{Indexed}[z, n] \leq rt[[n]] + \text{Indexed}[z, n - 1], \{n, 2, t - \text{Sort}[bn][[1]]\}\right] // \text{Flatten};$$

3.3.3 Restriction constraint of equality between debt and installment

To ensure that the final value of debt matches the total of all installments, the concept of amortization is employed, which utilizes the present value of debt and installments. This is achieved by equating the future value of debt at the end of the period with the sum of installments. In Equation 4, the left-hand side of the equation is set to zero, as shown:

$$\sum_{t'=b_{n(i)}}^{t} (D_{ni}^{t'} + A_{ni}^{t'} - V_{ni}^{t'}) \prod_{t''=t'+1}^{t} (1 + i_n^{t''}) + p_n \prod_{t'=b_{n(i)}}^{t} (1 + i_n^{t'}) = 0, \qquad i \in x$$

Equation 4

In section $\sum_{t'=b_{n(i)}}^{t}(D_{ni}^{t'} + A_{ni}^{t'} - V_{ni}^{t'})$ Starting with the sum from the release time $b_{n(i)}$ to total time at the end of the loan t, of the penalties of the minimum payment $\boldsymbol{D_{ni}^{t'}}$ for each loan each month, added to the penalties of maximum payment $\boldsymbol{A_{ni}^{t'}}$ each loan each month, subtracted from the amount of money to be paid in each loan each month $\boldsymbol{V_{ni}^{t'}}$.

In section $\prod_{t''=t'+1}^{t}(1 + i_n^{t''})$ That is then multiplied by the accumulation factor for the interest, which is the product for all available loans from time $t'' = t' + 1$ to time t, of 1 plus the interest $\boldsymbol{i_n^{t''}}$ of loan n at month t''.

In section $\boldsymbol{p_n \prod_{t'=b_{n(i)}}^{t}(1 + i_n^{t'})}$ that is then added to m_j the principle amount of loan j multiplied by the product of the accumulation factor, giving its amount at compound interest after t periods, for the interest from the release time $\boldsymbol{p_n}$ to time t, of 1 plus the interest $\boldsymbol{i_n^{t'}}$ Of loan n at month $\boldsymbol{t'}$.

Mathematica code

```
con4 =
 Table[ Sum[(Indexed[d, {i, t'}] + Indexed[a, {i, t'}] - Indexed[v, {i, t'}]) Product[(1 + Indexed[itn, {i, t''}]), {t'', t' + 1, t}], {t', bn[[i]], t}] +
   pn[[i]] Product[(1 + Indexed[itn, {i, t'}]), {t', bn[[i]], t}] == 0, {i, 1, x}] // Flatten;
```

$$\sum_{t'=b_{n(i)}}^{t} (V_{ni}^{t'} - D_{ni}^{t'} - A_{ni}^{t'}) \prod_{t''=t'+1}^{t} (1 + i_n^{t''}) = p_n \prod_{t'=b_{n(i)}}^{t} (1 + i_n^{t'})$$

Equation 5

Equation 4 can be rearranged as shown in Equation 5. When the installment amount on the left-hand side equals the final value of the debt on the right-hand side, it signifies that the debt has been fully repaid. As a result, Equation 4 holds true for the entire repayment period, from the loan release time $\boldsymbol{b_n}$ to the time t when the loan is completely paid off. By integrating this equation into the minimization objective function, it removes the need to know the exact completion time for any of the loans in advance.

$$\text{con5} = \text{Flatten}[\text{Table}[\text{Indexed}[y, \{i, \text{bn}[[i]]\}] == \text{pn}[[i]], \{i, 1, x\}]];$$

3.3.4 Restriction for remaining loan balance

The initial condition for loan balance $\boldsymbol{Y_n^{b_n}}$ is defined by the following constraints, which set the balance at time $\boldsymbol{b_n}$ (the loan start time) equal to the principal amount p_n, as shown in the equation.

$$\boldsymbol{Y_n^{b_n} = p_n \qquad i = \{1 \dots . x\}}$$

Equation 6

<u>Mathematica code</u>

```
con5 = Flatten[Table[Indexed[y, {i, bn[[i]]}] == pn[[i]], {i, 1, x}]];
```

The second restriction is defined by the following constraints, which determine the balance $\boldsymbol{Y_n^{b_n}}$ of loan n at time t. This equation allows for the determination of the amount owed by the debtor at any given time t for each loan n.

$$\boldsymbol{Y_n^{b_n}} = \sum_{t'=\text{bn}}^{t'=\text{bn}} (\boldsymbol{D_{ni}^{t'} + A_{ni}^{t'} - V_{ni}^{t'}}) \prod_{t''=t'+1}^{\boldsymbol{n}} \left(\boldsymbol{1 + i_j^{t''}}\right) + \boldsymbol{p_n} \prod_{t'=\text{bn}+1}^{n} (\boldsymbol{1 + i_n^{t'}})$$

$$\boldsymbol{i = \{1 \dots . x\} \quad , \quad b_n = \{1 \dots \dots . t\}}$$

Equation 7

<u>Mathematica code</u>

```
con6 =
 Table[( Sum_{t'=bn[[i]]}^{n-1} (Indexed[d, {i, t'}] + Indexed[a, {i, t'}] - Indexed[v, {i, t'}]) Product_{t''=t'+1}^{n} (1 + Indexed[itn, {i, t''}]) +
     pn[[i]] Product_{t'=bn[[i]]+1}^{n} (1 + Indexed[itn, {i, t'}]) ) == Indexed[y, {i, n}], {i, 1, x}, {l, bn[[i]] + 1, t}] // Flatten;
```

In section $\sum_{t'=\text{bn}}^{t'=\text{bn}}(\boldsymbol{D}_{ni}^{t'} + \boldsymbol{A}_{ni}^{t'} - \boldsymbol{V}_{ni}^{t'})$ of equation 7, the balance $\boldsymbol{Y}_{n}^{b_n}$ of loan n at time t is calculated as follows. The sum at the time t', which could be any month between the start date for the loan and one month before the last month of loan repayment, of $\boldsymbol{D}_{ni}^{t'}$ penalties for paying less than the minimum payment, plus $\boldsymbol{A}_{ni}^{t'}$ of paying more than the maximum payment minus the amount of money that should be paid.

In section $\prod_{t''=t'+1}^{n}\left(\mathbf{1} + \ \boldsymbol{i}_j^{t''}\right)$of equation 7, then it is multiplied by the product of the accumulation factor at the time t'' between $t' + 1$ and t.

In section $\boldsymbol{p_n} \prod_{t'=\text{bn}+1}^{l}(\mathbf{1} + \boldsymbol{i}_n^{t'})$of equation 7, added to the product of the principal amount of the loan and the accumulation factor at the time t' between the release month $\boldsymbol{p_n}$ and the final month *t*.

3.3.5 Constraint for a positive balance of a loan

To account for loans with a positive balance at time *t*, a constant *F* is introduced. This constant, set to a very large value, ensures that variables $\boldsymbol{H}_n^t$ = 1 in such cases.

$$\boldsymbol{Y}_{n-1}^{t}(\mathbf{1} + \ \boldsymbol{i}_n^t) \leq \boldsymbol{F}\boldsymbol{H}_n^t \ , \quad \boldsymbol{S} = \{\mathbf{1} \ldots. \boldsymbol{n} \ \} \quad , \ \boldsymbol{b_n} = \{\mathbf{1} \ldots \ldots . . \boldsymbol{t}\}$$

Equation 8

Mathematica code

```
con7 = Table[Indexed[y, {i, n - 1}] * (1 + itn[[i, n]]) ≤ 10^50 * Indexed[h, {i, n}],
    {i, 1, x}, {j, bn[[i]] + 1, t}] // Flatten;
```

3.3.6 Deficit Payment

Usually, loans are based on fixed monthly installments. $\boldsymbol{M}_n^t$that has to be paid each month. If $\boldsymbol{V}_n^t$ is not sufficient to cover the installment, then variable $\boldsymbol{D}_n^t$ registers the penalty to be paid. For loan *n* at time t, the binary value $\boldsymbol{H}_n^t$ is multiplied by $\boldsymbol{M}_n^t$ the amount due for debt subtracted by $\boldsymbol{V}_n^t$ The amount paid multiplied by one plus the default interest rate should be less than or equal to the penalty being paid.

$$(\boldsymbol{H_n^t M_n^t} - \boldsymbol{V_n^t})(\mathbf{1} + \boldsymbol{c_n^t}) \leq \boldsymbol{D_n^t} \qquad \boldsymbol{b_n} = \{\mathbf{1} \dots \dots \boldsymbol{t}\}$$

Equation 9

Mathematica code

```
con8 =
  Table[(Indexed[h, {i, n}] * mn⟦i⟧ - Indexed[v, {i, n}]) * (1 + cn⟦i, n⟧) ≤ Indexed[d, {i, n}],
    {i, 1, x}, {n, bn⟦i⟧ + 1, t}] // Flatten;
```

