American University in Cairo
AUC Knowledge Fountain

Theses and Dissertations | Student Research

2-1-2017

Improved fault-tolerant PMU placement using algebraic connectivity of graphs

Mahmoud El Hosainy

Follow this and additional works at: https://fount.aucegypt.edu/etds

Recommended Citation

APA Citation
El Hosainy, M. (2017).*Improved fault-tolerant PMU placement using algebraic connectivity of graphs* [Master's Thesis, the American University in Cairo]. AUC Knowledge Fountain.
https://fount.aucegypt.edu/etds/1372

MLA Citation
El Hosainy, Mahmoud. *Improved fault-tolerant PMU placement using algebraic connectivity of graphs*. 2017. American University in Cairo, Master's Thesis. *AUC Knowledge Fountain*.
https://fount.aucegypt.edu/etds/1372

This Master's Thesis is brought to you for free and open access by the Student Research at AUC Knowledge Fountain. It has been accepted for inclusion in Theses and Dissertations by an authorized administrator of AUC Knowledge Fountain. For more information, please contact thesisadmin@aucegypt.edu.

THE AMERICAN UNIVERSITY IN CAIRO

School of Sciences and Engineering

Improved Fault-Tolerant PMU Placement using Algebraic Connectivity of Graphs

A thesis submitted in partial fulfillment of the requirements for the Degree of the
Master of Science
in
Electronics and Communications Engineering
ECNG Department, School of Sciences and Engineering

By:

Mahmoud Mostafa El Hosainy

Under the supervision of:

Prof. Karim Seddik

Prof. Ayman Elezabi

September 2017

Cairo, Egypt

The American University in Cairo

Department of Electronics and Communications Engineering (ECNG), School of Sciences and Engineering (SSE)

Improved Fault-Tolerant PMU Placement using Algebraic Connectivity of Graphs

A thesis submitted by:

Mahmoud Mostafa El Hosainy

In partial fulfillment of the requirements for the degree of

Master of Science in Electronics and Communications Engineering

has been approved by :

Prof. Karim Seddik	**Prof. Ayman Elezabi**
Thesis Supervisor	Thesis Co-Supervisor

Acknowledgements

First and foremost, I feel always indebted to Allah, the Most Gracious, Most Merciful. I wish to thank my thesis advisors Professor Karim Seddik and Professor Ayman Elezabi for their support, their valuable suggestions saving no effort or time during the whole work, and their continuous guidance throughout the research work without which this work could not have been accomplished.

I wish to express my sincere gratitude to my parents, Fatma Fadel and Mostafa El Hosainy, and my sister Maryam for their unconditional trust, never ending support, endless patience, and their continuous encouragement that pushed me forward to accomplish this work. They helped me get through sometimes agonizing periods in the most positive way. They were my primary inspiration and motivation needed to complete this dissertation.

To my lovely wife Zaynab..... You are my life....

The American University in Cairo

Abstract

School of Sciences and Engineering

Electronics and Communications Engineering Department

Master of Science

Improved Fault-Tolerant PMU Placement using Algebraic Connectivity of Graphs

By Mahmoud El Hosainy

Due to perpetual and innovative technological advancements, the need for reliable and stable power generation and transmission has been increasing dramatically over the years. Smart grids use advanced technologies to provide self-monitoring, self-checking and self-healing power networks, including smart metering devices capable of providing accurate measurements of the network's power components. Among the most important metering devices in this context are "Phasor Measurement Units (PMUs)". PMUs are metering devices that provide synchronized measurements of voltage, current and phase angle differences using signals from the GPS satellites. However, due to the high cost of such advanced metering devices, studies were performed to determine the minimum number of PMUs required and their strategic placements in the power networks to provide full system observability.

In this thesis, we consider fault-tolerant PMU placement aiming to minimize the number of PMUs while maintaining system observability under various contingencies. Conventionally, the optimal number of PMUs in a system is determined based on the system's connectivity matrix under no contingency. This thesis considers fault- tolerant PMU placement under single and double branch failures. We propose algebraic connectivity, or Fiedler value, to identify the worst- case branch failures in terms of connectivity degradation. The proposed PMU placement accounts for this worst-case and covers a large percentage of other single and double branch failures. Furthermore,

we propose the usage of Fiedler vector to provide a PMU placement that would ensure that the system remains fully observable during system partitioning into separate sub-systems.

The resulting placements are compared with those obtained without considering connectivity degradation or system partitioning in terms of the percentages of observable systems during any single and double branch failures. The proposed PMU placements have increased percentages of fully observable systems in the event of any single or double branch failures compared to non—contingency based placement, with a reasonable increase in number of PMUs, and for some placement approaches no increase in PMUs is needed for providing a higher percentage of fully observable systems.

Contents

Acknowledgments ..... **III**

Abstract ..... **IV**

Contents ..... **VI**

List of figures ..... **IX**

List of tables ..... **XI**

Abbreviations ..... **XII**

1. Introduction **1**

1.1. Historical overview **1**

1.2. Phasor measurements **3**

1.3. Example of PMU implementation **5**

1.3.1. State estimation **5**

1.3.2. Static state estimation **6**

1.3.3. Fault detections **6**

1.3.4. Wide Area Monitoring System (WAMS) **6**

1.4. Thesis objectives and description **7**

2. Literature Review **10**

2.1. Rules of network observability **10**

2.2. Optimal PMU Placement problem formulation **11**

2.3. Heuristic Method **13**

2.3.1. Depth First Search (DFS) **13**

2.3.2. Domination set **14**

2.3.3. Greedy algorithm **15**

2.4. Meta-Heuristic Methods **16**

2.4.1. Genetic Algorithm **16**

2.4.2. Particle Swarm Optimization (PSO) **16**

2.5. Deterministic Methods…….. 17

2.5.1. Integer Linear Programming..…….. 17

2.5.2. Binary search ……. 18

3. PMU placement with contingencies…….. 20

3.1 Effect of Zero-injection buses …… 20

3.2 No PMU at zero-injection buses …… 22

3.3 Loss of measurement contingency ……. 23

3.4 Branch failure contingency …… 24

3.5 Measurement limitations …… 26

4. PMU Placement using Algebraic Connectivity of Graphs……. 29

4.1 Algebraic connectivity of graphs…… 29

4.2 Proposed PMU Placement formulation……..... 31

4.3 Improved PMU Placement using different approaches ……... 33

4.3.1. Approach 1 - PMU Placement to overcome single branch failure corresponding to the lowest Fiedler value……... 34

4.3.2 Approach 2 - PMU Placement to overcome single branch failure corresponding to the second lowest Fiedler value …….. 38

4.3.3 Approach 3 - PMU placement to overcome double branch failure corresponding to the lowest Fiedler value and the second lowest Fiedler value……..... 42

4.3.4 Approach 4 - PMU placement to overcome double branch failure corresponding to the lowest Fiedler value……... 46

4.3.5 Approach 5 - PMU placement to overcome system partitioning corresponding to Fiedler vector …….. 50

4.3.6 Approach 6 - PMU placement to overcome single branch failure corresponding to the lowest Fiedler value coupled with zero injection effect..... 55

5. Conclusion and future work …….. 58

6. Bibliography ……..... 60

List of figures

Figure 1.1	Sinusoidal waveform and its phasor representation
Figure 1.2	Phasor measurement unit and its function block diagram
Figure 1.3	Signals received by PMUs
Figure 2.1	The first observability rule
Figure 2.2	The third observability rule
Figure 2.3	Flowchart of depth first algorithm
Figure 2.4	6-bus system with domination set
Figure 2.5	Greedy algorithm
Figure 2.6	Binary search algorithm to determine the minimum number of PMUs required to make the system observable
Figure 3.1	9-bus system with PMU placement indicating zero injection effect
Figure 3.2	9-bus system with no PMU placement at zero injection buses
Figure 3.3	9-bus system optimal PMU placement guarding against single loss of measurements.
Figure 3.4	9-bus system optimal PMU placement guarding against single branch failure
Figure 3.5	9-bus system optimal PMU placement considering measurement limitations
Figure 4.1	Graph with algebraic connectivity (A) 0.9249; (B) 0.1531
Figure 4.2	IEEE 7-bus system
Figure 4.3	Flowchart of the algorithm used for Approach 1
Figure 4.4	Performance of PMU Placement Approach 1 for (A) All single branch failures and (B) All double branch failure combinations
Figure 4.5	Flowchart of the algorithm used for Approach 2
Figure 4.6	Performance of PMU Placement Approach 2 for (A) All single branch failures and (B) All double branch failure combinations
Figure 4.7	Flowchart of the algorithm used for Approach 3

Figure 4.8	Performance of PMU Placement Approach 3 for (A) All single branch failures and (B) All double branch failure combinations
Figure 4.9	Flowchart of the algorithm used for Approach 4
Figure 4.10	Performance of PMU Placement Approach 4 for (A) All single branch failures and (B) All double branch failure combinations
Figure 4.11	IEEE 7-bus system partitioned to two sub-systems
Figure 4.12	Flowchart of the algorithm used for Approach 5
Figure 4.13	Performance of PMU Placement Approach 5 for (A) All single branch failures, and (B) All double branch failure combinations.
Figure 4.14	Performance of PMU Placement Approach 6 in case of single and double branch failures

List of tables

Table 2.1	Examples of optimal PMU placement methods and algorithms
Table 2.2	Connectivity of 6-bus system
Table 3.1	PMU placement buses for the IEEE standard test systems
Table 4.1	Optimal PMU Placement under normal conditions
Table 4.2	Improved PMU Placement covering single branch failure corresponding to the lowest Fiedler value
Table 4.3	Number of PMUs required to maintain full observability in case of single branch failure as per Approach 1
Table 4.4	Improved PMU Placement covering single branch failure corresponding to the second lowest Fiedler value
Table 4.5	Number of PMUs required to maintain full observability in case of single branch failure as per Approach 2
Table 4.6	Improved PMU placement covering double branch failure corresponding to the lowest Fiedler value and the second lowest Fiedler value
Table 4.7	Number of PMUs required to maintain full observability in case of single branch failure as per Approach 3
Table 4.8	Improved PMU Placement covering double branch failure corresponding to the lowest Fiedler value
Table 4.9	Number of PMUs required to maintain full observability in case of single branch failure as per Approach 4
Table 4.10	Improved PMU Placement in case of system partitioning
Table 4.11	Number of PMUs required to maintain full observability in case system partitioning as per Approach 5
Table 4.12	Optimal PMU Placement with and without zero injection effect
Table 4.13	Improved PMU placement to overcome single branch failure corresponding to the lowest Fiedler value with and without Zero injection Effect

Abbreviations

AI	Artificial Intelligence
A/D	Analogue to digital converter
CT	Current Transformer
DFS	Depth First Search
DFT	Discrete Fourier Transform
GA	Genetic Algorithm
GPS	Global positioning system
IEEE	Institute of Electrical and Electronics Engineers
ILP	Integer Linear Programming
KCL	Kirchhoff's Current Law
OPP	Optimal PMU Placement
PDC	Phasor Data Concentrator
PMU	Phasor Measurement Unit
PSO	Particle Swarm Optimization
PT	Potential Transformer
RTU	Remote terminal units
SCADA	Supervisory control and data acquisition
WAMS	Wide Area Monitoring System

Chapter 1

Introduction

Power failure is a serious problem facing many countries around the world in 21st century. This causes financial losses for power companies estimated to be billions of dollars, as well as inconvenience to private and business customers. This maybe due to the low investment in the infrastructure and maintenance of power systems. Also, power transmission and distribution system become overloaded by the continuous increase in electricity consumption.

In order to make the power systems more reliable, stable, and controllable; state estimation of the transmission network is necessary [1]. The Phasor Measurement Unit (PMU) is a device responsible for the detection of voltage and current waveforms that are synchronized with a clocking signal obtained continuously from the global positioning system (GPS) satellite. Integrated with the GPS receiver [2], the base station housing the Phasor Data Concentrator (PDC), which is responsible for analyzing the PMU data, is able to receive the synchronous data from each PMU in real time

1.1 Historical Overview

Traditionally, power networks were monitored via the measurement of the phase angle between the voltage phases and current phases [3]. This is based on the fact that the sine of the angle difference between voltages at the two terminals of a distribution line is indicative of the power flow in this line.

Following huge losses in the Northeast blackout [4], in addition to the conventional method [5], supervisory control and data acquisition (SCADA) system were used to measure phasors by means of remote terminal units (RTUs). Primarily used in the industrial sector, SCADA system uses programmable logic controllers (PLCs) and RTUs as microcomputers capable of gathering information from a variety of information gathering technologies such as end devices, sensors, factory machinery,…etc. and then transmit the gathered data to central computers in the

network's backbone that have SCADA system software installed. The SCADA software in turn sorts, processes and displays the collected data using the programmed user interface to help operators interpret and analyze the data to make important decisions. Following the same principle used in industrial applications but on a larger scale, SCADA system was then used to collect data from power grids relating to the voltages, currents, frequencies and power flow through the system at different locations in the network; and sends the data to centralized locations where the operators can monitor the performance of the power grid. Thus, problems related to the operation of the power network are identifiable. However, these calculations are elaborate and cannot be performed in real-time. Thus, a new concept was introduced to face this problem, called static state estimation (will be discussed in section 1.3.2).

In the 1980s, a communication channel was implied in the phase angle measurement equipment to enable the synchronization of the reference signals. This channel was based on LORAN-C (a navigation system with the receiver able to determine its position through low frequency radio signals), The Geostationary Operational Environmental Satellite (GOES) satellite transmissions and radio transmissions of low frequency time signal utilized mainly by the Swiss time reference system (HBG) [4]. However, this communication channel faced the problem of zero crossing of the phase voltage, thus researchers measured local phase angle with respect to the time reference (accurate only in the order of 40 microseconds). This signifies that these devices cannot be suitable for monitoring power networks in time synchronization.

In 1990s, GPS was implied in the novel phase angle measurement equipment "PMU" [6]. Since transmission of high frequency signals to the control stations is provided through the presence of 24 satellites orbit at a height of twelve thousand miles from the earth, continuous and synchronized monitoring of the state of power systems is available. This overcomes the communication channel's problem having high precision time, ranging from 1 to 10 nanoseconds [7]. Also, GPS receiver supplies a unique pulse signal in one-second intervals. Furthermore, devices are less likely to be influenced by weather conditions and/or geography. Thus, installing GPS receivers into various devices can allow for real-time synchronization of power status and fault analysis [8].

1.2 Phasor Measurements

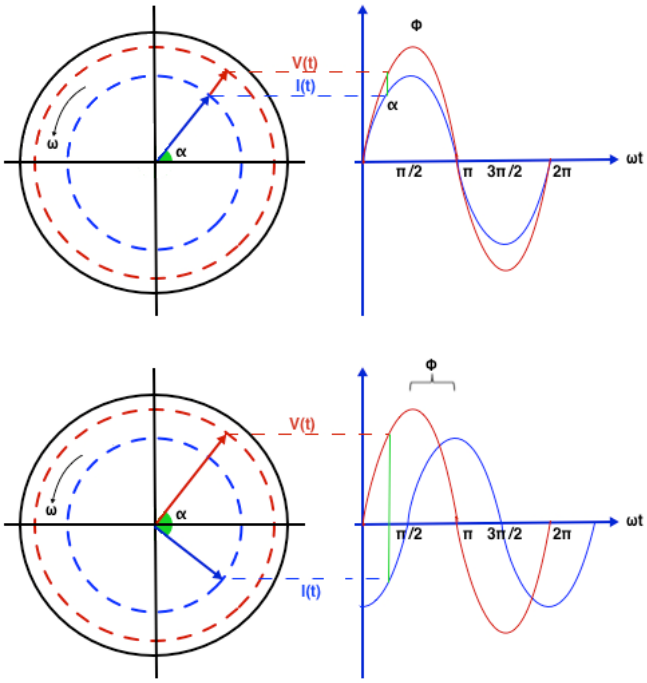


Figure 1.1 Sinusoidal waveform and its phasor representation

A phasor represents a sinusoidally varying quantity (current or voltage) by means of a line rotating about a point in a plane. The length of this line represents the magnitude of this quantity, and the angle between this line and a reference line represents the phase of this quantity. In Figure 1.1, the distance between two sinusoidal peaks of the signals is defined as the phase angle (Φ). Also, it can be converted to an angular measurement in the phasor representation.

The PMU receives the waveforms of the current and voltage from standard Current Transformer (CT) and Potential Transformer (PT) respectively (Figure 1.2). These analogue signals are isolated, filtered and sampled by means of anti-aliasing filters. These analogue signals are then transformed into digital output by mean of Analogue/Digital (A/D) converter. By means of GPS receiver clocks and phase locked oscillator, these signals are synchronized from different locations, covering all the power network. Then, the phasor microprocessor calculates from these sampled data the local positive sequence, fundamental frequency, voltage, and current phasors by the recursive Discrete Fourier Transform (DFT) algorithm. Finally, synchronized phasors are exported via the standard communications ports or modems.



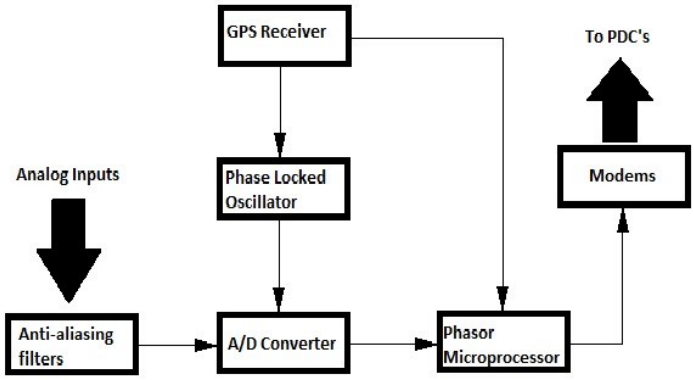


Figure 1.2 Phasor Measurement Unit [9] and its function block diagram [10]

Thus, by operating PMU program, PMU allows for real time monitoring of the status of the power network providing information of pre-fault or post-fault conditions. For example, it can be inferred from Figure 1.3 that a signal decay on the transmission line is responsible for the difference between Signal #1 and Signal #2 amplitudes. In Figure 1.3, the distance between the sinusoidal peak of the signal and the time reference (Y-axis) is defined as the phase angle (δ).