3.3.7 Overdraft

For default payment, the minimum payment is a percentage $\boldsymbol{o_n}$ of the balance $\boldsymbol{Y_n^t}$, and the penalization $\boldsymbol{D_n^t}$. The percentage of the minimum payment $\boldsymbol{o_n}$ is multiplied by the balance of debt multiplied by its interest $\boldsymbol{Y_n^{t-1}}(1 + i_n^t)$ subtracted from the amount to be paid that month $\boldsymbol{V_n^t}$ that, in turn, is multiplied by the default interest $(1 + \boldsymbol{c_n^t})$, the total should be less than or equal to $\boldsymbol{D_n^t}$. It is computed as follows:

$$\left(\boldsymbol{o_n Y_n^{t-1}}(1 + i_n^t) - \boldsymbol{V_n^t}\right)(1 + \boldsymbol{c_n^t}) \leq \boldsymbol{D_n^t} \qquad \boldsymbol{t} = \{\boldsymbol{b_n} \dots \dots \boldsymbol{T}\}$$

Equation 10

Mathematica code

```
con9 =
  Table[((on⟦i⟧ * Indexed[y, {i, n}] * (1 + itn⟦i, n⟧)) - Indexed[v, {i, n}]) * (1 + din⟦i, n⟧) ≤
    Indexed[d, {i, n}], {i, 1, x}, {n, bn⟦i⟧ + 1, t}] // Flatten;
```

3.3.8 Extra payment from the principal (admin fees)

An administration fee can also be applied if paying more than the monthly installment. the fee $\boldsymbol{ai_n}$, referring to the second binary equation determining if the penalty $\boldsymbol{A_n^t}$ applies, in that case, they will be handled inequality using constant $\boldsymbol{F}$. In Equation 11, the amount to be paid is subtracted by the binary variable that the loan exists multiplied by the initially established installment that should be less than or equal to the binary value that $\boldsymbol{V_n^t}$ is more than $\boldsymbol{M_n^t}$ it is multiplied by the constant $\boldsymbol{F}$.

$$\boldsymbol{V_n^t - H_n^t M_n^t \leq Q_n^t F} \quad , \quad \boldsymbol{n} = \{\mathbf{1} \ldots \ldots \boldsymbol{x}\} \ , \boldsymbol{t} = \{\boldsymbol{b_n} \ldots \ldots \ldots \boldsymbol{t}\}$$

Equation 11

Mathematica code

```
con10 = Table[(Indexed[v, {i, n}] - Indexed[h, {i, n}] * mn[[i]]) ≤ 10^50 * Indexed[q, {i, n}],
    {i, 1, x}, {n, bn[[i]] + 1, t}] // Flatten;
```

In equation 12, the value of the amount of the penalty $\boldsymbol{A_n^t}$ that is equal to the binary value $\boldsymbol{Q_n^t}$ multiplied by $\boldsymbol{ai_n}$ the fixed penalty.

$$\boldsymbol{A_n^t = Q_n^t ai_n} \ , \quad \boldsymbol{n} = \{\mathbf{1} \ldots \ldots \boldsymbol{x}\} \ , \boldsymbol{t} = \{\boldsymbol{b_n} \ldots \ldots \ldots \boldsymbol{t}\}$$

Equation 12

Mathematica code

```
con11 = Table[Indexed[a, {i, n}] == Indexed[q, {i, n}] * ain[[i]], {i, 1, x}, {n, bn[[i]] + 1, t}] //
    Flatten;
```

It is evident in the model that a penalty is computed in the final month of loan repayment if the debtor pays an amount less than the prescribed installment. To address this issue, additional constraints can be introduced to the problem formulation. In Equation 13, it is stated that the variable $\boldsymbol{V_n^t}$ represents the amount to be paid for loan $\boldsymbol{n}$ in month $\boldsymbol{t}$. This amount is subject to the penalty$\boldsymbol{D_n^t}$if the payment is less than the minimum required, and the penalty$\boldsymbol{A_n^t}$ if the payment exceeds the established installment. These constraints are applicable when $\boldsymbol{t}$ is prior to the loan release time and $\boldsymbol{n}$ belongs to the set $\boldsymbol{S}$. Ultimately, the variables are defined as follows:

$$\boldsymbol{V_n^t = D_n^t \ = A_n^t = 0} \ , \qquad \boldsymbol{S} = \{\ \mathbf{1} \ldots \ldots \boldsymbol{x}\}, \quad \boldsymbol{n} = \{\mathbf{1} \ldots \ldots \boldsymbol{b_n} - 1\}$$

Equation 13

Mathematica code

```
con12 = Flatten[Table[Indexed[v, {i, n}] == Indexed[d, {i, n}] == Indexed[a, {i, n}] == 0,
    {i, 1, x}, {n, 1, bn[[i]] - 1}]];
```

In Equation 14, it states that $\boldsymbol{V_n^t}$ the amount of money that should be paid for loan n and month t, $\boldsymbol{D_n^t}$ the penality for paying less than the minimum payment, $\boldsymbol{A_n^t}$ the penalty for paying more than the established installment, and $\boldsymbol{B_j^t}$ The loan balance is greater than zero when t equals months from *0* to *T,* and *n* belongs to *S.*

$$\boldsymbol{V_n^t \geq 0} \quad \boldsymbol{, D_n^t \geq 0} \quad \boldsymbol{, A_n^t \geq 0} \quad \boldsymbol{, Y_n^t \geq 0}$$

Equation 14

<u>Mathematica code</u>

```
con13 = {v ≥ 0, d ≥ 0, z ≥ 0, a ≥ 0, y ≥ 0};
```

In Equation 15 it states that $\boldsymbol{H_n^t}$ and, $\boldsymbol{Q_n^t}$ are binary variables where t equals months from *0* to *t* and *n* belongs to *S.*

$$0 \leq \boldsymbol{H_n^t} \leq 1 \quad , \quad 0 \leq \boldsymbol{Q_n^t} \leq 1$$

Equation 15

<u>Mathematica code</u>

```
bin = {0 ≤ h ≤ 1, 0 ≤ q ≤ 1};
```

3.3.9 The Solution of the Model

The following code represents the final line in the Mathematica code. It compiles all the constraints, objective function, and variables necessary to solve the model effectively."

Mathematica Code

```
solution = LinearOptimization[obj,
  {con2, con3, con4, con5, con6, con7, con8, con9, con10, con11, con12, con13, , bin},
  {v ∈ Matrices[{x, t}, NonNegativeReals], s ∈ Vectors[t, NonNegativeReals],
   d ∈ Matrices[{x, t}, NonNegativeReals], a ∈ Matrices[{x, t}, NonNegativeReals],
   y ∈ Matrices[{x, t}, NonNegativeReals], h ∈ Matrices[{x, t}, Integers],
   q ∈ Matrices[{x, t}, Integers]}, {"PrimalMinimumValue", "PrimalMinimizerRules"},
  MaxIterations → Infinity, Tolerance → 10^(-20), Method → "Gurobi"]
```

The objective of the optimization model is to minimize the total payment of the debt incurred by construction companies, taking into account penalties and default rates. This model is specifically designed to provide construction companies with an optimal repayment plan based on their current financial situation. However, if there are changes in the company's income or interest rates, the initially determined optimal repayment schedule may no longer be suitable or effective. To address this, it becomes necessary to recalculate the optimization model in order to generate a repayment plan that reflects the companies updated financial circumstances. By doing so, the company can ensure they have an effective repayment strategy that aligns with their current financial situation. The optimization model utilizes linear programming techniques to find the optimal solution. It considers various factors and constraints to determine the repayment plan that minimizes the debtor's total annual payments.

Chapter 4

Model verification

Table 3 below presents the results of a sensitivity analysis conducted on the optimization model. The purpose of this analysis is to assess the performance and reliability of the model by examining its output under different scenarios and variations in input parameters. Let's discuss the verification of the optimization model based on the provided table 3.

Table 3 Results of Verification trials on the model.

No.	Total amount of debts (number)	Short term Bank loans (number)	Medium term lease (number)	T (months)	Optimization Results in Millions EGP
1	24	12	12	120	353.0148
2	20	10	10	72	421.196
3	16	8	8	72	472.72
4	14	7	7	72	560.496
5	12	6	6	73	519.766
6	12	6	6	74	520.536
7	12	6	6	76	521.06
8	12	6	6	80	522.28
9	12	6	6	85	523.914
10	12	6	6	90	525.452
11	12	6	6	95	527.484
12	12	6	6	110	534.223
13	12	6	6	120	539.603
14	4	2	2	18	130.467
15	4	2	2	12	101.465
16	4	2	2	6	69.3877
17	3	2	1	24	67.2864
18	2	1	1	24	39.1287
19	2	1	1	12	39.1287

Verifying the optimization model consisted of evaluating its efficacy in various scenarios. Table 3 provides insight into the model's limitations and behavior. Next is analyzing the verification results in greater depth and expounding on the findings.

The optimization model underwent extensive calculations to assess its capabilities and limitations. It successfully handled a time frame of up to 120 months without encountering any issues, showcasing its effectiveness in managing longer repayment periods. However, when the scenario involved 120 months and 24 loans, the obtained results displayed inaccuracies. Notably, the errors in the results became more pronounced as the number of loans increased, indicating the model's limitations in handling larger loan quantities. Consequently, it can be concluded that the optimal capacity for the model, utilizing Mathematica on a standard laptop (Windows, i7 with 16 GB of RAM and SSD), is 72 months and 20 loans. The model exhibited satisfactory performance within these defined boundaries by generating precise and reliable outcomes.

In addition, the model was evaluated in scenarios with fewer loans and shorter repayment periods. Simulations were conducted with as few as two loans and as few as six months, and the model produced acceptable results. This demonstrates the model's adaptability, as it can manage small-scale scenarios effectively.

However, note that running the model with a larger number of loans or prolonging the duration may necessitate a workstation with a powerful processor or an alternative to Mathematica. The Complexity of calculations and size of data sets may exceed the capabilities of a standard laptop, necessitating the use of more advanced computational resources.

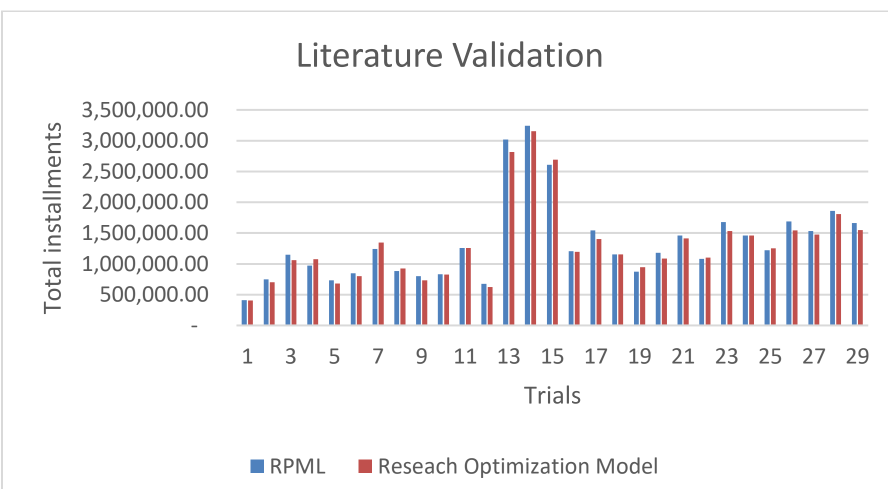
Chapter 5

Literature Validation

The optimization model underwent a rigorous validation process using a selected dataset obtained from a literature review focused on personal loans. The dataset was executed with the model, and its results were compared to those from the literature review. This validation aimed to confirm the model's ability to generate comparable outcomes, ensuring its accuracy and reliability.

The validation process demonstrated that the optimization model produced results consistent with the literature review and a research-based mixed integer linear programming (RPML) approach. These findings validate the model's performance and increase confidence in its predictions and practical applicability, specifically for analyzing data related to Egyptian construction companies. The validation results, as shown in Figure 7, exhibit minimal variation between the model and the literature review, with a mean difference of 2.33 percent and a standard deviation of 6.32 percent.

Figure 10. Validation of the Optimization Model with Literature Data



With the successful completion of the validation phase, the optimization model is now ready for utilization with real-world data from Egyptian construction companies. This data will encompass industry-specific standards, best practices, and relevant information regarding loan debt repayment, interest rates, repayment periods, lending mechanisms, penalties, early repayment fees, and more.

Utilizing the optimization model to analyze real-world data will provide valuable insights into debt repayment patterns within the Egyptian construction industry. These insights can be leveraged to optimize debt repayment strategies, reduce debt burden, and maintain financial stability. Additionally, the model enables accurate forecasting of future repayment patterns, empowering businesses with precise projections of their financial obligations.

Furthermore, the model's capabilities extend to identifying potential challenges during the repayment process, such as cash flow shortages or late payment fees. This proactive identification allows businesses to address these issues in a timely manner and avoid additional financial burdens.

The validation of the optimization model, as presented in Figure 7, confirms its precision, reliability, and applicability for analyzing debt repayment patterns in the Egyptian construction industry. By utilizing the model with real-world data, construction companies can optimize their debt repayment strategies, enhance financial stability, and foster long-term success and growth.

Chapter 6

The market validation and interviews

An investigation was carried out to analyze construction companies' actual debt repayment procedures. The research involved conducting interviews with finance executives, finance managers, and treasury managers of construction companies. The study's primary aim was to understand how construction companies manage their debt repayment process and the available financing options.

The survey revealed that not all construction companies finance their initiatives with loans, but medium and large businesses typically take out loans. In most construction projects, the project proprietor provides the construction company with an advance payment of approximately 20% of the project's value. However, seeking external financing options to bridge the funding gap is crucial for companies that cannot fund the project independently.

Debt financing is one of the most prevalent funding sources in the construction industry. It provides a reliable and tax-deductible funding source, making it an attractive financing alternative for many construction firms. The survey findings provided a better understanding of the complexities of the debt repayment process and offered insights into how construction companies handle their finances. By carefully analyzing these factors, construction companies can make informed financial decisions that will positively impact their business.

6.1 Interview outcomes

In the following sections, the outcomes of the interview will be presented in detail.

6.1.1 Managing Debt and Financing Options

In Egypt, medium and large construction companies rely heavily on loans to manage their finances and fund their projects. These loans can be categorized as direct or indirect, depending on the nature of project receivables. Direct loans are taken out against specific project receivables, while indirect loans are based on the company's financial position. Moreover, loans can be unrestricted, meaning they can be used for any purpose, or restricted, meaning they can only be used for a specific purpose related to the project.

The finance department plays a critical role in managing debt in the construction industry. The finance department is responsible for identifying the appropriate financing options for a particular project and determining the optimal debt structure. The treasury department is responsible for managing the company's cash flow and ensuring sufficient liquidity to meet the company's financial obligations.

Given the importance of debt financing in the construction industry, it is essential to understand the different financing options available to construction companies in Egypt. This understanding can help companies identify the most appropriate financing option for a particular project and manage their debt effectively. By exploring the various financing options available to construction companies in Egypt and the role of finance and treasury departments in managing debt, this research aims to provide insights that can help construction companies navigate the complex world of debt financing.

6.1.2 Project Financing and Letters of Guarantee (LGs)

Cash requirements are standard in the construction industry, and they can arise due to several reasons, such as project financing, fixed asset purchases, and equipment acquisition. The treasury department of construction companies plays a critical role in managing cash requirements. During the tender phase, the department evaluates whether project financing is required and whether there may be a cash flow deficit during the project's tenure. If necessary, they obtain financing for Letters of Guarantee (LGs), essential for winning construction contracts.

Payments for LGs are often split into two components. The first component is secured by work completed within one to one and a half years, which is the typical contract duration. The second component is the down payment for the LG, and its value decreases as the project progresses. It is crucial to managing cash requirements effectively to ensure timely project completion without any financial difficulties.

In Egypt's construction industry, the treasury department must carefully evaluate the cash requirements of each project. The department should determine the optimal financing options for LGs and other cash requirements. These options include direct and indirect loans and other forms of credit, such as credit lines and overdraft facilities. A well-planned and structured approach to cash management is essential to avoid financial difficulties that may delay project completion. By managing the cash requirements effectively, the treasury department can help construction companies achieve their financial goals and ensure the timely completion of their projects.

6.1.3 Short-term and Medium-term Loans

Loans between 15 and 20 million EGP are available to fund salaries, government payments, and indirect costs. These short-term loans can be used for small initiatives or to cover receivables delays of two to three months, allowing companies to pay subcontractors and suppliers. This loan may be repaid and reapplied three to four times yearly.

In the past, short-term revolving loans, also known as overdraft facilities, were available to construction companies, but this is no longer true. Banks have ceased offering overdraft facilities to construction companies, and now they only rely on short-term loans without overdraft facilities. These short-term loans range from six to twenty-four months and are typically provided by banks based on the company's most significant deficit point.

Banks typically offer medium-term loans (MTL) and have a duration of two to five years. However, currently, banks do not offer this type of loan MTL to construction companies; it is offered more to industrial companies as well as long-term loans LTL

6.1.4 Lease Agreements

Leasing companies have become a popular option in Egypt for construction firms seeking to acquire assets without the usual constraints of traditional debt financing. Leasing agreements typically range from 3 to 7 years, with the option to purchase the asset for a nominal fee, usually 1 pound, at the end of the lease period.

These leasing companies offer two main financial leasing options: direct lease and sales leaseback. Under a direct lease agreement, the construction company agrees to purchase equipment worth 20 to 30 million pounds from both the vendor and the leasing company. The leasing company pays the vendor directly, and the construction company pays back the leasing company in monthly installments plus interest. In contrast, a sales leaseback arrangement involves the construction company selling its existing assets to the leasing company, which then resells the assets to the construction company in installments with interest.