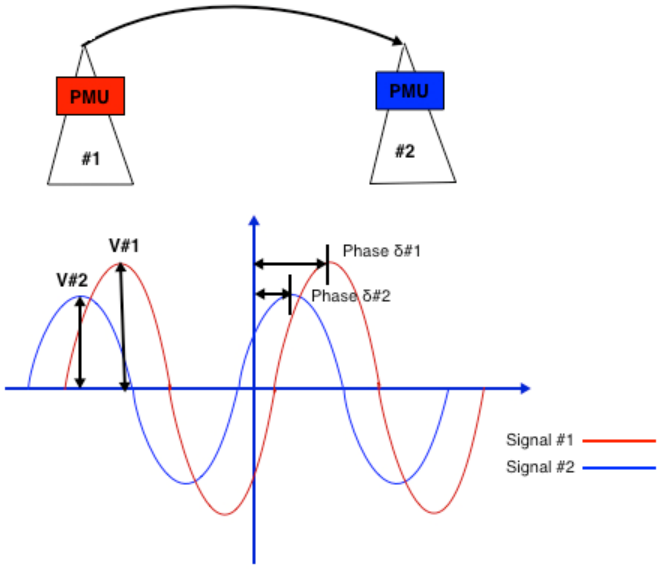


Figure 1.3 Signals received by PMUs

1.3 Example of PMU Usages

Thanks to the integration of phasor technology and the PMU device into electric power networks, power networks have become more controllable, stable, and reliable. In this section, we will focus on the advantages of integrating PMUs in power networks.

1.3.1 State Estimation

State estimation of power network functions is based on the scheduling generation and interchange; monitoring outages and scheduling alternatives; supervising scheduled outages; scheduling frequency and time corrections; coordinating bias settings; and emergency restoration of system [4]. This can be achieved either by state estimation algorithms [11], or by means of PMUs with extreme precision, time synchronization, and excellent performance. Measuring state estimation is achieved through complex bus voltages [12] that enable the estimation of bus voltage magnitudes and angles by using line flow measurement (both real and reactive power).

Thus, using the bad data processing technique, detection, identification, and correction of measurement errors is possible [13, 14] either as part of the state estimation process or as a post-estimation procedure. However, this necessitates a well-designed measurement system in order to detect errors in both redundant and critical measurements. Errors in redundant measurements can be statistically detected,

however, errors in critical measurements cannot be detected (since the system becomes unobservable by removing this measurement from the measurement system). Thus, any critical measurement must be observed by multiple measurement devices to be able to detect any error by bad data processing. This can be achieved by adding extra PMUs to the conventional measurements, hence, all bad data in the system become detectable [15].

1.3.2 Static state estimation

Static state estimation is a prediction method of the future behavior of a power network (e.g. one minute period) based on its present condition. This can be achieved by calculating the static and dynamic behaviors from the information gathered by PMUs. This allows the network operators to take the necessary measurements to deal with the problem before it happens. For example, if there is no change in the system during the monitoring period; the operator recognizes that there is a possibility of losing the static stability.

1.3.3 Fault Detection

Faults occurring in the overhead transmission line can be either permanent or temporary fault [16, 17]. Permanent fault may occur following a break in transmission line or a power generator malfunction, this makes a huge difference in signal during the pre-fault and post-fault moment. This can be easily located by detection devices.
While the temporary fault may occur following insulator flashover, this may lead to full insulator breakdown when occurring frequently. This emphasizes the importance of PMU-based fault location technique [18-20]. This is achieved through monitoring the synchronized fault voltages, calculating the line currents between these nodes, and forming node injection currents at two terminals of the faulted line. Thus, calculation of fault nodes or fault locations can be inferred.

1.3.4 Wide Area Monitoring System (WAMS)

Nowadays, WAMS is considered the most advanced method to detect and avoid pervasive blackout. It aims to maintain the dynamic stability in the overhead

transmission line network, based on the PMU [21]. This is implemented by synchronizing and recording the acquired data from systems in distributed locations through new computing and communication technology. Upon their delivery to the central control station, these data are measured and analyzed from any point of the power network.

In addition to its ability to monitor the static stability of the network (as traditional SCADA), WAMS enables the controllers to recognize unusual activities within the power network such as instability in the network voltage, to analyze the network oscillation, and to perform time-stamp for fault localization.

1.4 Thesis Objectives and Description

The optimal PMU placement is a complex optimization task for power system networks. In this thesis, the main objective is to use the theory of algebraic connectivity of graphs, also known as Fiedler value [22], to design a "fault-tolerant" PMU placement algorithm capable of achieving full system observability in the case of system breakdown caused by worst-case single and certain other single and multiple branch failures. By modelling the grid as a connected graph, and using tools from the algebraic connectivity of graphs, we are able to identify the most critical branches, whose failure would lead to the worst case degradation in connectivity and thus have a negative impact on the ability of the system to remain observable. By identifying the branches, a PMU placement done using various techniques to provide and maintain full system observability in the case of the failure of the selected branches specifically, as well as increasing the system reliability by achieving a higher percentage of fully observable systems in the case of any single or multiple branch failures that can occur in the grid. Furthermore, a variation to the proposed improved PMU placement is discussed such as incorporating branch failure contingency based on the system's Fiedler value with other types of contingencies. An extension to the use of the algebraic connectivity of graphs is also demonstrated by utilizing the concept of the Fiedler vector to generate a fault-tolerant PMU placement designed to maintain full observability in the case of system partitioning caused by the failure of the minimum number of branches in the grid. The Institute of Electrical and Electronics Engineers (IEEE) 7, 14, 30, 57 and 118

bus test systems will be used to examine the proposed methods adequately and each result will be compared to existing methods.

The outlines of this thesis are as follows:

Chapter 1: A historical overview of PMU Technology is introduced, and the application of the PMU, for both state estimation and fault detection, is discussed in detail. The outlines of the thesis are also presented in this chapter.

Chapter 2: This chapter reviews the existing optimized PMU placement methods. The existing methods can be presented in three categories, namely: Heuristic Method, Meta-Heuristic Method; and Deterministic Method. Furthermore, the chapter reviews different contingencies that can be included when deciding on the optimal placement.

Chapter 3: This chapter reviews the most commonly used contingencies and constraints that can be used to enhance the performance of the system in terms of guaranteeing observability under various conditions. Also, the principles of graph theory and the algebraic connectivity of graphs, also known as Fiedler Value, shall be described. Furthermore, the incorporation of the algebraic connectivity of graphs with the formulation of the optimal PMU placement problem is be justified.

Chapter 4: This chapter presents the proposed method, adding a contingency constraint representing the use of algebraic connectivity of graphs, also known as Fiedler value, to the optimal PMU placement problem to provide the improved fault-tolerant placement of PMUs. This novel method shall be compared to other basic fault-tolerant solutions for the optimal PMU placement problem guarding against branch failures. The results are examined using IEEE 7-bus, IEEE 14-bus, IEEE 30-bus, IEEE 57-bus and IEEE 118-bus systems. The results obtained using the proposed approaches are tabulated, and a comparison of the total number of PMUs needed to achieve a fault-tolerant and fully observable system is presented for each approach, followed by a comprehensive discussion of the corresponding results.

Chapter 5: In this chapter, a brief summary of the thesis is outlined, incorporating the main achievement of the proposed improved PMU placement methodologies utilizing

the different approaches to the algebraic connectivity of graphs. The resultant approachs' ability to achieve guarantee a fault-tolerant system in the case of single and double branch failures is summarized. Furthermore, the chapter concludes with a brief discussion of possible additions and modifications to be implemented in future works in the field of PMU placement.

Chapter 2

Literature Review

As discussed in Chapter 1, network full observability is crucial for preventing blackouts and having reliable, controllable, and stable system. Hence, some rules were formulated in order to analyze the network observability in the light of the branch current and node voltage laws.

2.1 The Rules of Network Observability

Rule 1: The voltage phasor of PMU-installed buses and all their incident branches are directly measured by the PMU. These measurements are known as 'direct measurements'. For example, in Figure 2.1, upon installing a PMU on Bus D, the following parameters can be measured directly: voltage in bus D (V_D); current in lines AD, BD, and CD (I_{AD}, I_{BD} and I_{CD}).

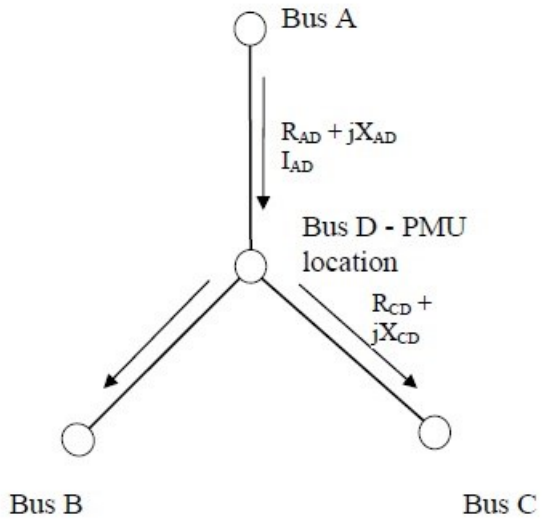


Figure 2.1 The first network observability rule

Rule 2: By applying Ohm's law, if the voltage phasor at one end of branch current is known as well as resistivity/ impedance of AD, BD, and CD transmission lines ($R_{AD} + jX_{AD}$, $R_{BD} + jX_{BD}$, $R_{CD} + jX_{CD}$), the voltage phasor at the other end of this branch can be calculated by equations 2.1 to 2.3. These measurements are known as 'pseudo measurements'.

$$V_A = V_D + I_{AD}(R_{AD} + jX_{AD}) \quad (2.1)$$

$$V_B = V_D - I_{BD}(R_{BD} + jX_{BD}) \quad (2.2)$$

$$V_C = V_D - I_{CD}(R_{CD} + jX_{CD}) \quad (2.3)$$

Rule 3: By applying Ohm's law, if the voltages at both ends of a branch are known, the branch current can be computed by equations 2.4 to 2.7. These measurements are known as 'pseudo measurements'.

$$V_D = V_A - I_{AD}(R_{AD} + jX_{AD}) \quad (2.4)$$

$$V_D = V_B + I_{BD}(R_{BD} + jX_{BD}) \quad (2.5)$$

$$V_D = V_C + I_{CD}(R_{CD} + jX_{CD}) \quad (2.6)$$

$$I_{AD} = I_{BD} + I_{CD} \quad (2.7)$$

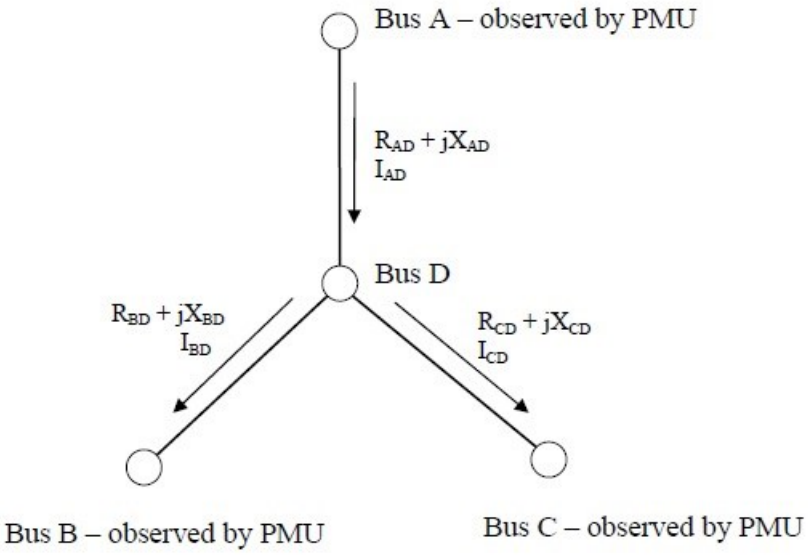


Figure 2.2 The third observability rule

For example, in Figure 2.2, PMUs on Buses A, B, and C measure their voltages, while the currents in AD, BD, and CD branches and Bus D voltage can be calculated.

2.2 Optimal PMU Placement problem formulation

Since any critical measurement must be observed by multiple measurement devices to be able to detect any error by bad data processing, adding extra PMUs makes the system more detectable [15, 23, 24]. This is why Phadke A. G. suggested that adding

PMUs in all substations can significantly improve network reliability [1]. However, adding PMU device costs $19,000 USD, hence, it is very costly to install PMUs in all locations [25]. This necessitates good planning to reduce the unit fees and maintenance costs. This can be achieved through minimizing the number of PMUs to be installed while having a completely topologically observable system. This is known as "Optimal PMU Placement (OPP) problem".

PMU placement problem is formulated as follows:

$$\text{Minimize} \sum_{k=1}^{N} x_k \tag{2.8}$$

Subject to

$$\mathrm{A.X} \geq \mathrm{B} \tag{2.9}$$

where N is the number of buses in a given system, X is a vector whose elements x_k is a binary value representing the presence of a PMU at bus k so that x_k is equal to 1 if there is a PMU at bus k and equal to zero otherwise, A is the binary connectivity matrix of the system in question and B is an observability column vector with number of elements equal to N with values representing the degree of observability of each bus. The binary connectivity matrix A in (2.9) is defined as

$$A_{ij} = \begin{cases} 1, & i = j \\ 1, & \text{buses } i \text{ and } j \text{ are connected} \\ 0, & \text{otherwise} \end{cases}$$

The sum of the elements of vector X represents the total number of PMUs that are needed to achieve the full system observability. The objective function represents PMU installations that can be extended to consider PMU installation costs. In such a case, x_k will be replaced by $c_k x_k$ where c_k is the installation cost at bus k. However, employing $c_k x_k$ in the objective function has no effect on the linear format of proposed model.

Due to its combinatorial nature, it is very difficult to solve the optimal PMU placement problem especially for large networks. This is why several approaches have been proposed solve this OPP problem. In the sequel, we will give a quick overview of the

techniques that have been proposed to solve this problem, such as: Heuristic methods, Meta-Heuristic methods and Deterministic methods [26] (Table 2.1).

Table 2.1 Examples of Optimal PMU Placement (OPP) Methods and algorithms [26]

Heuristic method	Meta-Heuristic method	Deterministic methods
Depth-First Search algorithm (DFS) [27, 28] Domination set [29, 30] Greedy algorithm [31]	Genetic Algorithms (GA) [32, 33] Particle Swarm Optimization (PSO) [34-36]	Integer Linear Programming (ILP) [37-39] Binary Search Algorithm (BSA) [40]

2.3 Heuristic method / Approximation algorithm

Heuristic method is a rapid method for solving OPP problems using reduced computational time and memory space, thus, it is regarded as practical but not optimal. It is useful when optimal solutions cannot be achieved in reasonable time [41].

2.3.1 Depth First Search algorithm (DFS)

As a criterion for placing PMUs, DFS installs PMUs on the branches with the largest number of connected branches (if they are many with the same number, one must be chosen randomly).

Then, this algorithm expands from the nodes on which PMU is placed to the nodes connected to PMU-placed nodes (with pseudo-measurement voltage) through the measurement of current branches, or the pseudo-measurement of branches connecting two nodes of known voltages, and then to all the nodes (Figure 2.3). These expanded nodes create a metrical tree; and observability is only reached if the tree contains all the nodes.

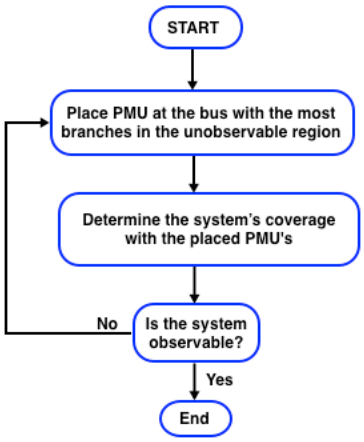


Figure 2.3 Flowchart of Depth First Algorithm [27]

The main disadvantage of DFS is that it takes only into consideration the expanding depth, and this leads to increased unwanted redundant measurements.

2.3.2 Domination set

Domination set is a set of electrical nodes/buses on which PMU is placed that dominates other electrical nodes for observing the full power system [29]. Full observability is only achieved if every electrical node is either included in the dominating set or adjacent to one or more electrical nodes in the dominating set [42]. The aim is to map the smallest dominating set [39]. For example, the optimal PMU location (nodes with black color) identified by the domination set of a 6-bus system is shown in Figure 2.4. This system is fully observable as explained in Table 2.2.

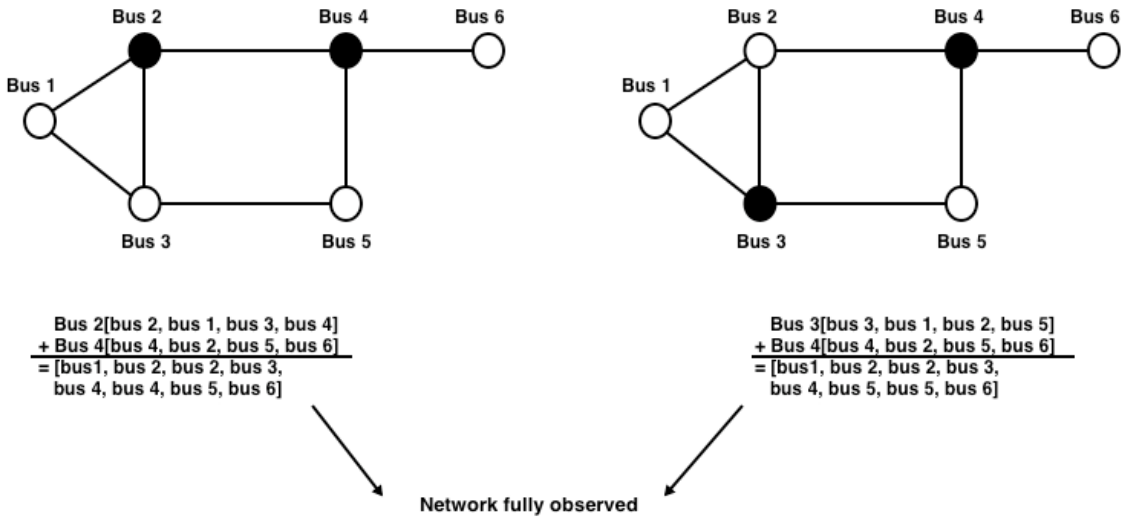


Figure 2.4 6-bus system with Domination Set

Table 2.2 Connectivity of 6-bus system

Bus	Degrees	Connectivity
1	2	Bus 2, Bus 3
2	3	Bus 1, Bus 3, Bus 4
3	3	Bus 1,Bus 2, Bus 5
4	3	Bus 2,Bus 5,Bus 6
5	2	Bus 4,Bus 3
6	1	Bus 4

2.3.3 Greedy algorithm

As criterion for placing PMUs, Greedy algorithm installs at each stage PMUs at the nodes/ buses with the largest number of uncovered buses (Figure 2.5). Lacking sophisticated optimization algorithms, Greedy algorithm is regarded as fast, easy (minimizing the computational efforts), and adaptable method [43].