Leasing companies also offer their clients additional services such as factoring and reverse factoring. Reverse factoring, or supply chain financing, is a service that leasing companies commonly offer customers. Essentially, a financial institution offers to pay a company's suppliers on its behalf in exchange for a discount on the amount owed. The company then repays the financial institution. This service can be particularly beneficial for construction companies, which often have payment terms of 90 days or longer. Construction companies issue payment requests monthly to their suppliers and subcontractors and then give those requests to the leasing company. The leasing company typically provides 90% of the value of those payment requests, which they use to pay the suppliers and subcontractors. This allows the construction company to maintain its cash flow, as it can receive payment from the leasing company immediately instead of waiting for its customers to pay them.

Leasing companies may take an assignment of proceeds to ensure a steady cash flow. This means the construction company assigns one of its project's revenue streams to the leasing company. This guarantee can be canceled at any time, according to the agreement. If a construction company is unable to pay the total value of its installment to the leasing company or bank, a penalty of 2% may be imposed until the total value is paid.

It's important to note that leasing companies in Egypt no longer offer short-term revolving loans or overdraft facilities to construction companies. Instead, construction firms rely on short-term loans without overdraft facilities. Medium-term loans, typically ranging from 2 to 5 years, are also not offered to construction companies.

Leasing companies in Egypt, including EFG Hermes, Global Lease, Ghabour, and Finlease, offer their clients a range of financial services, including factoring and direct and sales leaseback. Factoring is a service that enables businesses to sell their accounts receivable or unpaid invoices in exchange for immediate cash. On the other hand, leasing is a form of financing that allows businesses to use assets like equipment or vehicles for a fixed period in exchange for regular lease payments. Leasing companies can be either bank-backed or independent entities.

It's worth noting that construction companies in Egypt have an I-Score, which determines their credibility in taking out loans. Early retirement of the loan is rarely applied, and administration fees for early payment are typically between 3% to 5%.

6.1.5 Role of Finance and Treasury Departments

Construction companies' finance and treasury departments play a crucial role in managing debt and financing options. They evaluate the organization's financial condition, monitor debt levels, and assess available financing options to determine the most suitable form of debt. By effectively managing debt, these departments can ensure the long-term financial stability of the construction company.

In the construction industry in Egypt, several financing options are available to manage debt, including short-term and medium-term loans, lease agreements, and project financing with Letters of Guarantee (LGs). The treasury department is responsible for loan acquisition and punctual repayment in larger organizations. The finance and treasury departments are responsible for determining the organization's most suitable form of debt. They are essential in managing debt and ensuring the company's financial health.

6.1.6 Loan Evaluation and Terms

Financial institutions, particularly banks, evaluate loan requests from construction companies on a per-project basis, regardless of whether the company works on all its projects simultaneously. The loan amount, repayment terms, and interest rates are negotiable and may vary depending on the project's scale, duration, and potential risk. The construction company's creditworthiness, assessed by its I-Score, may also be considered when evaluating its loan application.

The I-Score is a tool financial institutions use to evaluate a company's creditworthiness based on its financial performance, credit history, and other relevant factors. A higher I-Score indicates a stronger financial position and increases the likelihood of loan approval, while a lower score may result in unfavorable loan terms or even rejection. Construction companies should aim to maintain a good I-Score by keeping their financial records up to date, meeting payment obligations on time, and minimizing their outstanding debts.

6.1.7 Early Repayment and Proceeds Assignment

Lenders may charge administration fees if construction companies repay their loans early. However, suppose the company is unable to pay the total amount on time. In that case, it may incur a penalty of up to 2% of the installment value until the outstanding balance is settled. In some cases, leasing companies may agree to assign proceeds to ensure a stable cash flow. However, this guarantee may be terminated at any time based on the terms and conditions of the agreement. It is crucial for construction companies to carefully evaluate their financial situation and repayment capabilities before opting for early repayment to avoid incurring any additional costs.

6.1.8 Interest Rates and Loan Amount

Bank loans and leasing are two common financing options available to construction companies. The interest rates for bank loans typically range from 2% to 5% above the corridor rate, while leasing interest rates usually range from 3.5% to 5%. However, established and reputable construction companies may be eligible for lower interest rates.

Depending on the specific circumstances, construction companies may be able to obtain financing of up to 300 million Egyptian pounds. However, it is generally recommended that loans should not exceed 20% to 30% of the organization's annual revenue to avoid excessive debt burden. There is typically a 1% to 2% annual commitment charge if the company does not use the entire available credit facility for revolving cash flow.

When securing loans for construction projects or businesses, it is imperative to consider various factors. Besides maintaining a debt-to-equity ratio between 60 and 40 percent, evaluating the loan's duration and repayment schedule is crucial. Moreover, businesses should assess their capacity to make regular payments and avoid penalties and fees associated with late payments. Seeking guidance from financial advisors and lenders is essential to determine the most appropriate loan options for specific projects or businesses.

Due to limited cash flow, construction companies often do not employ structured repayment strategies, such as snowball or avalanche. Repayments are typically limited to small amounts at a time. Construction firms face one of the most significant obstacles when project payments are used as loan collateral. If the project proprietor pays late or not at all, the lender may still expect payment in installments. Continuous negotiations occur throughout the loan period, and the size of the business and its relationship with the lender often determine the outcome. In case of disputes, the parties may submit to arbitration; however, in most cases, they adhere to the contract's terms. Even if construction companies have the funds to repay their loans, some treasurers may choose to retain the funds for psychological reasons. Therefore, construction companies must maintain a positive rapport with their financiers to mitigate potential issues that may arise during the loan period. It is also crucial for construction companies to maintain a debt-to-equity ratio of approximately 60:40 and ensure that loans do not exceed 20-30% of annual revenue, which are essential best practices.

6.2 Market Validation

After discussing the potential benefits of using a loan repayment optimization model with finance executives, finance managers, and treasury managers, there was considerable interest in trying this mechanism to reduce total debt and improve cash flow. While some were hesitant to adopt the model for psychological reasons, such as a preference to keep cash on hand, the potential benefits of reducing debt costs and increasing cash flow were acknowledged.

Using such a model could also facilitate negotiations with creditors, whether banks or leasing companies. All finance executives confirmed that they do not currently use any techniques, such as debt snowball or debt avalanche, which provides an opportunity to utilize this model.

Implementing the loan repayment optimization model would involve a detailed analysis of the company's current loan portfolio, considering factors such as interest rates, repayment schedules, and total outstanding debt. The model would then suggest an optimized payment plan, considering cash flow projections, budget constraints, and other relevant financial considerations.

One of the main advantages of using such a model is that it can help construction companies to prioritize loan payments, reducing the total cost of debt and freeing up cash flow for other business activities. Additionally, the model can be adjusted as needed, allowing for ongoing optimization of loan repayment strategies.

6.3 Summary

Interviews with finance executives, finance managers, and treasury managers in the Egyptian construction industry provided valuable insights into the financing environment and practices. Important results from the interviews include:

1. Construction companies in Egypt have access to a variety of financing options, including bank loans, leasing agreements, and project financing with Letters of Guarantee (LG). EFG Hermes, Global Lease, Ghabour, and Finlease are examples of leasing companies

that offer factoring, direct lease, sales leaseback, and supply chain financing.

2. Effective cash flow management is essential for construction companies to avoid financial difficulties that could delay project completion. The treasury department plays a crucial role in determining optimal financing options and evaluating financial needs.

3. Short-Term and Medium-Term Loans: Short-term loans between 15 and 20 million EGP can be used to finance salaries, government payments, and indirect costs. Rather than construction companies, industrial companies typically have access to 2- to 5-year bank loans for medium-term financing.

4. Lease Agreements: Leasing companies provide construction companies with direct lease and sales leaseback options, allowing them to acquire assets without the use of conventional debt financing. Typical lease terms range from three to seven years, and additional services such as factoring and reverse factoring are also available.

5. Finance and treasury departments are responsible for managing debt and assessing financing alternatives. They play a crucial role in sustaining the company's financial stability and ensuring that the selected debt form is suitable.

6. Loan Evaluation and Terms Financial institutions evaluate loan requests based on a project's scope, duration, potential risk, and the company's creditworthiness as determined by its I-Score. Companies in the construction industry should maintain a high I-Score by maintaining accurate financial records and meeting payment obligations.

7. Early Repayment and Proceeds Assignment If the total amount is not paid on time, early loan repayment may incur administration fees or penalties. Leasing companies may accept an assignment of proceeds to ensure a constant cash flow, but this assurance is terminable per the agreement.

8. Interest Rates and Loan Amount: Interest rates on bank loans and leasing options vary based on the corridor rate and the company's reputation. To avoid excessive debt burden, loan quantities should generally not exceed 20% to 30% of annual revenue.

9. Loan Repayment Strategies and Market Validation Typically, construction companies do not utilize structured repayment strategies, and negotiations with creditors continue throughout the loan period. To reduce debt and enhance cash flow, the implementation of a loan repayment optimization model was of great interest. This model allows for the prioritization and modification of loan payments.

These interviews highlighted the significance of prudent financial management, the availability of financing options, the function of finance and treasury departments, and the potential advantages of optimizing loan repayment strategies in the Egyptian construction industry.