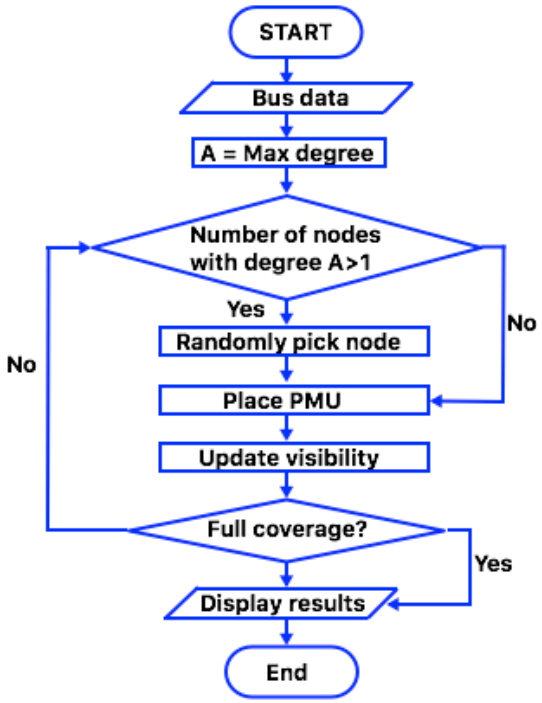


Figure 2.5 Greedy algorithm [44]

2.4 Meta-heuristic methods

As a way to improve the heuristic method, meta-heuristic iterative local method was proposed. It involves intelligent search processes dealing with discrete variables and non-continuous cost functions [45]. Basically, it is based on two stages: stage-I is concerned by the topological observability, while stage-II is concerned by the numerical observability. This is achieved through combining randomized and local optimization algorithms, thus the computational burden of meta-heuristic methods is high.

2.4.1 Genetic Algorithm (GA)

Inspired by the evolution of the living beings (reproduction, natural selection and diversity of the species), GA is regarded as direct, and parallel method.

GA is based on the random generation of an initial population of individuals/ chromosomes; the assignment of a certain fitness value (by means of selection operator) to each of these individuals during each iteration step/generation [46]. By this way, individuals are selected, then offspring candidates are produced and this process is repeated with these candidates as the next generation of parents. Thus, each individual is chosen the expected number of times and is approximately proportional to its relative performance in population.

Genetic algorithms use two operators: crossover and mutation.

- Crossover takes two individuals/ parents, and by combining parts of them, it produces one or two new individuals / offspring. Simply, a random crossover point is selected, then the operator exchanges substrings before and after this point.
- Mutation is regarded as an arbitrary modification that can prevent premature convergence by randomly sampling new points in the search space.

2.4.2 Particle Swarm Optimization (PSO)

Regarded as an artificial intelligence (AI) technique, PSO can be applied for extremely difficult numeric maximization and minimization problems. It is based on having a population of solutions/ particles, and iteratively trying to improve their measure of

quality (minimal PMUs with full observability of system) moving them around in the search-space (different PMU positions). This can be achieved by applying simple mathematical formulae over the particle's position and velocity. Searching for better positions, each particle is moving according to its local best known position, and is guided toward the best known positions in the search-space leading to the best solution.

2.5 Deterministic methods

Deterministic algorithms are considered as the most practical method thanks to their efficiency on real machines. Hence, they are the most studied and familiar kind of algorithms [47]. They predict the behavior given a certain parameter by means of a mathematical function, where an initial state of a system is used to determine the next state until a desired outcome is reached.

2.5.1 Integer Linear Programming (ILP)

ILP is an algorithm aiming to find the minimum solution (number of PMUs) to a function given certain constraints (all variables should be integer). These constraints are expressed as linear equalities or inequalities.

OPP can be expressed by Integer Linear Programing as follows:

$$\text{Minimize} \quad \sum_{k=1}^{N} x_k \tag{2.10}$$

Subject to

$$A.X \geq B \tag{2.11}$$

where N is the number of buses in a given system, X is a vector whose elements x_k is a binary value representing the presence of a PMU at bus k so that x_k is equal to 1 if there is a PMU at bus k and equal to zero otherwise, A is the binary connectivity matrix of the system in question and B is an observability column vector with number of elements equal to N with values representing the degree of observability of each bus. In addition to the monitoring of network full observability, Integer Linear Programming

takes into consideration different contingencies in power networks (such as measurement losses and branch failures) as constraints to the minimization function [37]. Thus, each required PMU is comprehensively studied in each scenario.

2.5.2 Binary Search Algorithm (BSA)

BSA takes into consideration all the possible combinations of PMUs location (N_{sol}) and narrows them down to the optimal solution by means of the binary search formulation [40, 48]. N_{sol} can be calculated as follows:

$$N_{sol} = \binom{P}{N_{PMU}} = \frac{P!}{N_{PMU}!\,(P - N_{PMU})!} \tag{2.12}$$

$$N_{PMU} = [(N + s/2)/3] \tag{2.13}$$

where P is the total number of candidate buses for a PMU placement; N_{PMU} is the initial number of PMUs; N is the total number of buses in the system; and s is the number of unknown power injections.

From (2.13), increasing the total number of buses in the power system increases the computational time in a non-linear relationship. Hence, in attempt to reduce it, a theoretical upper bound of the minimum number of PMUs needed to make the system observable is defined by (2.14) [37].

$$N_{PMU}^{ub} = [\frac{N+s/2}{3}] \tag{2.14}$$

If all the combinations of PMU locations does not lead to fully observable system, one more PMU must be added (N_{PMU}= (N_{PMU}+1)). However, if all the combinations of PMU locations lead to fully observable system, one PMU must be removed (N_{PMU}= (N_{PMU}-1)) [40]. The search is repeated until having the minimum number of PMUs necessary for making the system fully observable. Thus, exhaustive testing for all combinations of (N_{PMU}-1) must be performed before concluding that N_{PMU} is the minimum number of PMUs (Figure 2.6).

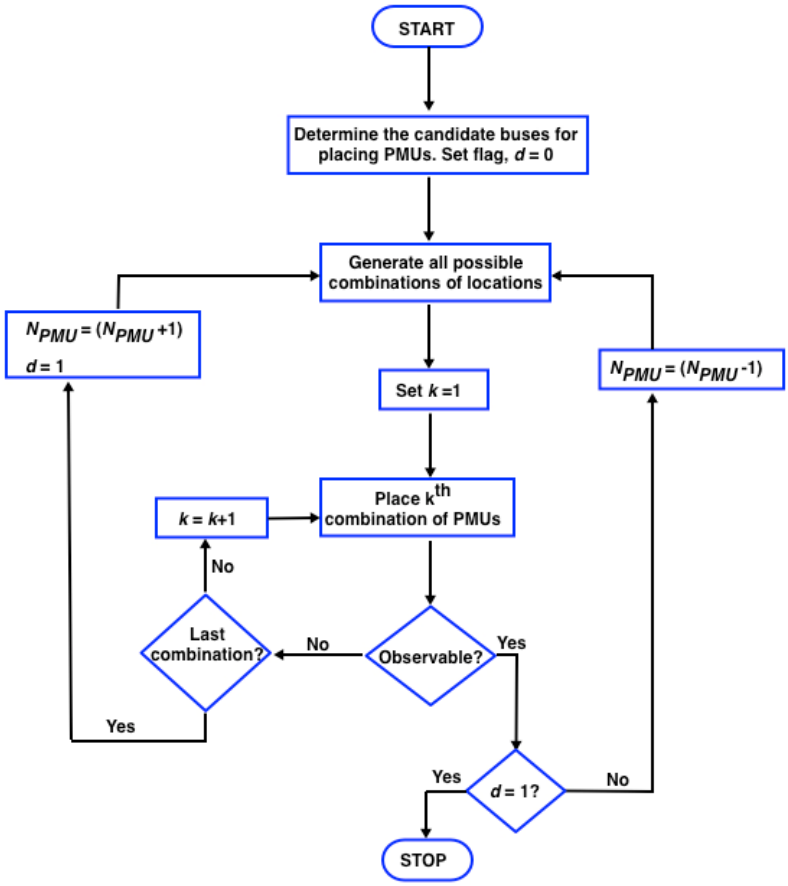


Figure 2.6 Binary search algorithm to determine the minimum number of PMUs required to make the system observable [40].

BSA can be used for assessing the complete observability of the system under normal operating conditions, or the outage of a single transmission line or a single PMU.

Chapter 3

PMU placement with contingencies

Most of the reported approaches for solving the optimal PMU placement problem ignore the power system contingencies. These contingencies take into account the losses of measurements and/or branch failures[49]. Thus, incorporating contingencies in the optimal PMU placement problem will account for more reliable results and ensure network observability. This was used for conventional measurement devices [50-52]. Also, taking into account single line contingencies, Milosevic model found observable network solutions [33]. Some problems faced PMU placement with contingencies such as being computationally expensive as the case of using heuristic approach during single measurement losses and line contingencies [53], or being time consuming as the case of considering single branch failures in Chakrabarti model [40]. Other models had considered PMU outages but branch failure contingencies were not incorporated [39]; while others consider both the outages of single lines and PMUs [38]. With the objective of minimizing the cost function, the following sections will present the different contingencies that can be taken into account, such as the effect of zero injection buses, loss of measurement contingency, branch failure contingency, measurement limitations.

3.1 Effect of Zero-Injection Buses

In any power network configuration, power is either injected, consumed or transmitted through the network. Buses that are responsible to transmit power through the network transmission lines without injecting or consuming power are called “zero injection buses”. As such as per Kirchhoff’s Current Law (KCL), the directional sum of the current transmitted through zero injection buses is equal to zero. Furthermore, by knowing the value and direction of all currents passing through a zero injection bus except one, the value and direction of the unknown current is identifiable. Assuming the resistance of all transmission lines connected to the zero injection bus is known, the voltage at the bus connected to the zero injection bus through the transmission line with the unmeasured current can also be identified, allowing that bus to be observable even when the current transmitted through the zero injection bus is unknown.

The zero-injection bus rules for assessing the network observability are:

1) When buses, which are incident to an observable zero-injection bus, are all observable except one, the unobservable bus will also be identified as observable by applying the KCL at zero-injection bus.

2) When buses incident to an unobservable zero-injection bus are all observable, the zero-injection bus will also be identified as observable by applying the KCL at zero-injection bus.

Thus, a single unobservable bus among a zero-injection bus and its incident buses can be made observable by making the others observable [54].

Using this simplification, the proposed objective function formulation considering the effect of zero-injection buses is presented as

$$fi \geq 1, \forall i \in I \tag{3.1}$$

$$fi = \sum_{j \in I} a_{ij} x_j + \sum_{j \in I} a_{ij} z_j y_{ij}, \forall i \in I \tag{3.2}$$

Expression (3.6) is the same as (3.3) with auxiliary binary variables y_{ij} added to zero-injection buses and those incident to zero-injection buses. Parameter z_j is a binary parameter that is equal to 1 if bus *j* is a zero-injection bus and 0 otherwise.

$$\sum_{i \in I} a_{ij} y_{ij} = z_j, \forall i \in I \tag{3.3}$$

When bus *j* is a zero-injection bus, the right hand side of (3.3) is equal to one. Therefore, exactly one auxiliary variable of buses which are incident to bus *j* or the auxiliary variable of bus *j*, would be equal to 1. When bus *j* is a nonzero-injection bus, the right hand side of (3.3) is zero. So all auxiliary variables of buses which are incident to bus *j* would be equal to zero. Thus, (3.2) and (3.3) together ensure that one of the buses which are incident to a zero-injection bus, or the zero-injection bus itself, will be observable when the others buses are observable.

Obviously, all zero-injection buses would have at least one auxiliary variable. For each

non zero-injection bus, the number of auxiliary variables is equal to the number of zero-injection buses that are incident to that bus. While for each zero-injection bus, the number of auxiliary variables is equal to the number of zero-injection buses that are incident to that bus plus one.

Applying this model on nine-bus network necessitates the installation of two PMUs at buses 5 and 8 (Figure 3.1).

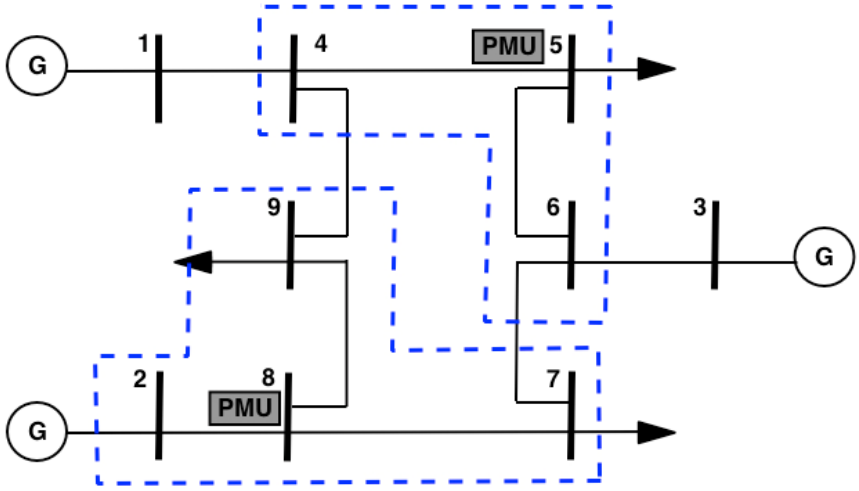


Figure 3.1. 9-bus system with PMU placement indicating zero injection effect.

In this case, installing PMU at bus 5 makes buses 4 and 6 observable; and installing PMU at bus 8 makes buses 2, 7, and 9 observable. Also, based on the zero injection effect of bus 4 and 6, buses 1 and 3 became also observable. This indicates that the network is entirely observable.

3.2 No PMU at Zero-Injection Buses

PMUs at zero-injection buses measure current phasors of corresponding lines; so the KCL at zero-injection bus provides no additional information but will help find the optimal solution. While removing PMUs from zero-injection buses will reduce the search space and enhance the solution speed.

The lack of PMUs at zero-injection buses is enforced by adding the following constraint:

$$z_j x_j = 0, \forall j \in I \qquad (3.4)$$

Constraint (3.4) states that when no PMU is placed at zero-injection buses, where z_j is equal to 1, the value of x_j would be zero. While at other buses, where z_j is zero, the value of x_j could be either zero or one.

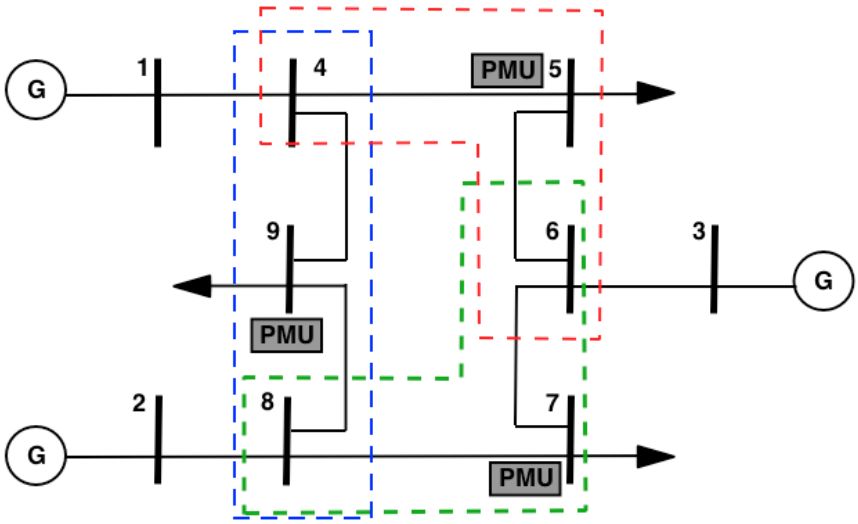


Figure 3.2. 9-bus system with no PMU placement at zero injection buses.

Applying this model on nine-bus network necessitates the installation of three PMUs at buses 5, 7, and 9. By this way, buses 4, 6, and 8 are observable; and buses 1, 2, and 3 are made observable through the zero- injection effect of buses 4, 8, and 6.

3.3 Loss of measurement contingency

The aim of this contingency is to make sure that all buses will remain observable even in case of unexpected failure of a single PMU either due to device or communication link failures [55]. In (3.2), the loss of single measurement can be modeled by modifying the inequality.

$$fi + \sum_{j \in I} a_{ij} y_{ij} \geq 2, \forall i \in I \qquad (3.5)$$

Here, if bus i is not observable by zero-injection buses,

$$\sum_{j \in I} a_{ij} y_{ij} = 0$$

Thus, $fi \geq 2$ indicates that bus i needs at least two observability sources. However, if

bus i is observable by a zero-injection bus,

$$\sum_{j \in 1} a_{ij} y_{ij} = 1$$

Thus, the observability of bus i would be robust for any loss of PMUs buses.

Applying this model on nine-bus network necessitates the installation of four PMUs at buses 4, 5, 7, and 8.

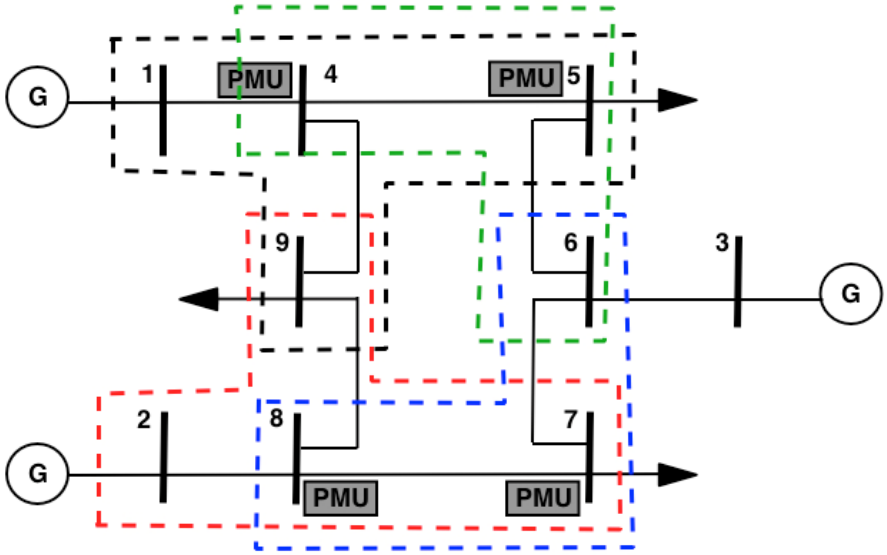


Figure 3.3. 9-bus system optimal PMU placement guarding against single loss of measurements.

Buses 4,5,7,8 are observable by means of their associated PMUs; and buses 1, 6, 2, and 9 are observable since they are adjacent to observable buses. Also, through the zero-injection effect of bus 6, bus 3 is also observable. Since buses (5, 6, and 7), and (4, 9, and 8) are observable during any single PMU outages, buses 3, 1, and 2 are also observable during any such conditions. If any of the PMUs is lost, the buses will still be observable by considering the three other PMUs.

3.4 Branch Failure Contingency

In order to consider branch failure contingency, the connectivity parameters, auxiliary variables, and observability functions are replaced outage of line k set of constraints ($k \in K$).

$$f_i^k \geq 1, \forall i \in I, \qquad \forall \kappa \in K \tag{3.6}$$

$$f_i^k = \sum_{j \in 1} a_{ij}^k x_j + \sum_{j \in 1} a_{ij}^k z_j y_{ij}^k, \qquad \forall i \in I, \forall \kappa \in K \tag{3.7}$$

$$\sum_{i \in I} a_{ij}^k y_{ij}^k = z_j, \qquad \forall j \in I, \forall \kappa \in K \tag{3.8}$$

The binary connectivity parameter when line k is out is defined as follows:

$$a_{ij}^k = \begin{cases} 0, & \text{line } k \text{ is between buses } i \text{ and } j \\ a_{ij}, & \text{Otherwise} \end{cases} \tag{3.9}$$

Applying this model on nine-bus network necessitates the installation of four PMUs at buses 1, 2, 3, and 6.