Table 4 provides a summary of important information regarding financing options, cash flow management, loan evaluation, early repayment, and interest rates in the construction industry of Egypt. It discusses various loan types (short-term, medium-term), lease agreements, the function of the finance and treasury departments, loan evaluation criteria, and the effect of early repayment. Table 4 attempts to provide a concise overview of these topics in order to facilitate comprehension of the Egyptian construction industry's financial landscape.

Table 4. Summary of Financing and Loan Management in Egypt's Construction Industry

Topic	Key Points
Financing Options	Bank loans: Up to 300 million EGP
	Leasing agreements: 3 to 7 years
Cash Flow Management	Short-term loans: 15-20 million EGP (repayable 3-4 times yearly)
Short-term and Medium-term Loans	Short-term loans: 6-24 months
Lease Agreements	Direct lease: Equipment purchase worth 20-30 million EGP with monthly installments
	Sales leaseback: Construction company sells existing assets, repurchases them in installments
Loan Evaluation and Terms	Administration fees for early payment: 3-5%
Early Repayment and Proceeds Assignment	Penalty for late payment: Up to 2%
Interest Rates and Loan Amount	Bank loan interest rates: 2-5% above the corridor rate
	Leasing interest rates: 3.5-5%

Chapter 7

The model Validation

In the previous sections, the optimization model was presented, showcasing its effectiveness in managing loan repayments for construction companies. This section now shifts focus to model validation, an essential step in assessing the accuracy and effectiveness of the model.

The case study provided an opportunity to evaluate the performance of the optimization model in various scenarios and with different dataset sizes. By applying the optimization model to the case study data and comparing its outcomes with conventional methods, the effectiveness of the model was assessed. Additionally, the model's performance was examined under different conditions, such as varying interest rates, loan terms, and cash flow projections, to ensure its robustness in practical applications.

The validation results confirm that the optimization model is highly effective in optimizing loan repayments and reducing debt costs for construction companies. By considering the company's unique financial situation and loan structure, the model identifies optimal repayment strategies, leading to significant reductions in debt costs and improvements in cash flow.
Moreover, the validation study results demonstrate that the optimization model can be applied to large-scale construction projects with complex loan structures and repayment schedules. The model handled the computational complexity of such projects, providing accurate and efficient solutions.

The validation study provides further evidence of the optimization model's efficacy in optimizing loan repayments for construction companies. The model can handle large-scale projects and complex loan structures, leading to a significant reduction in debt costs and an increase in cash flow. The validation study also highlights the importance of model validation in evaluating the accuracy and effectiveness of mathematical models and ensuring their suitability for real-world applications.

7.1 Scenarios

The study focuses on analyzing a specific subset of debts, chosen for their prevalence and distinct legal and financial attributes. Within the wide range of debt categories, the primary focus will be on two main types of debt, namely:

1- Short-term loans to pay for working capital or other expenses.
2- Medium-term lease for equipment

The current study was carried out by randomly generating scenarios using Mathematica. A total of 90 scenarios were generated, 30 with four debts and 24 months, 30 with eight debts and 48 months, and 30 with 12 debts and 72 months. Those scenarios were generated using a uniform set of parameters, as shown in Table 5. The parameters were selected through interviews with financial executives, mainly treasury managers, working for construction companies. This strategy ensured that the study accurately reflected the situation on the ground and was not predicated on speculative possibilities. Table 5 describes the many parameters that were employed in the investigation. The first three lines of the table represent the principle, yearly interest, and annual default interest in that order. The fourth line indicates the penalty when the debtor pays more than the required installment. This penalty is a critical component of any loan arrangement, and it was included in the study to account for its impact on the repayment schedule. This strategy facilitated the solution process using the methodology devised for the study. Finally, the table's final line gives the optimal loan maturity dates. This analysis component is crucial because it helps determine the repayment schedule and overall loan feasibility. It also emphasizes the significance of having explicit repayment terms and constraints in any loan agreement.

It is worth noting that corporate loans in Egypt are computed using the corridor rate. The corridor rate was 16% at the time of writing this research; thus, all interest rates were calculated using that figure. This information is critical for comprehending the context in which the study was carried out. This study was carried out using a solid approach that included factors based on real-world information. The study is relevant and applicable to Egyptian construction companies due to the inclusion of certain types of loans, fines, payback schedules, and interest rates based on the corridor rate.

Table 5. Intervals from which the scenario parameters were uniformly picked.

	STL Bank	MTL lease
Principle p_n million	(20,150)	(10,75)
Interest i_n^t (annual)	(17.5%,19%)	(19%,20.5%)
Default interest di_n^t (monthly)	(1%,2%)	(1%,2%)
Admin fees ai_n(on payment)	(3%,5%)	(3%,5%)
Final Month e_n	(6,24)	(12,48)

A simple amortization formula was used to calculate the designated installments M_n^t. To determine the monthly incomes r_n^t , I had to consider several factors to estimate the monthly installment payments the construction company would make. The estimate was based on a construction firm with a yearly revenue of one billion Egyptian pounds. It is widely agreed that the money utilized to pay back debt should not exceed 30% of annual revenue. In this scenario, the highest sum that may be given to debt payments was assessed to be 300 million Egyptian pounds, equating to 25 million monthly. However, I calculated the actual amount available using a random number generator to imitate a real-world scenario where the amount available for debt repayment is not always fixed. I chose 20 million Egyptian pounds as the starting point, multiplied by a value between -0.25 and 0.25. This method meant that the payment predictions were more realistic, as the availability of funds for debt repayment can fluctuate based on the company's financial status. I captured the unpredictability of the real world and provided more accurate estimations by including a random component. It is worth noting that the range of the random number generator was carefully chosen. The lower bound of -0.25 ensures that the amount available for debt repayment does not fall below a specific threshold. In contrast, the upper bound of 0.25 keeps the estimate from being overly optimistic.

7.2 Results

Table 6 presents the repayment policy of the Optimization model for multiple loans, with average results displayed for three different loan groups. The first row corresponds to the group of four loans with a repayment duration of 24 months. The second row corresponds to the group of eight loans with a repayment duration of 48 months, while the third row corresponds to the group of twelve loans with a repayment duration of 72 months.

Table 6. Average Results for the Three Different Group Scenarios

The total amount of debts	T in Months	Principle (Million EGP)	Total Amortization (million EGP)	Optimization model (Million EGP)	Percentage of saving
4	24	228.4	263.6672	252.3132	4.62%
8	48	328.5333	410.2962	345.8205	15.55%
12	72	444.6667	563.2075	461.764	17.63%

The average number of debts in millions of Egyptian pounds for each group is shown in the first column of the table. The second column displays the repayment duration in months for each group. The average principle for each group is shown in the third column, which is the total amount paid by the construction company. The fourth column displays the average payment to the bank using the standard amortization technique: the amount of money the construction company would be required to repay the bank if they used the Total Amortization option. The Optimization model's output is shown in the fifth column of the table, which is the amount of money the company would be required to repay the bank if they used the Optimization model. The sixth column displays the percentage of money saved using the Optimization model vs. the Total Amortization method.

According to Table 6, the Optimization model can significantly save construction companies. The Optimization model produces a payment of 252.3131667 million Egyptian pounds for the category with four loans and a repayment duration of 24 months, which is 4.62% less than the Total Amortization of 263.6671999 million Egyptian pounds. The Optimization model produces a payment of 345.8205 million Egyptian pounds for the category with eight loans and a repayment duration of 48 months, which is 15.55% less than the Total Amortization of 410.296232 million Egyptian pounds.

The Optimization model produces a payment of 461.764 million Egyptian pounds for the category with twelve loans and a repayment duration of 72 months, which is 17.63% less than the Total Amortization of 563.2074984 million Egyptian pounds.

Table 6 shows that the Optimization model's savings increase with the number of loans and the repayment length. According to Table 6, construction enterprises with twelve loans and a repayment period of 72 months saved the most (17.63%), followed by those with eight loans and a repayment period of 48 months (15.55%) and those with four loans and a repayment period of 24 months (4.62%). This trend can be explained by the Optimization model optimizes payment allocation across numerous loans to minimize overall interest paid. As a result, the bigger the potential savings, the more loans a construction company has, and the longer the repayment period.

Figure 8 depicts the results of applying the Optimization model to 4 loans with a repayment period of 24 months. Showing three different lines, first in blue is the principle of the loans, second in orange is the result of the Optimization model, and last in grey is the standard loan payment. The savings percentage ranges from 0.15% to 9.45%, with an average of 4.34%. While the savings may appear minor due to the lower number of loans and years of payment, they demonstrate the efficiency of the Optimization model approach in lowering loan payments.