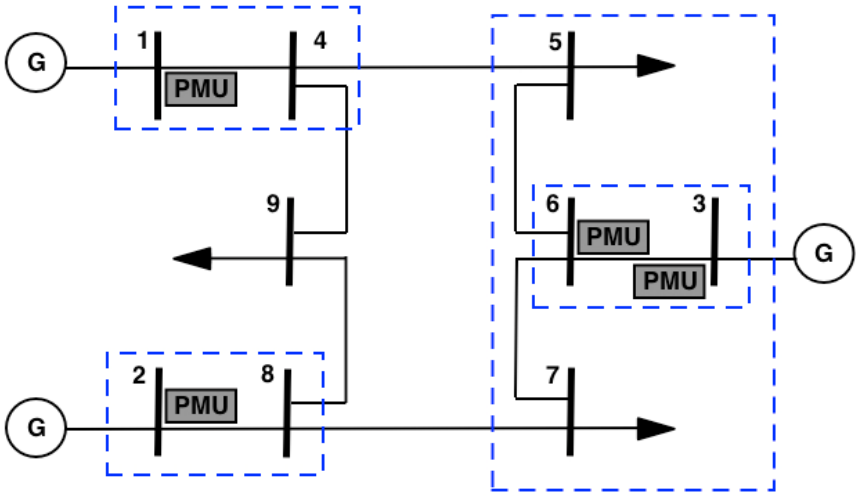


Figure 3.4. 9-bus system optimal PMU placement guarding against single branch failure.

Here, buses 1, 2, 3, and 6 are observable by means of their associated PMUs; buses 4, 5, 7, and 8 are observable by the PMU installed at bus 1, 6, and 2. Bus 4 has then redundant measurements by the PMU installed at bus 1 and by its zero-injection effect. Thus, when line 1 - 4 is on outage, bus 4 is made observable by its zero-injection effect. This is the same case with bus 5, and 7. Thus, when line 5–6 is on outage, it is made observable by zero-injection effect of bus 6. Also, when line 6–7 is on outage, it is made observable by the zero-injection of bus 8. Again, when line 2-7 is on outage, bus 8 is made observable by its zero-injection effect. Moreover, when either line 4–9 or line 8–

9 is on outage, bus 9 is made observable by the zero-injection of buses 4 and 8. Therefore, these four PMUs maintain the network observability during any single branch failures.

3.5 Measurement Limitations

The aim of this contingency is to take into consideration the extensive set of communication equipment required to communicate PMU measured data to a control center. Usually multi-channel PMUs are installed at buses with several measurements [9], [26]. We consider the effect of limited communication on PMU placements by substituting $\sum a_{ij}x_j$ with $\sum a_{ij}w_{ij}x_j$ in (3.3) and (3.6).

$$f_i = \sum_{j \in 1} a_{ij}w_{ij}x_j, \forall i \in I \tag{3.10}$$

$$\sum_{i \in 1} a_{ij}w_{ij} \leq w_j^{max}, \forall j \in I \tag{3.11}$$

$$w_{ij} \leq x_j, \ \forall i,j \in I \tag{3.12}$$

In (3.11), the binary variable w_{ij} represents the measurement at bus *i* using a PMU placed at bus *j*; and is considered as another constraint in order to limit the number of measurements associated with bus *j*.

As per (3.12), if a PMU is placed, the related binary measurement variables will be either zero or one; while if no PMU is installed, associated binary measurement variables will be zero.

In order to convert non-linear expression ($w_{ij}x_j$) to linear one, the nonlinear variable r_{ij} is expressed as a set of three linear inequalities with binary variables (as shown in (3.13) – (3.16)).

$$r_{ij} = w_{ij}x_j \tag{3.13}$$

$$r_{ij} \leq x_j \tag{3.14}$$

$$r_{ij} \leq w_{ij} \tag{3.15}$$

$$r_{ij} \geq x_j + w_{ij} - 1 \qquad (3.16)$$

Applying this model on nine-bus network necessitates the installation of three PMUs at buses 5, 7, and 9 so that each PMU measures its own voltage phasor and one current phasor associated with the lines incident to that bus. This ensures that each PMU has at most two measurements.

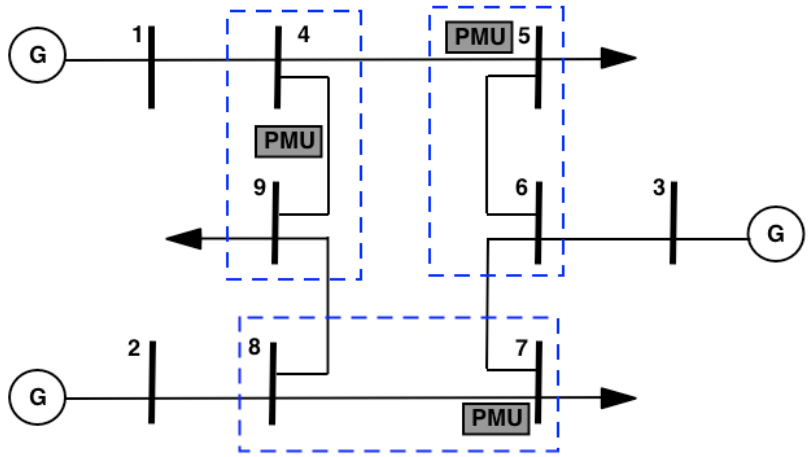


Figure 3.5. 9-bus system optimal PMU placement considering measurement limitations.

Here, buses 4, 6, and 8 are observable directly and buses 1, 2, and 3 are observable by the zero-injection of buses 4, 8, and 6, respectively.

When comparing Figures 3.2 and 3.5, it is obvious that the observability zone of each PMU is restricted by limited measurements.

To summarize the conclusions derived from IEEE standard test systems (Table 3.1),

. All buses must be observable by at least one source.

. According to the rules of zero-injection buses, three sources of observability are available (observability by PMUs, observability associated with the lines incident to that bus having PMU, and observability by zero injection effect).

. According to the rules of loss of measurement, all buses must have redundant measurements so they remain observable even if there is a single measurement loss.

. According to the branch failure rules, buses must be observable by means of other sources in case of branch failure.

Table 3.1. PMU placement buses for the IEEE standard test systems

Test system	Base case state	No PMU at zero injection	Branch failure	Loss of measurement	Branch failure or loss of measurement
IEEE 14-bus	2,6,9	2,6,9	1,3,6,8,9,11,13	2,4,5,6,9,10,13	1,2,4,6,8,9,10,13
IEEE 30-bus	3,5,10,12,18, 24,27	1,2,10,12,18,24, 29	1,3,5,10,11,13, 14,15,16,19,23, 26,30	1,3,5,7,10,12,13, 15,16,19,20,24, 25,27,29	1,3,5,7,10,11,12, 13,15,16,19,20, 23,24,26,27,30
IEEE 39-bus	3,8,11,16,20, 23,25,29	3,8,12,16,20, 23,25,29	3,8,16,24,26,28, 30,31,32,33,34, 35,36,37,38	3,6,8,9,12,14,16, 18,20,21,23,25, 26,29,34,36,37,38	3,6,8,9,10,16,18, 20,21,23,25,26, 29,30,31,32,33, 34,35,36,37,38
IEEE 57-bus	1,4,13,20,25, 29,32,38,51, 54,56	1,6,13,19,25, 29,32,38,51, 54,56	1,2,6,12,14,19, 21,27,29,30,32, 33,41,44,49,51, 53,55,56	1,2,4,6,9,12,14, 19,20,24,25,27, 29,30,32,33,38, 39,41,44,46,50, 51,53,54,56	1,2,4,6,9,12,14, 19,20,24,25,27, 29,30,32,33,36, 38,41,44,46,50, 51,53,54,56
IEEE 118-bus	3,9,11,12,17, 21,25,28,34, 37,40,45,49, 53,56,62,72, 75,77,80,85, 86,90,94,102, 105,110,114	3,8,11,12,17, 21,25,28,34, 35,40,45,49, 53,56,62,72, 75,77,80,85, 86,90,94,102, 105,110,114	1,6,10,11,12,15, 17,19,21,23,26, 27,29,34,35,39, 41,44,46,49,51, 53,56,57,59,61, 67,72,73,74,75, 76,78,80,83,85, 87,89,91,92,94, 96,100,101,105, 107,109,11,112, 113,114,116,117	1,3,5,7,9,10,11, 12,15,17,19,21, 22,26,27,28,29, 32,34,35,37,40, 41,43,45,46,49, 50,51,52,54,56, 59,62,66,68,70, 71,72,75,76,77, 79,80,83,85,86, 87,89,90,92,94, 96,100,101,105, 106,108,110,111, 112,114,117	1,3,5,7,8,10,11, 12,15,17,19,21, 22,24,25,27,28, 29,32,34,35,37, 40,41,44,45,46, 49,50,51,52,54, 56,59,62,66,68, 72,73,74,75,76, 77,78,80,83,85, 86,87,89,90,92, 94,96,100,101, 105,107,109,110, 111,112,115,116,117

In the following chapters, this thesis will present further enhancement on solving the optimal PMU placement problem in the event of branch failures by using Algebraic connectivity of graphs (Fiedler value) to identify the worst- case single branch failure in terms of connectivity degradation.

Chapter 4

PMU Placement using Algebraic Connectivity of Graphs

4.1 Algebraic Connectivity of Graphs

Let G be a graph and let N denote the set of nodes and L the set of links, with N = |N| nodes and L = |L| links, respectively. The Laplacian matrix of G with N nodes is an N x N matrix Q = Δ - A where Δ = diag(D_i), D_i is the nodal degree of node i ∈ N and A is the adjacency matrix of G. The eigenvalues of Q are called the Laplacian eigenvalues. The Laplacian eigenvalues are all real and non-negative [56]: they are all contained in the interval [0, min{N, $2D_{max}$}], where D_{max} is the maximum nodal degree of G. The set of all N Laplacian eigenvalues $\mu N = 0 \leq \mu_{N-1} \leq ... \leq \mu_1$ is called the Laplacian spectrum of G. The second smallest Laplacian eigenvalue μ_{N-1} of Q is known as the algebraic connectivity [57].

Algebraic connectivity = $\min_{\substack{x \perp 1 \\ x \neq 0}} \frac{x^T L(G) x}{x^T x}$ [58]

where 0 = $(0,0,\ldots\ldots 0)^T$, and 1= $(1,1,\ldots.1)^T$ is an eigenvector of eigenvalue = 0. It is the only eigenvector if the graph G_n is connected. If G_n has C_m components, there should be C_m orthogonal eigenvectors each having non-negative elements where a positive entry indicates that the corresponding node belongs to the graph components derived by eigenvectors.

Unlike the traditional connectivity, the algebraic connectivity is dependent on the number of vertices, as well as the way in which vertices are connected. In random graphs, the algebraic connectivity decreases with the number of vertices, and increases with the average degree [59].

Algebraic connectivity is of great interest because of the following inequality developed by Fiedler: $\lambda_2(G) \leq \upsilon(G) \leq \eta(G)$ [60] which states that the algebraic connectivity of a graph G is less than or equal to the node connectivity which is less

than or equal to the edge connectivity [57]. Algebraic connectivity for certain types of networks is inversely proportional to node-connectivity and edge-connectivity.

Algebraic connectivity is considered one of the most popular eigenvalues of the Laplacian of a graph [57]. Considered as a good parameter to measure how well a graph is connected, algebraic connectivity accompanied by a measure of link density (interconnectedness of nodes) are basic measures of the robustness of a graph G. The denser the connections, the less vulnerable the network is to being disconnected. The algebraic connectivity must be greater than zero if *G* is connected. This is a corollary to the fact that the multiplicity of the eigenvalue 0 of Q is the number of disconnected components of *G* [57]. Whereas, if algebraic connectivity tends to be 1, the network tends to be fully connected. An example of algebraic connectivity based on the complexity of graph connection is shown in Figure 4.1. By applying that concept to the power grid and the rules of network observability previously explained, it can be deduced that the more connected a network is, the more observable it can be with fewer number of PMUs.

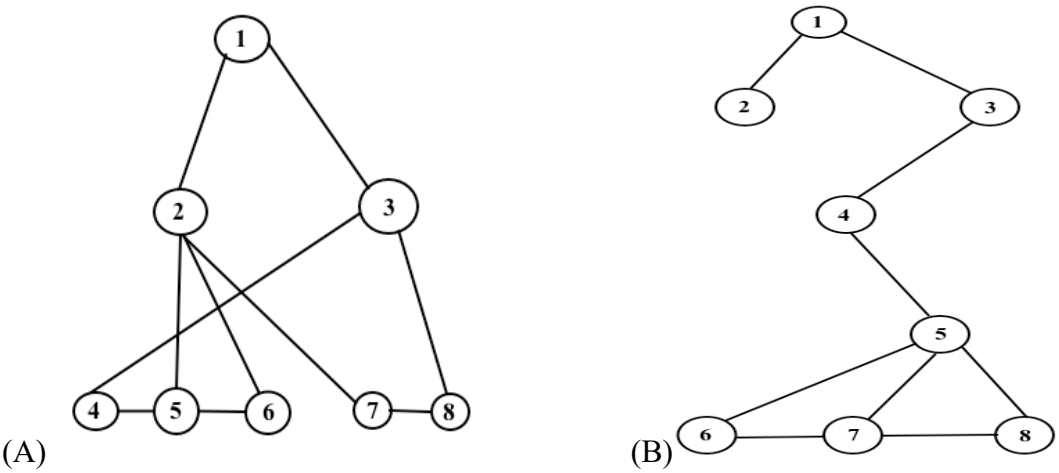


Figure 4.1 Graph with algebraic connectivity (A) 0.9249, (B) 0.1531

Applications of algebraic connectivity are diverse. It is used for surveys and books [56, 57, 61-67]; for application on trees [68-74]; for the study of the asymptotic behavior of algebraic connectivity for random graphs [75, 76]. Moreover, the algebraic connectivity is relevant for graph theory problems such as: the expanding properties of graphs, weighted graphs, absolute algebraic connectivity, isoperimetric number, genus and other invariants of a graph [56, 63, 77-83]; for the theory of elasticity [76]; for the

correspondence between continuous and discrete mathematics [84]; for the investigation of a bandwidth-type problem using the spectral parameter [56]; and for the combinatorial optimization problems: the problem of certain flowing process, the maximum cut problem and the traveling salesman problem [56, 79, 82, 85-89]. It is noteworthy that the most important application of algebraic connectivity is to the robustness of a graph since the larger the algebraic connectivity is, the more difficult it is to cut a graph into independent components ; whereas the node and the link connectivity (algebraic connectivity's classical upper bound) quantify the extent to which a graph can accommodate to node and link failures [57].

Due to its importance, Ghosh and Boyd [90] describe a method to maximize the algebraic connectivity over the convex hull of the Laplacian of graphs in a particular family which is a convex optimization problem. The eigenvectors corresponding to the algebraic connectivity are called Fiedler vectors [68-70]. These Fiedler vectors are used in algorithms for distributed memory parallel processors [91]. In graph theory, Fiedler vectors are used to determine the minimum number of edges in a graph that would partition the graph into subgraphs if removed [92]. The concept of graph partitioning using Fiedler vector is to separate the positive and negative value elements of the vector so that each subset represent a subgraph separated from the other set by removing the connecting edges between the two subgraphs.

By analyzing a connected system of buses as a graph, where its bus elements are the vertices and its branches are the edges connecting them, algebraic connectivity of graphs can be used to determine the level of connectivity of the system. Furthermore, Fiedler value can be used to identify branches that would have significant effects on the connectivity of a system and thus on its level of observability. By identifying such branches, we can determine an improved PMU placement for the tested system to ensure full observability even during the event of the failure of the branch singled out by the algebraic connectivity of graphs value [93].

4.2 Proposed PMU Placement Formulation

Integer linear programming (ILP) is used to obtain the minimal number of PMUs needed to achieve full system observability as well as the specific buses where the PMUs shall be installed. As a start, a binary connectivity matrix **A** shall be built to

represent the topological connections of the system with elements based on the rules and criteria explained in Chapter 3 of the thesis. As an example, the connectivity matrix for IEEE 7-bus system, shown in Figure 4.2, is described by the following matrix.

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}. \tag{4.1}$$

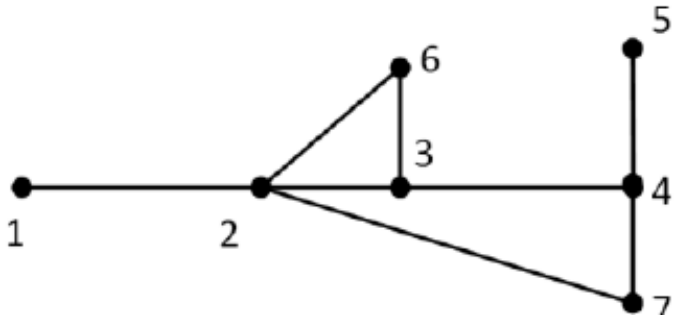


Figure 4.2 IEEE 7-bus system

To identify bus locations where PMUs should be installed, we seek the vector **X**:

$$[\mathbf{X}]_i = x_i = \begin{cases} 1 & if\ PMU\ is\ installed\ at\ bus\ i \\ 0 & otherwise. \end{cases} \tag{4.2}$$

Since the purpose of solving the optimal placement problem is to minimize the number of PMUs installed on buses in the system, the objective function can be written as follows:

$$\min_{X} \sum_{i=1}^{N} x_i$$

$$\text{subject to } \mathbf{A}.\mathbf{X} \geq \mathbf{B} \tag{4.3}$$

where *N* is the number of buses and the vector **B** is an observability column vector with number of elements equal to *N* with values representing the degree of observability of each bus. In the mentioned objective function, the cost of all PMUs is assumed to be equal. The lowest acceptable value for each observability vector element is 1, which indicates that the respective bus is observable through at least one PMU. For a bus to

be observable through more than one PMU, the corresponding value in the observability vector **B** shall be set to a value higher than one. Optimal PMU placement under normal conditions for several systems without any contingencies can be viewed in Table 4.1 below. This data is going to serve as a control with which all subsequent proposed approaches using algebraic connectivity of graphs shall be compared in terms of performance. The highest performance PMU placement shall be the one which offers the highest percentage of fully observable systems in the event of any single branch failure and any double branch failure combinations.