Figure 11 Loan Repayment Scenarios for 4 Loans over 24 Months

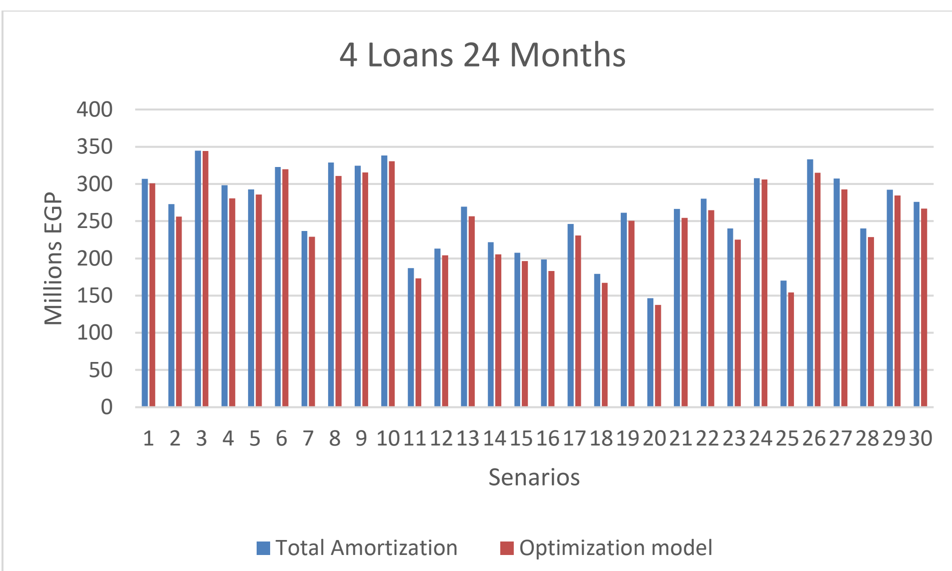


Figure 9 displays the results of applying the Optimization model to eight loans with a 48-month repayment duration. It can be observed that the proportion of savings fluctuates wildly, ranging from 7.46% to 21.56%. Some circumstances result in considerable savings over the Total Amortization, while others do not.

Figure 12   Loan Repayment Scenarios for 6 Loans over 48 Months

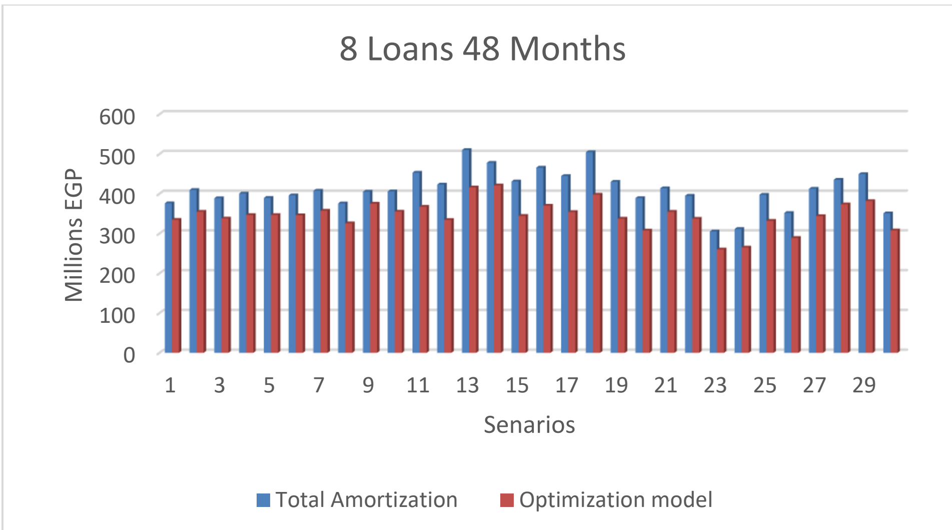
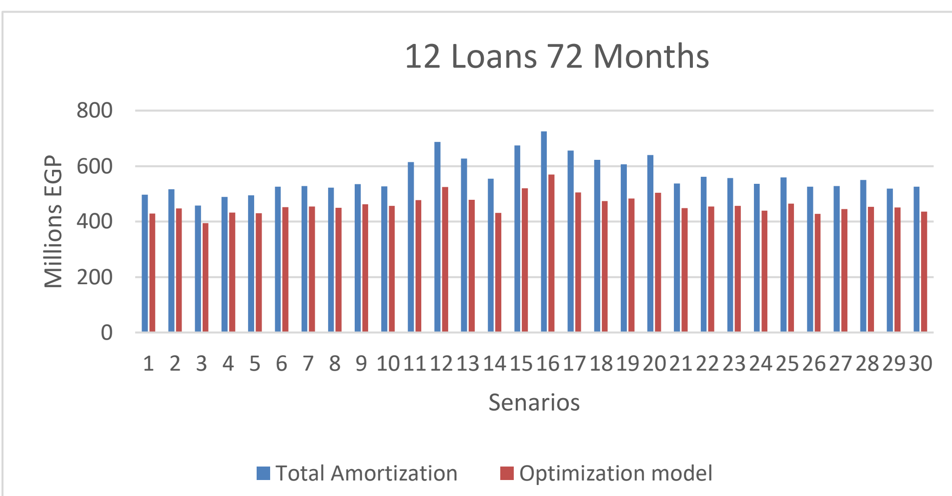


Figure 10 displays the results of applying the Optimization model to twelve loans with a 72-month repayment duration. The repayment plans with the highest saving percentage (over 20%) were achieved with more significant principal amounts and Optimization model repayment plans. The repayment plans with the lowest saving percentage (less than 14%) was achieved with smaller principal and standard repayment plans.

Figure 13.   Loan Repayment Scenarios for 12 Loans over 72 Months



Analyzing the results presented in Figures 8, 9, and 10, as well as Table 6, it is clear that there is a direct relationship between the number of loans, the number of years for repayment, and the amount of savings. The analysis shows explicitly that the more the number of loans and the number of years for payback, the greater the amount of savings that can be realized. This finding can be linked to more extended repayment periods and multiple debts, giving the company more repayment flexibility and resulting in lower overall borrowing costs and more considerable savings.

According to the findings in Table 7, there is a significant difference in savings between the two types of loans. The findings show that short-term bank loans result in an average savings of 9.51%, whereas medium-term leases result in an average savings of 15.99%. It is worth noting that holding both types of debt resulted in an average savings of 12.30%, comparable to the savings achieved by the two different loan types. The longer repayment duration of medium-term leases than bank loans accounts for higher savings. These findings support prior findings that a longer payback duration is associated with higher savings.

Table 7. Comparison of Loan Types in the Scenarios

Short term Bank loans	Medium-term lease	Principle	Total Amortization	Optimization model	Percentage of saving
available	0	342.4667	403.7629	362.046	9.51%
0	available	335.0333	435.6072	354.095	15.99%
available	available	324.1	397.8008	343.756	12.30%

Chapter 8

Conclusion and recommendations

The research aimed to implement on Mathematica a loan repayment optimization model for construction companies to improve their financial performance, reduce debt costs, and increase cash flow. A comprehensive literature review was conducted on loan repayment optimization models, financial performance indicators, and debt management strategies. Based on the literature review, a loan repayment optimization model for construction companies was implemented, considering their unique financial situation and cash flow projections.

Using a hypothetical case study, it was determined that the model effectively reduces debt costs and improves cash flow. Financial advisors and lenders were also interviewed to validate the model's viability and obtain market validation.

Principal contributions of the study include the implementation of a loan repayment optimization model tailored to the needs of construction companies and the identification of best practices for successful implementation, such as engaging with financial advisors and lenders, developing an optimized payment plan, monitoring performance, and maintaining positive relationships with creditors.

The findings of the research indicate that construction companies can substantially benefit from adopting a loan repayment optimization model to improve their financial performance, reduce debt costs, and increase cash flow. Successful model implementation requires careful consideration of the company's particular financial circumstance, regular monitoring, and ongoing optimization of repayment strategies. Recommendations for future research include investigating the model's efficacy in real-world construction company settings and determining the influence of external factors, such as economic conditions and industry trends, on loan repayment optimization.

Due to limited cash flow and complex repayment structures, managing loans and debt repayments is a major challenge for construction companies. However, adopting a loan repayment optimization strategy can help construction companies improve their financial performance, reduce debt costs, and increase cash flow.

To ensure the successful implementation of the model, it is crucial to engage with financial advisors and lenders to understand the benefits and potential risks of using such a mechanism. Financial advisors can provide valuable insights into the unique financial situation of each company and suggest the most suitable repayment strategies. They can also advise on the potential impact on the company's credit score and how the model may affect future borrowing opportunities.

It is essential to establish and maintain positive relationships with lenders. By working closely with lenders, construction companies can negotiate better loan terms and conditions and, in some cases, even restructure their repayment plans. The lender can also provide valuable feedback on the company's financial performance and assist in identifying potential repayment strategies.

The loan repayment optimization strategy provides construction companies with a unique opportunity to develop a repayment plan considering their unique financial situation and cash flow projections. The model identifies the best repayment strategy based on the company's financial position and loan structure. This can include prioritizing high-interest loans, consolidating debt, or restructuring the repayment plan.

Regularly monitoring loan repayment performance is crucial to ensure the model delivers the desired results. Construction companies should keep track of their loan payments, monitor their cash flow, and adjust the model as needed to optimize repayment strategies continually. This will help them meet their financial obligations on time, avoid late fees and penalties, and optimize their cash flow.

Maintaining positive relationships with creditors is also essential for construction companies.