Table 4.1 Optimal PMU Placement Under Normal Conditions

Test System	PMU Placement	
	No. of PMUs	PMU Location
IEEE 7-bus system	2	2,4
IEEE 14-bus system	4	2,6,7,9
IEEE 30-bus system	10	1,2,6,9,10,12,15,19,25,27
IEEE 57-bus system	17	1,4,9,20,24,27,29,30,32,36,38,39, 41,45,46,51,54
IEEE 118-bus system	32	1,5,9,12,13,17,21,23,26,28, 34,37,41,45,49,53,56,62,63,68, 71,75,77,80,85,86,90,94,101,105, 110,114

After the formulation of the optimal PMU placement problem using integer linear programming, the following step will be to incorporate the concept of algebraic connectivity to determine the branch failures to be taken into account as contingencies. Using the concept of Fiedler value as discussed in Chapter 3, it is possible to take a variety of branch failure contingencies, not only that of a single branch failure corresponding to Fiedler value. All simulations presented in this thesis is performed using MATLAB computer program.

4.3 Improved PMU Placement using Different Approaches

In the following sections, different approaches to determine branch failures shall be discussed including single branch failure using Fiedler value as a tool to select the branch whose failure causes the highest degradation in grid connectivity. We also

present the results for another approach where the branch to be accounted for its failure is selected using the third lowest eigenvalue of the Laplacian matrix which is the second lowest Fiedler value and compare the level of system observability of that case with that corresponding to Fiedler value. Furthermore, Fiedler value can also be used to determine more than just one branch failure affecting the system degradation, pushing the criteria to detect multiple branch failures that would have the greatest impact on system connectivity and whether those failures occur at the same time or simultaneously.

4.3.1 Approach 1 - PMU Placement to Overcome Single Branch Failure corresponding to the Lowest Fiedler Value

As explained in Chapter 3, Fiedler value is used to determine the level of connectivity of a system. As such, it can be used to identify the branch with the greatest impact on system connectivity providing a worst case event of system degradation by removing single branches one at a time from any given system and calculating the algebraic connectivity of that system.

After repeating the process for each branch in the system, a set of Fiedler values will be obtained, each value corresponding to a single branch failure in the system. Based on the concept of Fiedler value, we can deduce that the lowest Fiedler value in the set obtained corresponds to the single branch failure that has the worst impact on system connectivity, and in turn the improved PMU placement should include a contingency constraint to ensure full system observability even during the case of the single branch failure corresponding to the calculated lowest Fiedler value. A flow chart detailing the algorithm for the proposed approach is shown in Figure 4.3.

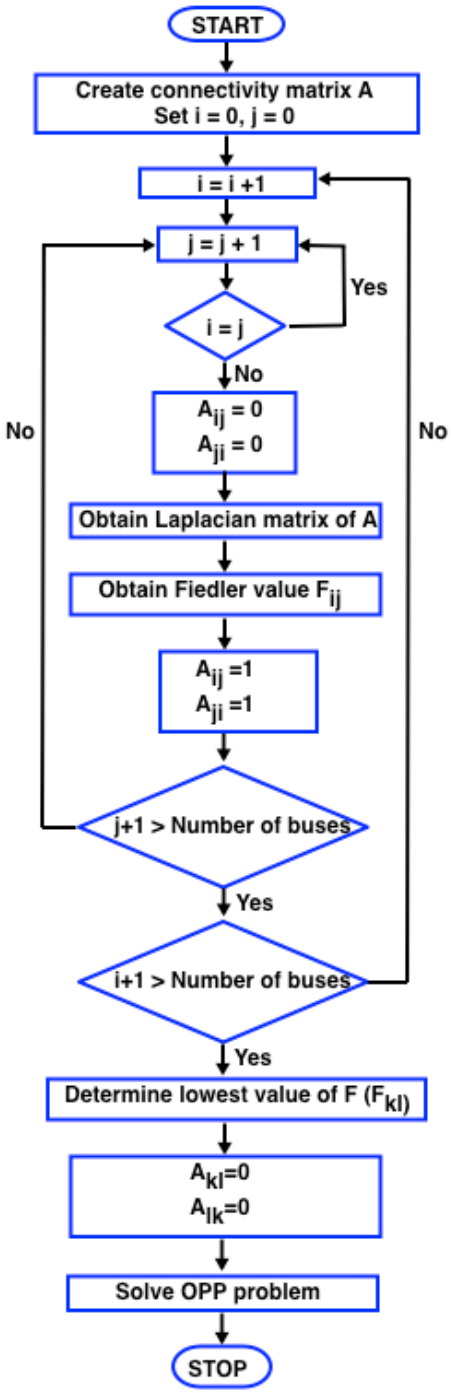


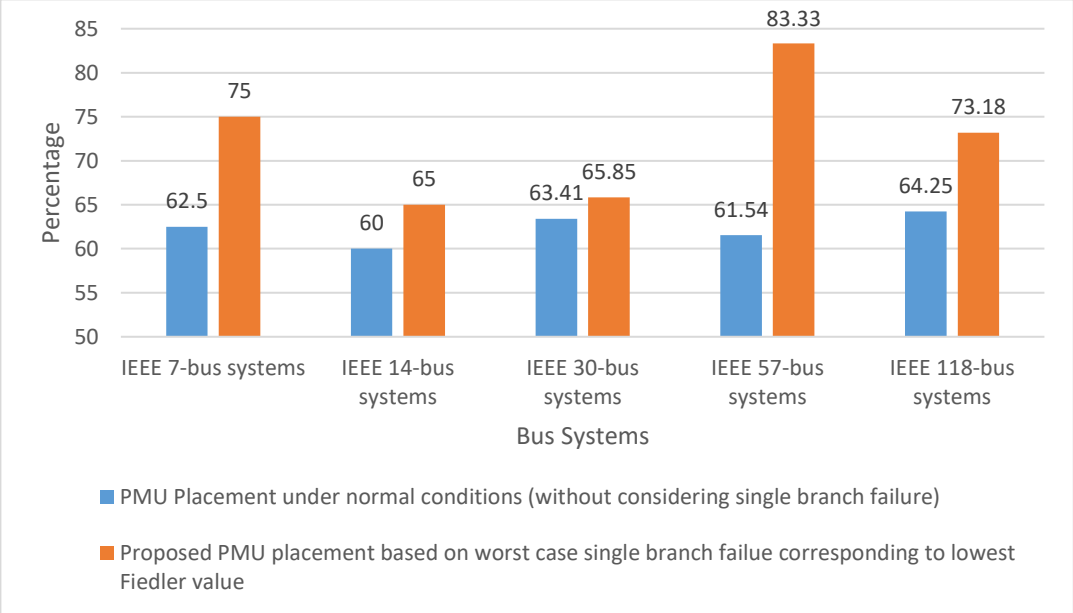
Figure 4.3 Flowchart of the algorithm used for Approach 1

Based on the optimal PMU placement problem formula taking into account single branch failure contingency corresponding to the lowest Fiedler value, Table 4.2 below presents the number of PMUs and their locations in the IEEE 7-bus, IEEE 14-bus, IEEE 30-bus, IEEE 57-bus and IEEE 118 systems to achieve full system observability.

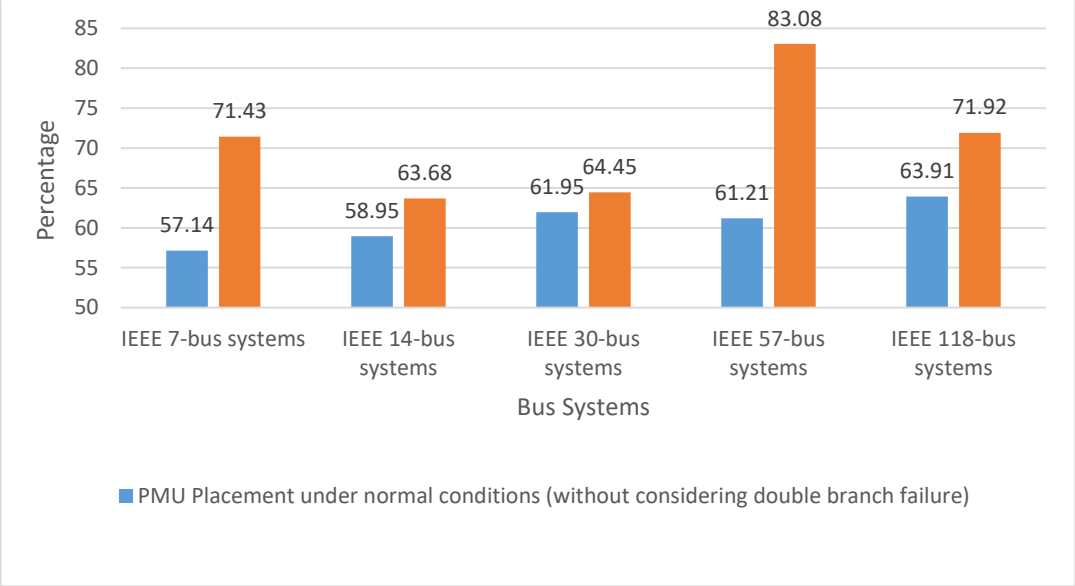
Table 4.2 Improved PMU Placement Covering Single Branch Failure Corresponding to Lowest Fiedler Value

Test System	PMU Placement	
	No. of PMUs	PMU Location
IEEE 7 bus system	3	2,3,5
IEEE 14 bus system	4	2,6,8,9
IEEE 30 bus system	10	1,2,6,10,11,12,15,19,25,27
IEEE 57 bus system	25	1,4,6,9,15,19,20,22,24, 28,29,30,32,33,34,36,37, 38,41,46,47,50,53,54,56
IEEE 118 bus system	38	2,5,9,11,12,15,17,21,23,25, 28,29,30,34,37,40,45,49, 52,56,62,63,64,68,70,71, 75,77,80,85,87,90,91,94, 101,105,110,114

After obtaining the improved PMU placement using the mentioned approach, the next step is to compare the resultant PMU placement with that of the optimal PMU placement done under normal conditions without any contingency consideration. The comparison shall be based on the percentage of fully observable systems during any single or double branch failure combinations that each placement can achieve. The results are presented in Figure (4.4 A and B) below.



(A)



(B)

Figure 4.4 Performance of PMU Placement Approach 1 for (A) All Single Branch Failures and (B) All Double Branch Failure Combinations

As evident in the results plotted above, the proposed system provides a higher percentage of fully observable systems in the event of both any single branch failure as well as any double branch failure combination while at the same time taking into consideration the failure of the branch with the highest impact on system connectivity. Furthermore, regarding the improved PMU placement for the IEEE 14-bus system and IEEE 30-bus system, the results show that proposed PMU placement method provides a higher percentage of fully observable systems for all single and double branch failures than the PMU placement method under normal conditions without the need of installing additional PMUs. The number of PMUs calculated by the proposed approach is compared to the number of PMUs required under normal conditions and to the number of PMUs that guarantees 100% of fully observable systems during any single branch failure that can occur by guaranteeing that each bus is observable by at least two PMUs is shown in Table 4.3.

Table 4.3 Number of PMUs required to maintain full observability in case of single branch failure as per Approach 1

Test System	No. of PMUs determined by Approach 1	No. of PMUs where every bus is observable by 2 PMUs
IEEE 7-bus system	3	5
IEEE 14-bus system	4	7
IEEE 30-bus system	10	14
IEEE 57-bus system	25	31
IEEE 118-bus system	38	50

While able to guarantee observability in the case of any single branch failure, the normal approach utilizing at least two PMUs to observe every bus requires a high number of PMUs to maintain observability.

4.3.2 Approach 2 - PMU Placement to Overcome Single Branch Failure corresponding to the Second Lowest Fiedler Value

In this approach, the effect of the single branch failure corresponding to the second lowest Fiedler value shall be studied. As explained previously, single branches shall be removed from a given system one at a time and the Fiedler value for the resultant system shall be calculated. After determining the required branch, the removal of which would result in a system with the second lowest Fiedler value, it is removed from the system and the optimal PMU placement problem is solved. A flow chart detailing the algorithm for the proposed approach is shown in Figure 4.5.

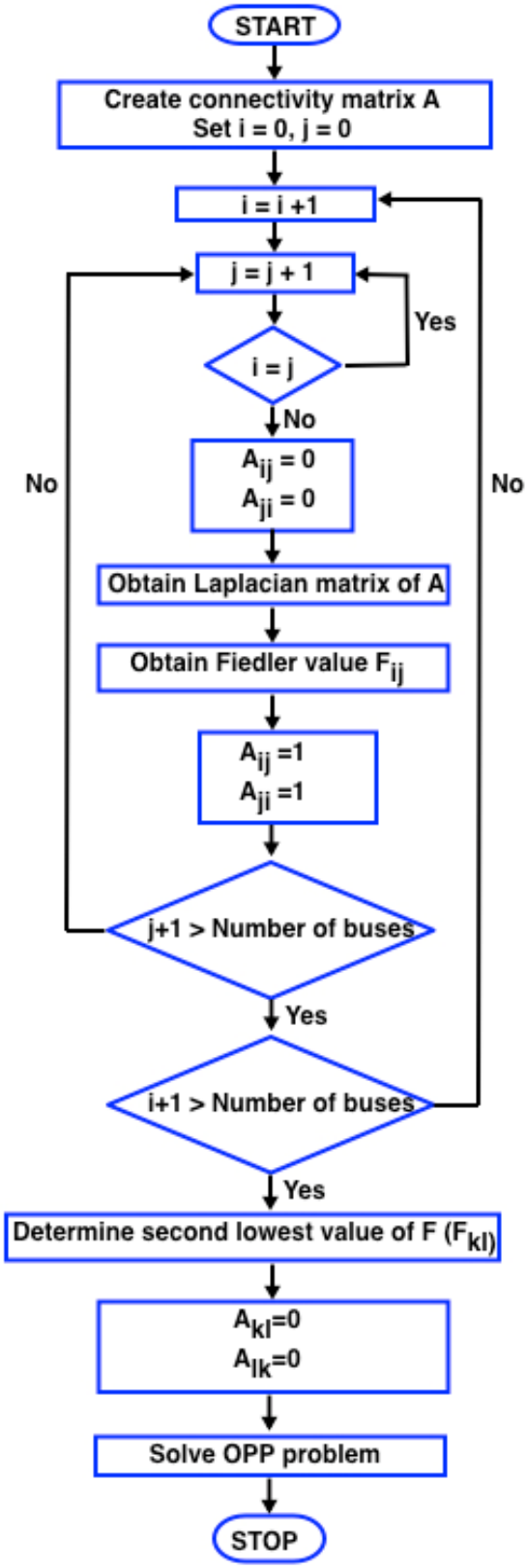


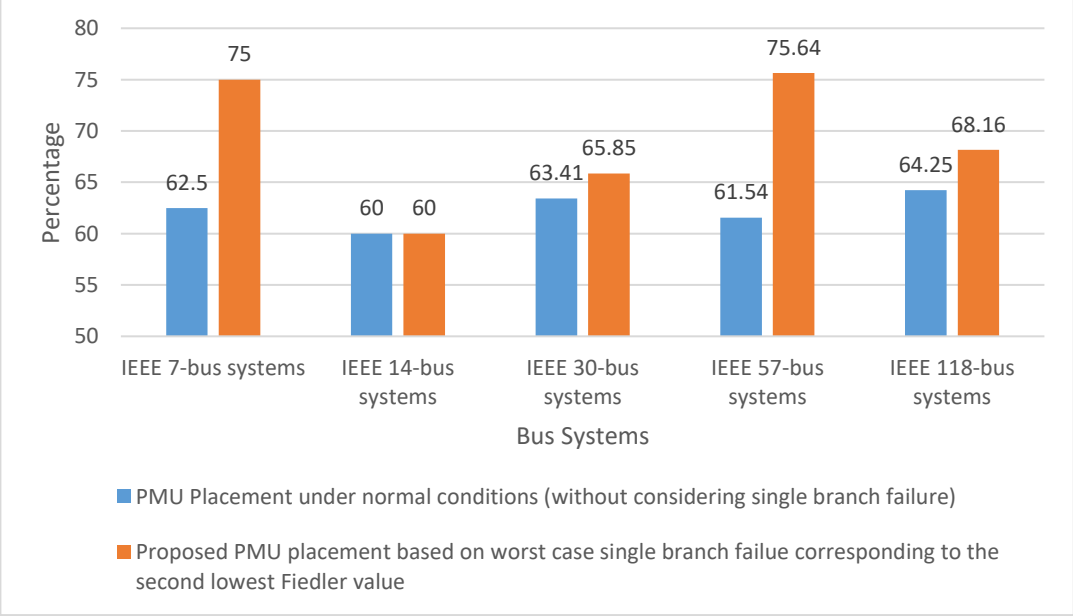
Figure 4.5 Flowchart of the algorithm used for Approach 2

The number of PMUs and their bus placement is demonstrated in Table 4.4 below.

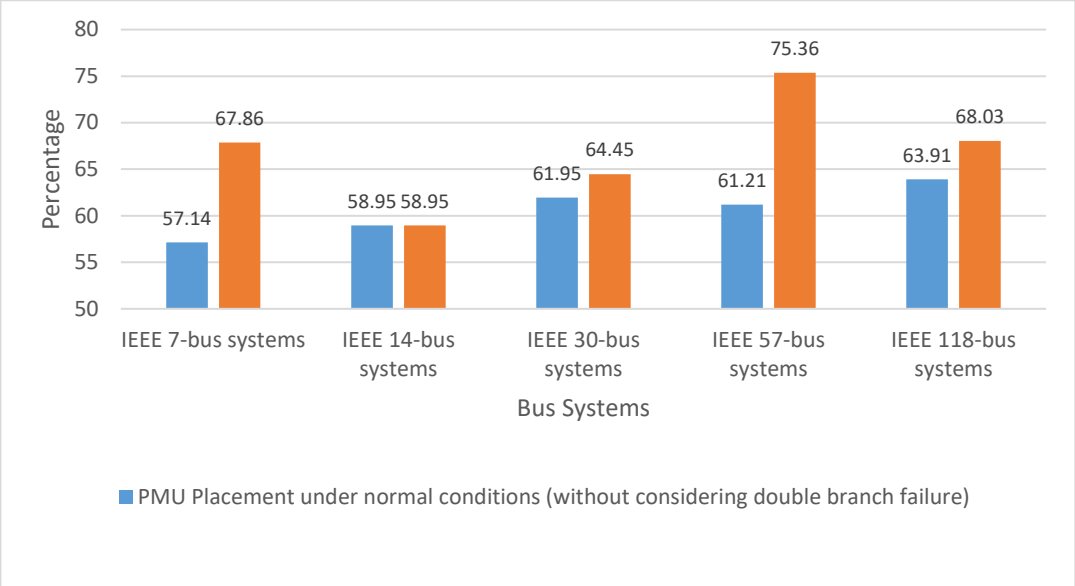
Table 4.4 Improved PMU Placement Covering Single Branch Failure Corresponding to the Second Lowest Fiedler Value

Test System	PMU placement	
	No. of PMUs	PMU Location
IEEE 7 bus system	3	1,2,4
IEEE 14 bus system	4	2,6,7,9
IEEE 30 bus system	11	1,2,6,9,10,12,13,15,19,25,27
IEEE 57 bus system	23	1,4,6,9,15,19,20,22,24,25,28,29,32,34, 37,38,41,46,47,50,53,54,56
IEEE 118 bus system	36	2,5,9,11,12,15,17,21,25,28,29, 34,37,40,45,49,52,56,62,64,68, 72,73,75,77,80,85,86,90,91,94, 101,102,105,110,114

After obtaining the improved PMU placement using the mentioned approach, the next step is to compare the resultant PMU placement with that of the optimal PMU placement done under normal conditions without any contingency consideration. The comparison shall be based on the percentage of fully observable systems during any single or double branch failure combinations that each placement can achieve. The results are presented in Figure (4.6 A and B) below.



(A)



(B)

Figure 4.6 Performance of PMU Placement Approach 2 for (A) All Single Branch Failures and (B) All Double Branch Failure Combinations

As evident in the results plotted above, the proposed system, while not as high performance as that of the PMU placement calculated using Fiedler value, still provides a higher percentage of fully observable systems in the event of both any single branch failure as well as any double branch failure combination while at the same time taking into consideration the failure of the branch with the second highest impact on system connectivity. However, this approach can be used to enhance the solution to the optimal PMU placement problem by incorporating the approach of PMU placement using the second lowest Fiedler value with the previous approach which takes into consideration the actual Fiedler value when solving the optimal PMU placement problem. The number of PMUs calculated by the proposed approach is compared to the number of PMUs required under normal conditions and to the number of PMUs that guarantees 100% of fully observable systems during any single branch failure that can occur by guaranteeing that each bus is observable by at least two PMUs is shown in Table 4.5.

Table 4.5 Number of PMUs required to maintain full observability in case of single branch failure as per Approach 2

Test System	No. of PMUs determined by Approach 2	No. of PMUs where every bus is observable by 2 PMUs
IEEE 7-bus system	3	5
IEEE 14-bus system	4	7
IEEE 30-bus system	11	14
IEEE 57-bus system	23	31
IEEE 118-bus system	36	50

While able to guarantee observability in the case of any single branch failure, the normal approach utilizing at least two PMUs to observe every bus requires a high number of PMUs to maintain observability.

4.3.3 Approach 3 - PMU Placement to Overcome Double Branch Failure corresponding to the Lowest Fiedler value and the Second Lowest Fiedler Value

In this approach, the effect of the double branch failure corresponding to both the lowest Fiedler value and the second lowest Fiedler value shall be studied. As per established protocol, after determining the two branches in a system where the removal of which would cause the greatest impact of system connectivity, both branches are removed and the optimal PMU placement problem is solved. A flow chart detailing the algorithm for the proposed approach is shown in Figure 4.7.

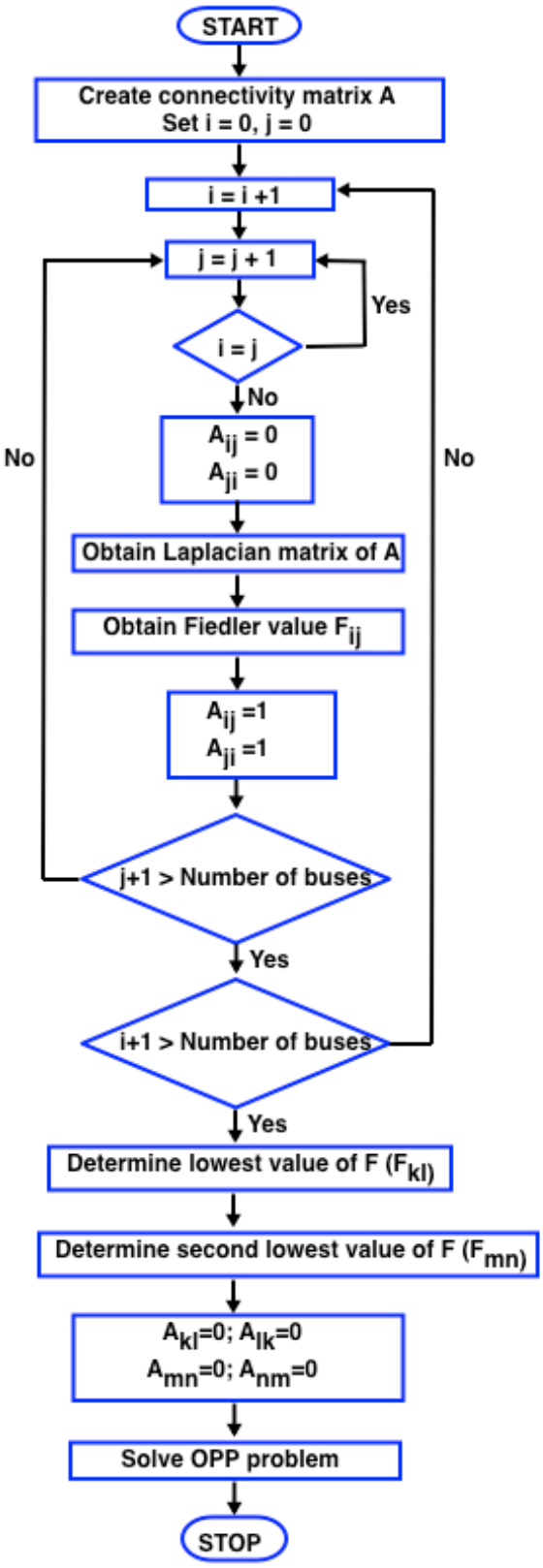


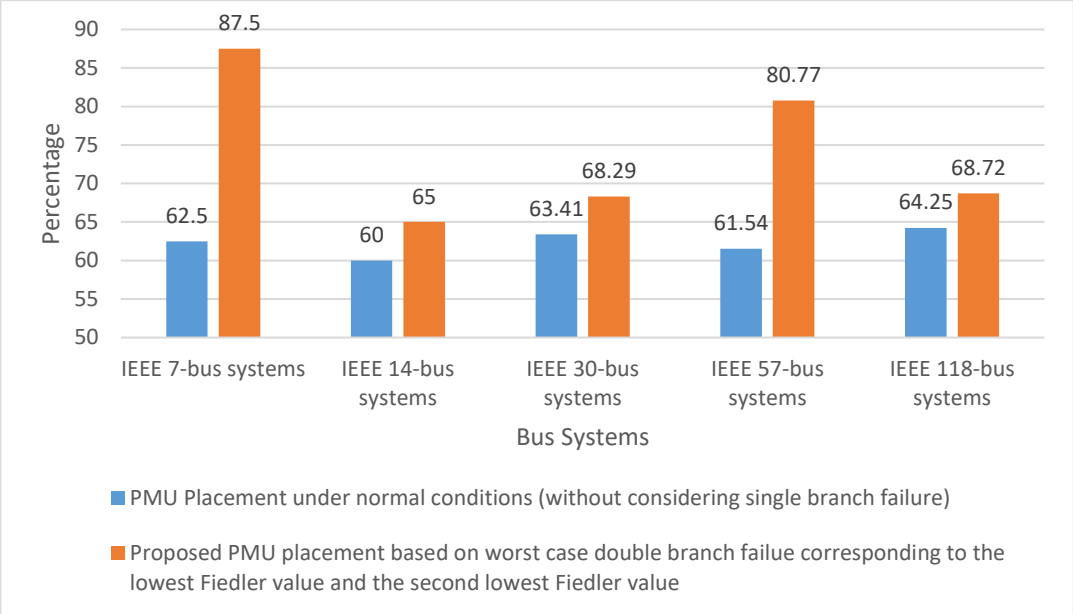
Figure 4.7 Flowchart of the algorithm used for Approach 3

The number of PMUs and their bus placement is demonstrated in Table 4.6 below.

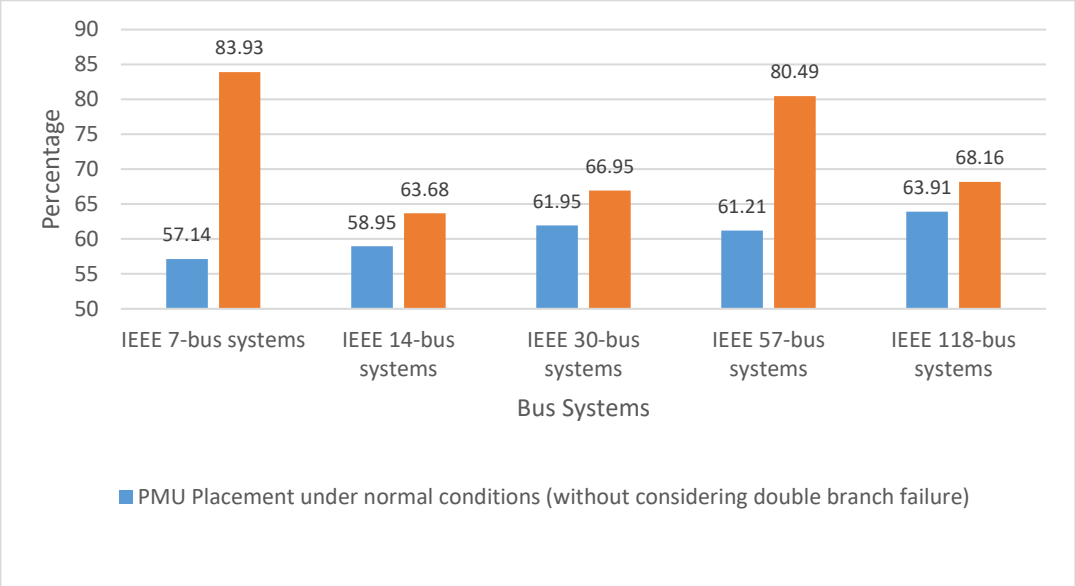
Table 4.6 Improved PMU Placement Covering Double Branch Failure Corresponding to the Lowest Fiedler value and the Second Lowest Fiedler Value

Test System	PMU placement	
	No. of PMUs	PMU Location
IEEE 7 bus system	4	1,2,3,5
IEEE 14 bus system	4	2,6,8,9
IEEE 30 bus system	11	1,2,6,10,11,12,13,15,19,25,27
IEEE 57 bus system	24	1,4,6,9,15,19,20,22,24,26,28,29,30,33, 34,37,38,41,46,47,50,53,54,56
IEEE 118 bus system	36	2,5,9,11,12,15,17,21,25,28,29, 34,37,40,45,49,52,56,62,64,68, 72,73,75,77,80,85,87,90,91,94, 101,102,105,110,114

After obtaining the improved PMU placement using the mentioned approach, the next step is to compare the resultant PMU placement with that of the optimal PMU placement done under normal conditions without any contingency consideration. The comparison shall be based on the percentage of fully observable systems during any single or double branch failure combinations that each placement can achieve. The results are presented in Figure (4.8 A and B) below.



(A)



(B)

Figure 4.8 Performance of PMU Placement Approach 3 for (A) All Single Branch Failures and (B) All Double Branch Failure Combinations

As evident in the results plotted above, the proposed system provides a higher percentage of fully observable systems in the event of both any single branch failure as well as any double branch failure combination while at the same time taking into consideration the double branch failure with the highest impact on system connectivity as indicated by the lowest and second lowest Fiedler values. . The number of PMUs calculated by the proposed approach is compared to the number of PMUs required under normal conditions and to the number of PMUs that guarantees 100% of fully observable systems during any single branch failure that can occur by guaranteeing that each bus is observable by at least two PMUs is shown in Table 4.7.

Table 4.7 Number of PMUs required to maintain full observability in case of double branch failure as per Approach 3

Test System	No. of PMUs determined by Approach 3	No. of PMUs where every bus is observable by 2 PMUs
IEEE 7-bus system	4	5
IEEE 14-bus system	4	7
IEEE 30-bus system	11	14
IEEE 57-bus system	24	31
IEEE 118-bus system	36	50

While able to guarantee observability in the case of any single branch failure, the normal approach utilizing at least two PMUs to observe every bus requires a high number of PMUs to maintain observability.

4.3.4 Approach 4 - PMU Placement to Overcome Double Branch Failure corresponding to the Lowest Fiedler value

In this approach, the effect of the double branch failure corresponding to the lowest Fiedler value shall be studied. The methodology used to determine single branch failure corresponding to the lowest Fiedler value will be used in this approach. However instead of eliminating just single branches one at a time and calculating the resultant Fiedler values, in this case all double branch failure combinations will be examined, each time calculating the corresponding Fiedler value. The number of double branch failure combinations can be expressed as *n choose 2* combinations, where *n* is the total number of branches in the system.

The PMU placement problem shall be solved based on eliminating the two branches with the lowest corresponding Fiedler and solving the optimal PMU placement problem with those branches excluded from the system. A flow chart detailing the algorithm for the proposed approach is shown in Figure 4.9.

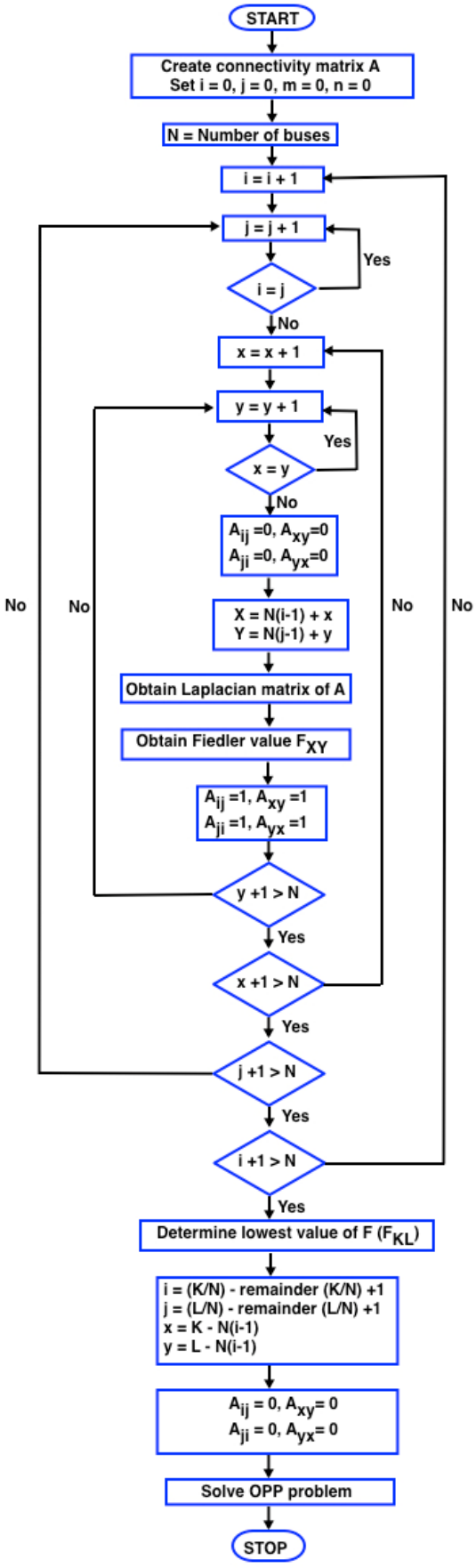


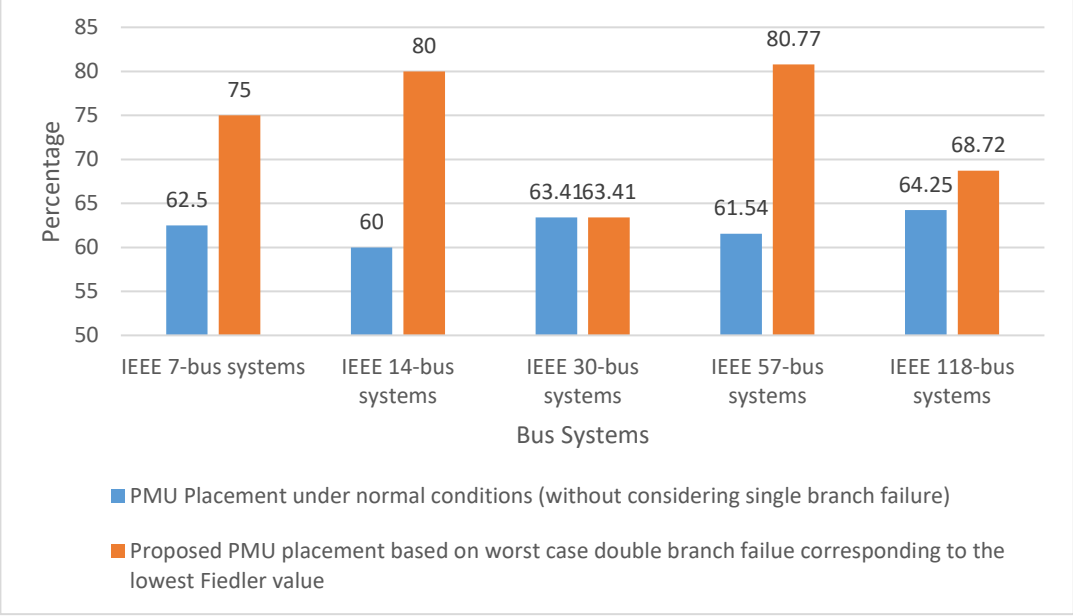
Figure 4.9 Flowchart of the algorithm used for Approach 4

The number of PMUs and their bus placement is demonstrated in Table 4.8 below.

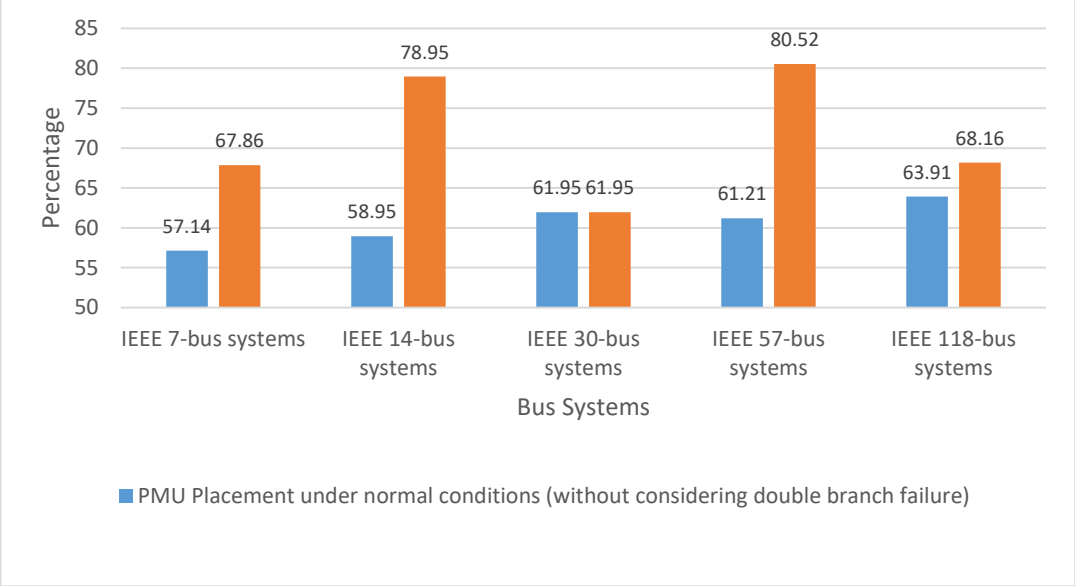
Table 4.8 Improved PMU Placement Covering Double Branch Failure Corresponding to the Lowest Fiedler value

Test System	PMU placement	
	No. of PMUs	PMU Location
IEEE 7 bus system	3	1,2,4
IEEE 14 bus system	5	2,6,7,10,13
IEEE 30 bus system	11	1,2,6,9,10,12,15, 18,20,25,27
IEEE 57 bus system	25	1,4,6,9,14,19,20,22,26,29,30, 32,33,34,36,37,41,44,45,48,50, 51,53,54,56
IEEE 118 bus system	38	2,5,9,11,12,15,17,21,23,25,28,29, 30,34,37,40,45,49,52,56,62,63, 64,68,70,71,75,77,80,85,87,90, 91,94,101,105,110,114

After obtaining the improved PMU placement using the mentioned approach, the next step is to compare the resultant PMU placement with that of the optimal PMU placement done under normal conditions without any contingency consideration. The comparison shall be based on the percentage of fully observable systems during any single or double branch failure combinations that each placement can achieve. The results are presented in Figure (4.10 A and B) below.