Timely loan payments will help them avoid late fees and penalties and foster a positive relationship with their lenders. They should also constantly communicate with their creditors, regularly updating them on the company's financial performance and any challenges they may face.

Adopting a loan repayment optimization strategy can help construction companies improve their financial performance, reduce debt costs, and increase cash flow. However, successful implementation requires careful consideration of the company's unique financial situation, regular monitoring, and adjustment of the model to optimize repayment strategies continually. By adhering to best practices such as consulting with financial advisors and lenders, developing an optimized payment plan, monitoring performance, and maintaining positive relationships with creditors, construction companies can ensure that they meet their financial obligations, strengthen their financial position, and lay the groundwork for future growth and success.

In addition, it is essential to note that while the loan repayment optimization model is an effective tool, it is not a replacement for sound financial management practices. Construction companies should still adhere to best practices such as maintaining a cash reserve, managing expenses, and developing a sound budget to ensure long-term financial stability. The model should be used as a complement to these practices to optimize debt repayment and cash flow.

The proposed loan repayment optimization model offers a plausible solution to the problem of construction company repayment policies for multiple loans. However, additional research is necessary to enhance and expand the model's applications.

Consideration of the impact of external factors on loan repayment, such as economic conditions, interest rate fluctuations, and inflation, is one area for future research. These variables can substantially affect the company's financial performance and the efficacy of its repayment strategy. Incorporating these external variables into the model can assist construction companies in making informed decisions and adjusting their repayment strategy accordingly.

Investigating the effect of various loan structures on the repayment policy for multiple loans is a potential avenue for future research. The current model implies a set of fixed loans with fixed repayment terms. However, construction companies may have loans with variable repayment terms, such as variable interest rates. The optimal loan structure for construction companies can be determined by analyzing the effect of these various loan structures on the efficacy of the repayment strategy.

Lastly, future research can investigate the application of machine learning and artificial intelligence to the loan repayment optimization model to enhance it. These technologies can aid in predicting future cash flow and financial performance, allowing for more precise and well-informed loan repayment strategy decisions.

Future research may investigate the impact of external factors on loan repayment, the impact of various loan structures, and the use of machine learning and artificial intelligence to enhance the loan repayment optimization model. These research areas can provide valuable insights into optimizing loan repayment strategies for construction companies, resulting in enhanced financial performance and stability.

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Appendix 1

Questions for the Interview

Company: ________________________
Name: _______________________
Position: ______________________

1. Who is responsible and how do you manage the loan payments in your company?
2. What type of loans do your company usually use in financing its activities, projects, and working capital?
3. Do you set aside a certain monthly provision to repay your debts?
4. What is the total number of individual debts do you have?
5. What types of debts do you currently have (corporate, project, equipment installments ..etc.)?
6. What are the penalties for missing a payment installment, or paying less than the installment amount for each loan type?
7. What are the administration fees that is to be paid in case of paying more than the installment amount for each loan?
8. What is the maturity periods of your loans?
9. Do you usually have a different maturity period for each type of loan?
10. What are the sizes of the loans that you currently have?
11. Is there is a certain approach that you use to prioritize the payment of debts?
    a. If yes, what is it?
    b. If no, do you know about the different types of debt repayment methods?
12. How much is your total current debt paid per month?
13. What is your list of installments to be paid within the next 24 months?

Appendix 2

Validation

No.	Literature Review	Model	Deviation
1	409,257.54	407,776.00	0.36%
2	747,958.59	702,337.00	6.50%
3	1,149,248.77	1,059,660.00	8.45%
4	972,841.47	1,078,280.00	-9.78%
5	731,797.69	680,918.00	7.47%
6	849,485.18	801,665.00	5.97%
7	1,240,330.14	1,345,430.00	-7.81%
8	885,604.16	926,514.00	-4.42%
9	802,355.61	730,430.00	9.85%
10	833,835.76	828,385.00	0.66%
11	1,255,923.36	1,256,340.00	-0.03%
12	677,956.98	622,038.00	8.99%
13	3,019,334.17	2,815,300.00	7.25%
14	3,243,614.92	3,152,960.00	2.88%
15	2,607,559.49	2,689,620.00	-3.05%
16	1,203,299.56	1,195,250.00	0.67%
17	1,543,640.26	1,404,970.00	9.87%
18	1,155,144.46	1,152,750.00	0.21%
19	871,573.37	946,976.00	-7.96%
20	1,181,037.25	1,083,610.00	8.99%
21	1,460,581.93	1,415,110.00	3.21%
22	1,080,942.28	1,104,220.00	-2.11%
23	1,677,519.77	1,534,220.00	9.34%
24	1,457,654.99	1,460,910.00	-0.22%
25	1,222,266.24	1,251,720.00	-2.35%
26	1,689,523.20	1,541,700.00	9.59%
27	1,532,595.35	1,474,110.00	3.97%
28	1,859,102.18	1,809,740.00	2.73%
29	1,664,971.56	1,547,280.00	0.00%

Appendix 3

Model Results

1. Four loans and 24 Months

The total amount of debts	Short term Bank loans	Medium-term lease	T	Principle	Total Amortization	Optimization model	Percentage of saving
4	4	0	24	264	306.9415	301	1.94%
4	4	0	24	237	272.912	256.171	6.13%
4	4	0	24	297	344.8474	344.321	0.15%
4	4	0	24	259	298.2994	280.652	5.92%
4	4	0	24	251	292.6155	285.594	2.40%
4	4	0	24	284	322.6763	319.684	0.93%
4	4	0	24	206	236.5717	229.004	3.20%
4	4	0	24	286	328.6118	310.907	5.39%
4	4	0	24	279	324.342	315.498	2.73%
4	4	0	24	299	338.2294	330.643	2.24%
4	0	4	24	162	186.9048	172.991	7.44%
4	0	4	24	185	213.2318	204.224	4.22%
4	0	4	24	231	269.2341	256.571	4.70%
4	0	4	24	192	221.7321	205.49	7.33%
4	0	4	24	178	207.6109	196.142	5.52%
4	0	4	24	172	198.6738	183.211	7.78%
4	0	4	24	212	246.2941	230.727	6.32%
4	0	4	24	152	179.0603	167.222	6.61%
4	0	4	24	224	261.1614	250.401	4.12%
4	0	4	24	125	146.3902	137.295	6.21%
4	2	2	24	234	266.5846	254.4	4.57%
4	2	2	24	244	280.3538	264.791	5.55%
4	2	2	24	207	240.2845	225.363	6.21%
4	2	2	24	266	307.6483	305.891	0.57%
4	2	2	24	150	170.1081	154.035	9.45%
4	2	2	24	293	333.074	314.85	5.47%
4	2	2	24	268	307.3569	292.452	4.85%
4	2	2	24	208	240.2185	228.546	4.86%
4	2	2	24	251	292.2048	284.3	2.71%
4	2	2	24	236	275.8421	267.019	3.20%

2. Eight loans and 48 Months

The total amount of debts	Short term Bank loans	Medium-term lease	T	Principle	Total Amortization	Optimization model	Percentage of saving
8	8	0	48	326	376.1107	334.498	11.06%
8	8	0	48	342	409.5696	354.772	13.38%
8	8	0	48	329	388.7566	337.87	13.09%
8	8	0	48	335	400.5507	346.628	13.46%
8	8	0	48	325	389.7373	346.638	11.06%
8	8	0	48	338	396.0621	345.939	12.66%
8	8	0	48	348	407.8283	357.344	12.38%
8	8	0	48	315	375.9106	325.694	13.36%
8	8	0	48	347	405.3367	375.116	7.46%
8	8	0	48	347	405.8823	355.112	12.51%
8	0	8	48	336	453.1454	367.314	18.94%
8	0	8	48	321	423.1855	334.146	21.04%
8	0	8	48	378	510.0002	416.056	18.42%
8	0	8	48	369	477.8538	421.055	11.89%
8	0	8	48	326	431.1824	344.37	20.13%
8	0	8	48	350	465.6697	370.198	20.50%
8	0	8	48	342	444.6087	354.137	20.35%
8	0	8	48	383	504.6471	397.896	21.15%
8	0	8	48	327	430.1782	337.428	21.56%
8	0	8	48	298	389.199	307.694	20.94%
8	4	4	48	344	413.9357	354.634	14.33%
8	4	4	48	316	395.0759	337.326	14.62%
8	4	4	48	249	305.3319	260.102	14.81%
8	4	4	48	257	311.4457	264.864	14.96%
8	4	4	48	322	397.6235	331.971	16.51%
8	4	4	48	274	351.7903	288.928	17.87%
8	4	4	48	332	412.6109	343.868	16.66%
8	4	4	48	344	435.1774	373.443	14.19%
8	4	4	48	348	449.5562	381.708	15.09%
8	4	4	48	288	350.9245	307.866	12.27%