(A)



(B)

Figure 4.10 Performance of PMU Placement Approach 4 for (A) All Single Branch Failures and (B) All Double Branch Failure Combinations

As evident in the results plotted above, the proposed system provides a higher percentage of fully observable systems in the event of both any single branch failure as well as any double branch failure combination while at the same time taking into consideration the double branch failure with the highest impact on system connectivity as indicated by the lowest Fiedler value. Furthermore, regarding the improved PMU placement for the IEEE 14-bus system and IEEE 57-bus system, the results show that proposed PMU placement method provides a significantly higher percentage of fully observable systems for all single and double branch failures than the PMU placement method under normal conditions with only a minor increase in the required number of PMUs. The number of PMUs calculated by the proposed approach is compared to the number of PMUs required under normal conditions and to the number of PMUs that guarantees 100% of fully observable systems during any single branch failure that can occur by guaranteeing that each bus is observable by at least two PMUs is shown in Table 4.9.

Table 4.9 Number of PMUs required to maintain full observability in case of double branch failure as per Approach 4

Test System	No. of PMUs determined by Approach 4	No. of PMUs where every bus is observable by 2 PMUs
IEEE 7-bus system	3	5
IEEE 14-bus system	5	7
IEEE 30-bus system	11	14
IEEE 57-bus system	25	31
IEEE 118-bus system	38	50

While able to guarantee observability in the case of any single branch failure, the normal approach utilizing at least two PMUs to observe every bus requires a high number of PMUs to maintain observability.

4.3.5 Approach 5 - PMU Placement to Overcome System Partitioning corresponding to Fiedler Vector

In the discussed approaches above, the lowest algebraic connectivity of graphs for a given connected system was used to identify the single or double branch failures that would have the worst impact on the system connectivity and thus its observability. In the event of failure of the branches deduced from the algebraic connectivity, the system connectivity would be affected but still remain connected to a certain degree.

The concept of Fiedler vector is a more severe approach to system connectivity as it can be used as explained previously to identify the specific number of branches that would cause partitioning of the system thus resulting in two separate subsystems [94]. In the following approach, Fiedler vector of a system is used to identify the minimum number of branches the failure of which would cause system partitioning and aid in providing a fault tolerant improved PMU placement that would guarantee full system observability even in the event of partitioning of the system. Based on the sign of the elements of the Fiedler vector of a system, the resultant subsystems can be identified.

By comparing the connections of these systems with the original connectivity matrix, the number and location of the branches to be severed can be deduced, thus improved PMU placement can be generated to overcome the failure of those branches by deleting the elements in the system's original connectivity matrix and solving the optimal placement problem for the resultant matrix.

Using the IEEE 7-bus system as an example to this approach, the Fiedler vector for such a system is [-0.5579, -0.2273, -0.0582, 0.29, 0.7117, -0.2029, 0.0445]. As it is observed from the Fiedler vector, two subsets can be formed from the resultant vector, the elements of which represent the buses in the IEEE 7-bus system, based on the sign on each vector element. In this case, the two subsets are {-0.5579, -0.2273, -0.0582, -0.2029} and {0.29, 0.7117, 0.0445}. Using the corresponding element number in the Fiedler vector, two subsets including the elements' orders in the vector can be created {1, 2, 3, 6} and {4, 5, 7}. The IEEE 7-bus system can then be modified to display the connections of each subset individually by removing all other branches in the system connecting both subsets, thus creating two sub-systems. The resultant system representation is shown in Figure 4.7 below.

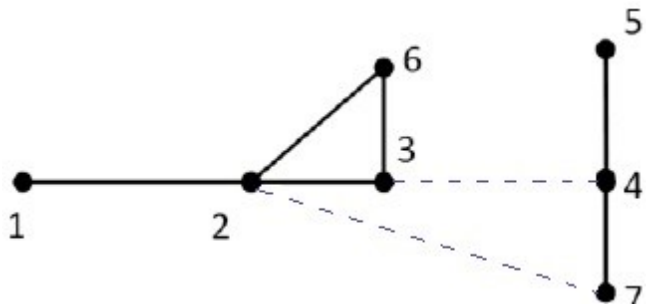


Figure 4.11 IEEE 7-bus system partitioned to two sub-systems

As shown in the figure above, by defining the two subsets as per Fiedler vector values, the branches between buses 3 and 4; and 2 and 7 were deleted to create the partitioned sub-systems. To obtain the improved PMU placement to overcome such partitioning, the connectivity matrix of the IEEE 7-bus system is modified to omit the branch connection between the sub-systems and the modified matrix is used to obtain the PMU's number and location in the system. A flow chart detailing the algorithm for the proposed approach is shown in Figure 4.12.

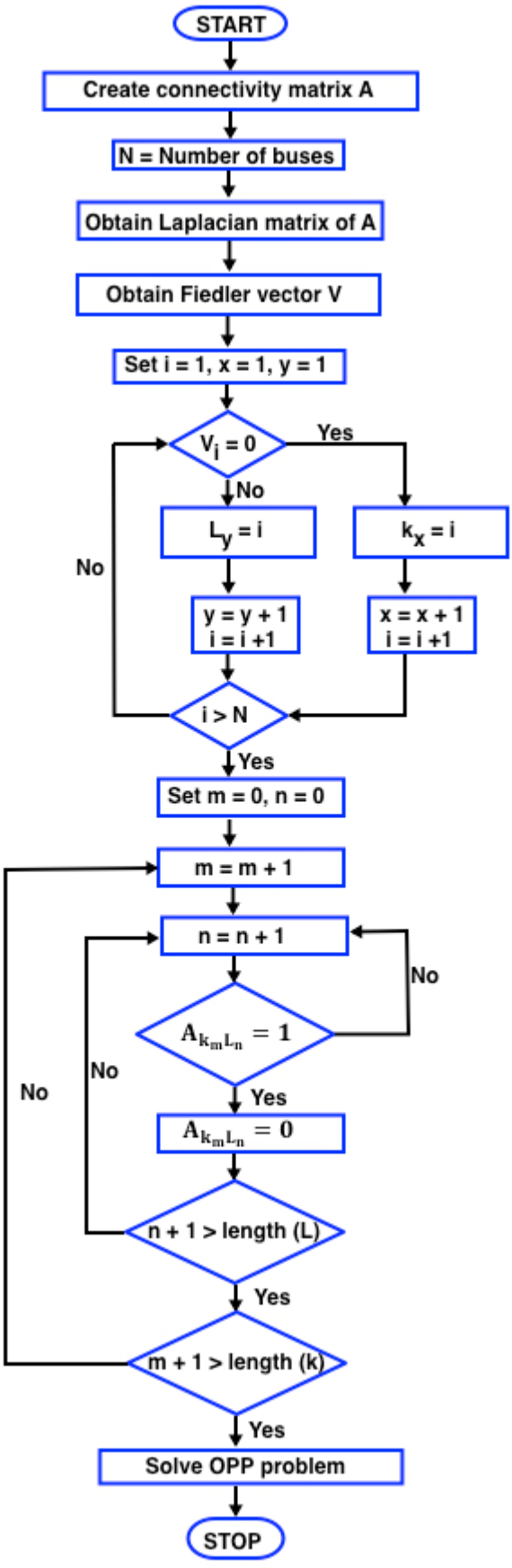


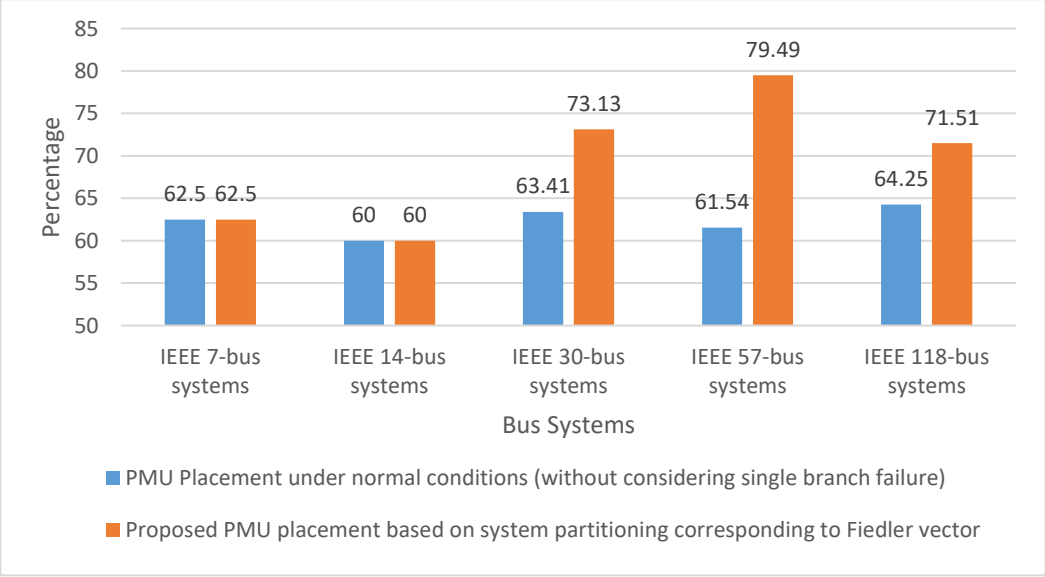
Figure 4.12 Figure 4.9 Flowchart of the algorithm used for Approach 5

The improved PMU placement in case of system partitioning for the IEEE 7-bus, IEEE 14-bus, IEEE 30-bus, IEEE 57-bus and IEEE 118 systems is presented in Table 4.10 below.

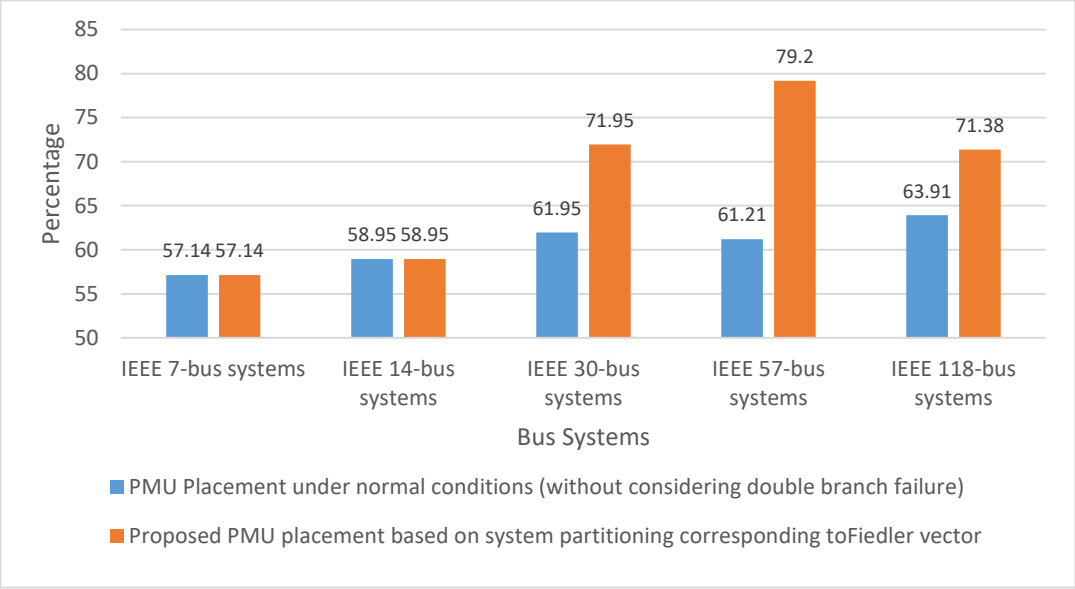
Table 4.10 Improved PMU Placement in case of System Partitioning

Test System	PMU placement		
	No. of branch failures	No. of PMUs	PMU Location
IEEE 7 bus system	2	2	2,4
IEEE 14 bus system	3	4	2,7,11,13
IEEE 30 bus system	11	13	1,2,6,9,10,12,16,19,23,24, 25,27,28
IEEE 57 bus system	15	27	1,4,8,10,11,13,19,20,23,25, 26,29,30,32,36,37,44,45,46, 47,49,51,52,53,54,55,56
IEEE 118 bus system	11	37	2,5,9,10,12,15,17,21,23,28, 29,30,34,37,40,45,49,52, 56,62,64,68,71,75,77,80,85, 86,87,89,92,94,100,105,110, 114,115

After obtaining the improved PMU placement using the mentioned approach, the next step is to check the percentage of fully observable systems in the case of any single or double branch failures that can occur in the system. The results are presented below in Figure (4.13 A and B).



(A)



(B)

Figure 4.13 Performance of PMU Placement Approach 5 for (A) All Single Branch Failures and (B) All Double Branch Failure Combinations

As evident in the results plotted above, the proposed system provides a higher percentage of fully observable systems for the IEEE 30-bus, IEEE 57-bus and IEEE 118-bus systems and maintaining the same percentage of fully observable systems in the case of the smaller test systems in the event of both any single branch failure as well as any double branch failure combination while at the same time taking into consideration the possibility of complete system partitioning. The number of PMUs calculated by the proposed approach is compared to the number of PMUs required under normal conditions and to the number of PMUs that guarantees 100% of fully observable systems during any single branch failure that can occur by guaranteeing that each bus is observable by at least two PMUs is shown in Table 4.11.

Table 4.11 Number of PMUs required to maintain full observability in case system partitioning as per Approach 5

Test System	No. of PMUs determined by Approach 5	No. of PMUs where every bus is observable by 2 PMUs
IEEE 7-bus system	3	5
IEEE 14-bus system	5	7
IEEE 30-bus system	11	14
IEEE 57-bus system	25	31
IEEE 118-bus system	38	50

While able to guarantee observability in the case of any single branch failure, the normal approach utilizing at least two PMUs to observe every bus requires a high number of PMUs to maintain observability.

4.3.6 Approach 6 - PMU Placement to Overcome Single Branch Failure corresponding to the Lowest Fiedler value coupled with Zero Injection Effect

In this approach, the effect of the single branch failure corresponding to the lowest Fiedler value while also taking into consideration the effect of zero injection buses shall be studied. As explained in Chapter 3, taking into consideration the effect of zero injection buses under normal conditions (without taking into consideration fault tolerance against any single branch failure) decreases the number of PMUs that are needed to be installed in a system to provide full observability by taking advantage of the indirect measurement using KCL. For the 9-bus test system used to explain the effect of zero injection buses in Chapter 3, below in Table 4.11 the number of PMUs required for full system observability under normal conditions is presented, with and without taking into consideration the zero injection effect.

Table 4.12 Optimal PMU Placement with and without Zero Injection Effect

Test System	PMU placement without considering zero injection effect		PMU placement considering zero injection effect	
	No. of PMUs	PMU Location	No. of PMUs	PMU Location
9-bus test system	3	4,6,8	2	6,9

As expected, while considering the effect of zero injection buses on the improved PMU placement, the number of PMUs required to achieve full observability is lower than when not considering the effect of zero injection buses. Next, the effect of the single branch failure contingency corresponding to the lowest Fiedler value shall be studied with and without the consideration of the effect of zero injection buses. The improved PMU placement for both approaches in presented in Table 4.12 below.

Table 4.13 Improved PMU Placement to Overcome Single Branch Failure corresponding to the Lowest Fiedler Value with and without Zero Injection Effect

Test System	PMU placement without considering zero injection effect		PMU placement considering zero injection effect	
	No. of PMUs	PMU Location	No. of PMUs	PMU Location
9-bus test system	4	2,4,6,9	3	2,6,9

While both approaches demonstrate the need for an additional PMU to overcome single branch failure corresponding to Fiedler value, the approach taking zero injection effect into consideration still provides the required system observability with fewer PMUs. However, the advantage of considering the effect of zero injection buses is only due to the fact that buses connected to the zero injection bus can be observable in the case that the zero injection bus and every other bus connected to it is observable through KCL. That advantage relies heavily on the ability of the system to stay connected and in the event of branch failures the level of observability could be greatly reduced. The percentage of fully observable systems in the case of any single

or double branch failures for the PMU placement presented above is demonstrated in Figure 4.14.

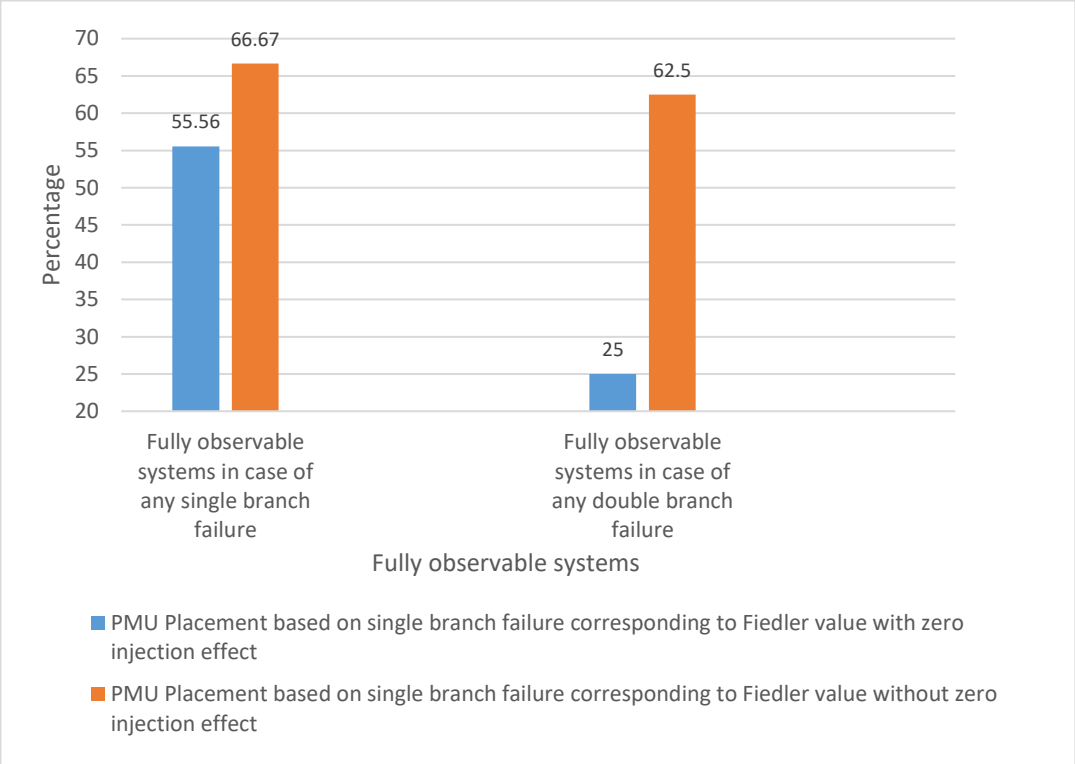


Figure 4.14 Performance of PMU Placement Approach 6 in case of Single and Double Branch Failures

As evident, the improved PMU placement approach taking into consideration the zero injection effect has a somewhat poor performance, especially in the case of contingency against any double branch failure, as the main advantage of KCL is challenged by the removal of branches from the system.