3. Twelve loans and 72 Months

The total amount of debts	Short term Bank loans	Medium-term lease	T	Principle	Total Amortization	Optimization model	Percentage of saving
12	12	0	72	416	496.5966	428.922	13.63%
12	12	0	72	435	516.052	447.426	13.30%
12	12	0	72	382	457.9106	394.619	13.82%
12	12	0	72	409	488.3575	432.896	11.36%
12	12	0	72	413	494.0121	429.606	13.04%
12	12	0	72	439	525.8276	451.428	14.15%
12	12	0	72	440	528.3172	453.932	14.08%
12	12	0	72	437	522.3209	449.842	13.88%
12	12	0	72	447	534.8293	462.601	13.50%
12	12	0	72	442	526.8719	457.031	13.26%
12	0	12	72	461	614.2394	477.506	22.26%
12	0	12	72	504	686.617	524.641	23.59%
12	0	12	72	460	627.3369	478.4	23.74%
12	0	12	72	418	554.3956	431.694	22.13%
12	0	12	72	502	674.5725	519.811	22.94%
12	0	12	72	550	724.823	570.004	21.36%
12	0	12	72	487	656.3933	505.238	23.03%
12	0	12	72	456	622.6854	474.041	23.87%
12	0	12	72	469	606.9334	483.533	20.33%
12	0	12	72	481	640.2561	503.419	21.37%
12	6	6	72	433	536.7761	448.291	16.48%
12	6	6	72	438	561.1025	454.489	19.00%
12	6	6	72	439	557.147	457.083	17.96%
12	6	6	72	426	535.9572	439.77	17.95%
12	6	6	72	441	559.0723	464.479	16.92%
12	6	6	72	412	525.1688	427.646	18.57%
12	6	6	72	431	528.2057	445.091	15.74%
12	6	6	72	438	549.6316	453.312	17.52%
12	6	6	72	423	518.5247	450.705	13.08%
12	6	6	72	411	525.2908	435.464	17.10%

Appendix 4

Sample Scenarios from the Excel Sheet for Loan Repayment Optimization in the Construction Industry

	No.	1	2	3	4	5	6	7	8	9	10
Scenarios	Total amount of debts	4	4	4	4	4	4	4	4	4	4
	Short term Bank loans	2	2	2	2	2	2	2	2	2	2
	Medium term lease	2	2	2	2	2	2	2	2	2	2
	T (Months)	24	24	24	24	24	24	24	24	24	24
1	Vector (n) with the principal of all the loans	99	126	48	58	95	159	20	91	56	33
		24	33	85	172	27	25	161	62	159	157
		71	16	51	24	11	51	38	33	25	19
		40	69	23	12	17	58	49	22	11	27
2	Matrix of (n x T) with the interest rates	0.015	0.015	0.015	0.015	0.015	0.015	0.016	0.015	0.016	0.016
		0.015	0.015	0.015	0.015	0.016	0.015	0.015	0.015	0.016	0.015
		0.016	0.016	0.016	0.016	0.017	0.016	0.016	0.017	0.016	0.016
		0.017	0.017	0.016	0.016	0.016	0.017	0.016	0.016	0.016	0.017
3	Matrix of (n x T) with the default interest rates	0.013	0.016	0.019	0.015	0.012	0.014	0.012	0.017	0.015	0.019
		0.011	0.019	0.019	0.020	0.014	0.012	0.012	0.013	0.014	0.018
		0.019	0.017	0.018	0.015	0.018	0.020	0.017	0.016	0.019	0.013
		0.018	0.019	0.012	0.016	0.018	0.019	0.016	0.015	0.014	0.014
4	Vector (n) with the percentage of the loan used to compute the minimum payment	0.071	0.053	0.063	0.063	0.071	0.071	0.048	0.059	0.048	0.050
		0.059	0.048	0.048	0.050	0.056	0.048	0.059	0.050	0.053	0.048
		0.053	0.071	0.053	0.059	0.067	0.048	0.071	0.056	0.067	0.059
		0.071	0.071	0.071	0.050	0.048	0.071	0.056	0.056	0.067	0.063
5	Vector (n) with the penalization for paying more than the stipulated	0.905	0.750	0.379	0.463	0.712	1.280	0.137	0.723	0.365	0.217
		0.195	0.182	0.486	1.031	0.196	0.169	1.003	0.391	1.115	0.804
		0.434	0.167	0.369	0.189	0.105	0.357	0.375	0.269	0.159	0.131
		0.376	0.669	0.241	0.084	0.114	0.490	0.285	0.170	0.077	0.201
6	Vector (T) with the amount of money that the loaner has per month for paying his debts	Generated randomly using Mathematica when creating the Scenarios									
7	Vector (n) with the realize date of the debts	10	5	8	8	10	10	3	7	3	4
		7	3	3	4	6	3	7	4	5	3
		5	10	5	7	9	3	10	6	9	7
		10	10	10	4	3	10	6	6	9	8
8	Vector (n) with the stipulated amount for each debt	7.914	7.665	3.410	4.099	7.588	12.675	1.125	6.125	3.150	1.938
		1.615	1.843	4.769	10.016	1.731	1.401	10.836	3.607	9.746	8.775
		4.372	1.284	3.144	1.626	0.834	2.884	3.052	2.137	1.894	1.284
		3.233	5.574	1.846	0.706	0.962	4.687	3.154	1.416	0.831	1.937

Appendix 5

Optimization Model on Mathematica

```
obj = Total[x, 2];

con4 = ∑_{t'=1}^{n} Indexed[x, {t', Sort[rj][[1]]}] + Indexed[s, 1] ≤ ft[[1]] ;

con5 = Table[∑_{t'=1}^{n} Indexed[x, {t', Sort[rj][[1]] + j - 1}] + Indexed[s, j] ≤ ft[[j]] + Indexed[s, j - 1], {j, 2, t - Sort[rj][[1]]}] // Flatten;

con6 = Table[∑_{t'=rj[[i]]}^{t} (Indexed[c, {i, t'}] + Indexed[p, {i, t'}] - Indexed[x, {i, t'}]) ∏_{t''=t'+1}^{t} (1 + Indexed[itj, {i, t''}]) + mj[[i]] ∏_{t'=rj[[i]]}^{t} (1 + Indexed[itj, {i, t'}]) == 0, {i, 1, n}] // Flatten;

con7 = Flatten[Table[Indexed[b, {i, rj[[i]]}] == mj[[i]], {i, 1, n}]];

con8 = Table[(∑_{t'=rj[[i]]}^{j-1} (Indexed[c, {i, t'}] + Indexed[p, {i, t'}] - Indexed[x, {i, t'}]) ∏_{t''=t'+1}^{j} (1 + Indexed[itj, {i, t''}]) + mj[[i]] ∏_{t'=rj[[i]]+1}^{j} (1 + Indexed[itj, {i, t'}])) == Indexed[b, {i, j}], {i, 1, n}, {j, rj[[i]] + 1, t}] // Flatten;
con9 = Table[Indexed[b, {i, j - 1}] * (1 + itj[[i, j]]) ≤ 10^50 * Indexed[z, {i, j}], {i, 1, n}, {j, rj[[i]] + 1, t}] // Flatten;
con10 = Table[(Indexed[z, {i, j}] * ej[[i]] - Indexed[x, {i, j}]) * (1 + htj[[i, j]]) ≤ Indexed[c, {i, j}], {i, 1, n}, {j, rj[[i]] + 1, t}] // Flatten;
con11 = Table[((pmj[[i]] * Indexed[b, {i, j}] * (1 + itj[[i, j]])) - Indexed[x, {i, j}]) * (1 + htj[[i, j]]) ≤ Indexed[c, {i, j}], {i, 1, n}, {j, rj[[i]] + 1, t}] // Flatten
con12 = Table[(Indexed[x, {i, j}] - Indexed[z, {i, j}] * ej[[i]]) ≤ 10^50 * Indexed[y, {i, j}], {i, 1, n}, {j, rj[[i]] + 1, t}] // Flatten;
con13 = Table[Indexed[p, {i, j}] == Indexed[y, {i, j}] * pcj[[i]], {i, 1, n}, {j, rj[[i]] + 1, t}] // Flatten;
con14 = Flatten[Table[Indexed[x, {i, j}] == Indexed[c, {i, j}] == Indexed[p, {i, j}] == 0, {i, 1, n}, {j, 1, rj[[i]] - 1}]];
con15 = {x ≥ 0, c ≥ 0, s ≥ 0, p ≥ 0, b ≥ 0};
bin = {0 ≤ z ≤ 1, 0 ≤ y ≤ 1};


Solution = LinearOptimization[obj, {con4, con5, con6, con7, con8, con9, con10, con11, con12, con13, con14, con15, bin},
  {x ∈ Matrices[{n, t}, NonNegativeReals], s ∈ Vectors[t, NonNegativeReals], c ∈ Matrices[{n, t}, NonNegativeReals], p ∈ Matrices[{n, t}, NonNegativeReals],
   b ∈ Matrices[{n, t}, NonNegativeReals], z ∈ Matrices[{n, t}, Integers], y ∈ Matrices[{n, t}, Integers]}, {"PrimalMinimumValue", "PrimalMinimizerRules"},
  MaxIterations → Infinity, Tolerance → 10^(-20), Method → "Gurobi"]
```