Chapter 5
Conclusion and Future Work

In this thesis, a fault-tolerant PMU placement methodologies based on tools from the algebraic connectivity of graphs is proposed. By describing the smart grid as a graph, the branch whose failure will have the highest effect on the system connectivity based on the lowest Fiedler value can be identified. With the understanding that the grid observability is directly related to the graph connectivity, the PMU placement problem can be formulated to make sure that the system will be fully observable under the failure of the branch that has the highest effect on the grid connectivity. Furthermore, the approach is further enhanced by taking into consideration the effect of the second lowest Fiedler value of a system as a standalone concept to identify single branch failures as well as incorporating the concept with the actual lowest Fiedler value to identify the double branch failure with the highest effect on the system connectivity.

As expected, comparing the proposed PMU placement with the PMU placement under normal conditions (without considering the effect of single or double branch failures), the results show a superiority in the performance of the proposed PMU placement in terms of the percentages of fully observable systems in case of any single or double branch failures with a reasonable increase in number of PMUs used. Furthermore, in some cases an increase in performance is obtainable by providing an alternate distribution of PMUs without increasing the number of PMUs to be installed in the system providing a net gain of performance at no additional cost.

In addition to using the concept of Fiedler value to determine improved PMU placement, the concept of using the Fiedler vector of a system is proposed to identify the branches in any given system whose failure would result in a complete system partitioning and propose an improved PMU placement approach to provide a contingency for that occurrence. The performance of the approach is also tested against conventional PMU placement in terms of evaluating the percentage of fully observable system in case of any single or double branch failure that could occur in the system.

Fiedler value and Fiedler vector to identify branch failures can be used in tandem with other contingencies such as the ones explained in Chapter 3. As an example of the effect of such contingency merger to determine the optimal PMU placement, the effect of zero injection buses was added as a constraint to the optimal PMU placement problem to overcome single branch failure corresponding to the lowest Fiedler value of the system. The performance of such merger of contingencies was reviewed in terms of the percentage of systems that remain fully observable in the event of any single branch failure. As it was demonstrated, the effect of zero injection buses contingency on improved PMU placement did not provide a decent performance as it was only able to fully observe a small percentage of systems in the event of any double branch failure. A further deduction can be interpreted in this area that not all contingency considerations provide an overall increase in performance.

Further exploration is possible where more contingencies can be taken into consideration, such as limited number of PMU channels, different probabilities of branch failures, etc. Furthermore, Fiedler value can be used to determine more than double branch failure contingencies as any number of branches can be removed and the optimal PMU placement problem can be solved accordingly.

In addition, the effect of more than one additional contingency to the branch failures corresponding to Fiedler value can be used. For example in addition to using Fiedler value, the optimal PMU placement problem can be solved while considering the effect of measurement loss as well as limited or different PMU channel capacity. While it may not guarantee a higher performance like the case of combining the effect of zero injection buses with branch failures, it still presents an opportunity to further study the different effects of different combination of contingencies.

The work into using Fiedler vector to determine branch failure contingencies can also be enhanced by studying the effect of partitioning the system into more than two subsystems, while also studying those subsystems individually as if they were their own systems. This way all contingencies can be studied further as presented in this thesis but while incorporating the effect of Fiedler vector at the beginning.

Bibliography

1. Karimi, K.J., *Power system static state estimation with phasor measurements*. 1986, Cornell Univ., Ithaca, NY (USA).

2. Phadke, A.G., *Synchronized phasor measurements in power systems.* IEEE Computer Applications in power, 1993. **6**(2): p. 10-15.

3. Zhong, J., *Phasor measurement unit (PMU) placement optimisation in power transmission network based on hybrid approach.* 2012.

4. Phadke, A.G. and J.S. Thorp, *Synchronized phasor measurements and their applications*. Vol. 1. 2008: Springer.

5. Cheng, Y., X. Hu, and B. Gou. *A new state estimation using synchronized phasor measurements*. in *Circuits and Systems, 2008. ISCAS 2008. IEEE International Symposium on*. 2008. IEEE.

6. *USNO NAVSTAR Global Positioning System*, http://tycho.usno.navy.mil/gpsinfo.html. 2012.

7. Allan, D.W., N. Ashby, and C.C. Hodge, *The science of timekeeping*. 1997: Hewlett-Packard.

8. Liu, Y., et al., *State estimation and voltage security monitoring using synchronized phasor measurements.* 2001.

9. *Phasor Measurement Unit model*, http://www.macrodyneusa.com/model_1690.htm. 2011.

10. *Consortium for Electric Reliability Technology Solution*, http://www.phasor-rtdms.com/phaserconcepts/phasor_adv_faq.html. . 2012.

11. Allemong, J., L. Radu, and A. Sasson, *A fast and reliable state estimation algorithm for AEP's new control center.* IEEE Transactions on Power Apparatus and systems, 1982(4): p. 933-944.

12. Abbasy, N.H. and H.M. Ismail, *A unified approach for the optimal PMU location for power system state estimation.* IEEE Transactions on Power Systems, 2009. **24**(2): p. 806-813.

13. Handschin, E., et al., *Bad data analysis for power system state estimation.* IEEE Transactions on Power Apparatus and Systems, 1975. **94**(2): p. 329-337.

14. Chen, J. and A. Abur, *Placement of PMUs to enable bad data detection in state estimation.* IEEE Transactions on Power Systems, 2006. **21**(4): p. 1608-1615.

15. Dopazo, J., et al., *Implementation of the AEP real-time monitoring system.* IEEE Transactions on Power Apparatus and Systems, 1976. **95**(5): p. 1618-1629.

16. Yin, H. and L. Fan. *PMU data-based fault location techniques*. in *North American Power Symposium (NAPS), 2010*. 2010. IEEE.

17. Lin, Y.-H., C.-W. Liu, and C.-S. Chen, *A new PMU-based fault detection/location technique for transmission lines with consideration of arcing fault discrimination-part I: theory and algorithms.* IEEE Transactions on power delivery, 2004. **19**(4): p. 1587-1593.

18. Lin, Y.-H., C.-W. Liu, and C.-S. Chen, *A new PMU-based fault detection/location technique for transmission lines with consideration of arcing fault discrimination-part II: performance evaluation.* IEEE transactions on power delivery, 2004. **19**(4): p. 1594-1601.

19. Ibe, A. and B. Cory, *A travelling wave-based fault locator for two-and three-terminal networks.* IEEE transactions on power delivery, 1986. **1**(2): p. 283-288.

20. Takagi, T., et al., *Development of a new type fault locator using the one-terminal voltage and current data.* IEEE Transactions on Power apparatus and systems, 1982(8): p. 2892-2898.

21. De La Ree, J., et al., *Synchronized phasor measurement applications in power systems.* IEEE Transactions on Smart Grid, 2010. **1**(1): p. 20-27.

22. Fiedler, M., *Eigenvalues of nonnegative symmetric matrices.* Linear Algebra and its Applications, 1974. **9**: p. 119-142.

23. Raju, V.V.R. and S.J. Kumar. *An optimal PMU placement method for power system observability*. in *Power and Energy Conference at Illinois (PECI), 2016 IEEE*. 2016. IEEE.

24. Roy, B.S., A. Sinha, and A. Pradhan, *An optimal PMU placement technique for power system observability.* International Journal of Electrical Power & Energy Systems, 2012. **42**(1): p. 71-77.

25. *Keithley's Ultra-Fast Current-Voltage System Combines Three Essential Characterization Capabilities in One Chassis*, http://www.keithley.com/news/prod021810. 2012.

26. Manousakis, N., G. Korres, and P. Georgilakis. *Optimal placement of phasor measurement units: A literature review*. in *Intelligent System Application to Power Systems (ISAP), 2011 16th International Conference on*. 2011. IEEE.

27. Farsadi, M., H. Golahmadi, and H. Shojaei. *Phasor measurement unit (PMU) allocation in power system with different algorithms*. in *Electrical and Electronics Engineering, 2009. ELECO 2009. International Conference on*. 2009. IEEE.

28. CAi, T. and Q. Ai. *Research of PMU optimal placement in power systems*. in *International conference on system theory and scientific computation*. 2005.

29. Haynes, T.W., et al., *Domination in graphs applied to electric power networks.* SIAM Journal on Discrete Mathematics, 2002. **15**(4): p. 519-529.

30. Yuan, X.A. *A linear algorithm for minimum Phasor measurement units placement*. in *Innovative Smart Grid Technologies (ISGT), 2010*. 2010. IEEE.

31. Zhou, M., et al. *A preprocessing method for effective PMU placement studies*. in *Electric Utility Deregulation and Restructuring and Power Technologies, 2008. DRPT 2008. Third International Conference on*. 2008. IEEE.

32. Marin, F., et al., *Genetic algorithms for optimal placement of phasor measurement units in electrical networks.* Electronics Letters, 2003. **39**(19): p. 1403-1405.

33. Milosevic, B. and M. Begovic, *Nondominated sorting genetic algorithm for optimal phasor measurement placement.* IEEE Transactions on Power Systems, 2003. **18**(1): p. 69-75.

34. Kennedy, J., *Particle swarm optimization*, in *Encyclopedia of machine learning*. 2011, Springer. p. 760-766.

35. Sadu, A., R. Kumar, and R.G. Kavasseri. *Optimal placement of phasor measurement units using particle swarm optimization*. in *Nature & Biologically Inspired Computing, 2009. NaBIC 2009. World Congress on*. 2009. IEEE.

36. Ahmadi, A., Y. Alinejad-Beromi, and M. Moradi, *Optimal PMU placement for power system observability using binary particle swarm optimization and considering measurement redundancy.* Expert Systems with Applications, 2011. **38**(6): p. 7263-7269.

37. Aminifar, F., et al., *Contingency-constrained PMU placement in power networks.* IEEE Transactions on Power Systems, 2010. **25**(1): p. 516-523.

38. Chakrabarti, S., E. Kyriakides, and D.G. Eliades, *Placement of synchronized measurements for power system observability.* IEEE Transactions on Power Delivery, 2009. **24**(1): p. 12-19.

39. Dua, D., et al., *Optimal multistage scheduling of PMU placement: An ILP approach.* IEEE Transactions on Power Delivery, 2008. **23**(4): p. 1812-1820.

40. Chakrabarti, S. and E. Kyriakides, *Optimal placement of phasor measurement units for power system observability.* IEEE Transactions on power systems, 2008. **23**(3): p. 1433-1440.

41. Nazari-Heris, M. and B. Mohammadi-Ivatloo, *Application of heuristic algorithms to optimal PMU placement in electric power systems: An updated review.* Renewable and Sustainable Energy Reviews, 2015. **50**: p. 214-228.

42. Deo, N., *Graph Theory with applications to engineering and computer science*. 1974: Prentice -Hall Inc.

43. Lassonde, W., *Nathan Jensen Advisor: Professor Samee Khan Group SD0903.* Phasor measurement unit placements for complete observability using linear-time, quadratic-time, and subquadratic-time heuristics, NDSU ECE wiki, 2010.

44. Abdelaziz, A.Y., A.M. Ibrahim, and R.H. Salem. *Optimal PMU placement for complete observability using heuristic methods*. in *Proceedings of the 15th International Middle East Power Systems Conference, Alexandria, Egypt*. 2012.

45. Yuill, W., et al. *Optimal PMU placement: A comprehensive literature review*. in *Power and Energy Society General Meeting, 2011 IEEE*. 2011. IEEE.

46. Cormen, T.H., *Introduction to algorithms*. 2009: MIT press.

47. *The TOMLAB Optimization Environment*, http://tomlab.biz/. 2011.

48. Nazari-Heris, M. and B. Mohammadi-Ivatloo, *Optimal placement of phasor measurement units to attain power system observability utilizing an upgraded binary harmony search algorithm.* Energy Systems, 2015. **6**(2): p. 201-220.

49. Zhao, Y., et al., *Optimal PMU placement considering topology constraints.* International Journal of Electrical Power & Energy Systems, 2015. **73**: p. 240-248.

50. Park, Y.M., et al., *Design of reliable measurement system for state estimation.* IEEE Transactions on Power systems, 1988. **3**(3): p. 830-836.

51. Abur, A. and F.H. Magnago, *Optimal meter placement for maintaining observability during single branch outages.* IEEE Transactions on Power Systems, 1999. **14**(4): p. 1273-1278.

52. Magnago, F.H. and A. Abur. *A unified approach to robust meter placement against loss of measurements and branch outages*. in *Power Industry Computer Applications, 1999. PICA'99. Proceedings of the 21st 1999 IEEE International Conference*. 1999. IEEE.

53. Rakpenthai, C., et al., *An optimal PMU placement method against measurement loss and branch outage.* IEEE transactions on power delivery, 2007. **22**(1): p. 101-107.

54. Korres, G.N., et al., *Optimal phasor measurement unit placement for numerical observability in the presence of conventional measurements using semi-definite programming.* IET Generation, Transmission & Distribution, 2015. **9**(15): p. 2427-2436.

55. Huang, J. and N.E. Wu, *Fault-tolerant placement of phasor measurement units based on control reconfigurability.* Control Engineering Practice, 2013. **21**(1): p. 1-11.

56. Mohar, B., et al., *The Laplacian spectrum of graphs.* Graph theory, combinatorics, and applications, 1991. **2**(871-898): p. 12.

57. Fiedler, M., *Algebraic connectivity of graphs.* Czechoslovak mathematical journal, 1973. **23**(2): p. 298-305.

58. Chopade, P. and M. Bikdash. *Critical infrastructure interdependency modeling: Using graph models to assess the vulnerability of smart power grid and SCADA networks*. in *Emerging Technologies for a Smarter World (CEWIT), 2011 8th International Conference & Expo on*. 2011. IEEE.

59. Holroyd, M.J., *Synchronizability and connectivity of discrete complex systems*. 2006, College of William and Mary.

60. Watts, D.J. and S.H. Strogatz, *Collective dynamics of 'small-world'networks.* nature, 1998. **393**(6684): p. 440-442.

61. Chung, F.R., *Spectral graph theory*. 1997: American Mathematical Soc.

62. Godsil, C. and G. Royle, *Algebraic graph theory, volume 207 of Graduate Texts in Mathematics*. 2001, Springer-Verlag, New York.

63. Grone, R., R. Merris, and V.S. Sunder, *The Laplacian spectrum of a graph.* SIAM Journal on Matrix Analysis and Applications, 1990. **11**(2): p. 218-238.

64. Merris, R., *Degree maximal graphs are Laplacian integral.* Linear Algebra and its Applications, 1994. **199**: p. 381-389.

65. Merris, R., *Laplacian matrices of graphs: a survey.* Linear algebra and its applications, 1994. **197**: p. 143-176.

66. Merris, R., *A survey of graph Laplacians.* Linear and Multilinear Algebra, 1995. **39**(1-2): p. 19-31.

67. Mohar, B., *Laplace eigenvalues of graphs—a survey.* Discrete mathematics, 1992. **109**(1-3): p. 171-183.

68. Grone, R. and R. Merris, *Algebraic connectivity of trees.* Czechoslovak Mathematical Journal, 1987. **37**(4): p. 660-670.

69. Grone, R. and R. Merris, *Ordering trees by algebraic connectivity.* Graphs and Combinatorics, 1990. **6**(3): p. 229-237.

70. Kirkland, S. and M. Neumann, *Algebraic connectivity of weighted trees under perturbation.* Linear and Multilinear Algebra, 1997. **42**(3): p. 187-203.

71. McKay, B.D., *On the spectral characterisation of trees.* Ars Combin, 1977. **3**: p. 219-232.

72. Merris, R., *Characteristic vertices of trees.* Linear and Multilinear Algebra, 1987. **22**(2): p. 115-131.

73. Molitierno, J.J., M. Neumann, and B.L. Shader, *Tight bounds on the algebraic connectivity of a balanced binary tree.* 2000.

74. Rojo, O. and L. Medina, *Tight bounds on the algebraic connectivity of Bethe trees.* Linear algebra and its applications, 2006. **418**(2-3): p. 840-853.

75. Juhász, F., *The asymptotic behaviour of Fiedler's algebraic connectivity for random graphs.* Discrete mathematics, 1991. **96**(1): p. 59-63.

76. Shy, L. and B. Eichinger, *Large computer simulations on elastic networks: Small eigenvalues and eigenvalue spectra of the Kirchhoff matrix.* The Journal of Chemical Physics, 1989. **90**(9): p. 5179-5189.

77. Berman, A. and X.-D. Zhang, *Lower bounds for the eigenvalues of Laplacian matrices.* Linear Algebra and its Applications, 2000. **316**(1-3): p. 13-20.

78. Fallat, S. and S. Kirkland, *Extremizing algebraic connectivity subject to graph theoretic constraints.* Electronic Journal of Linear Algebra, 1998. **3**(1): p. 7.

79. Fiedler, M., *A property of eigenvectors of nonnegative symmetric matrices and its application to graph theory.* Czechoslovak Mathematical Journal, 1975. **25**(4): p. 619-633.

80. Fiedler, M., *Absolute algebraic connectivity of trees.* Linear and Multilinear Algebra, 1990. **26**(1-2): p. 85-106.

81. Fallat, S.M., S. Kirkland, and S. Pati, *Maximizing algebraic connectivity over unicyclic graphs.* Linear and Multilinear Algebra, 2003. **51**(3): p. 221-241.

82. Mohar, B., *Eigenvalues, diameter, and mean distance in graphs.* Graphs and combinatorics, 1991. **7**(1): p. 53-64.

83. Molitierno, J.J., *On the algebraic connectivity of graphs as a function of genus.* Linear algebra and its applications, 2006. **419**(2-3): p. 519-531.

84. Cheeger, J., *A lower bound for the smallest eigenvalue of the Laplacian.* Problems in analysis, 1970: p. 195-199.

85. Belhaiza, S., et al., *Variable neighborhood search for extremal graphs. XI. bounds on algebraic connectivity.* Graph theory and combinatorial optimization, 2005: p. 1-16.

86. Cvetković, D., M. Čangalović, and V. Kovačević-Vujčić, *Optimization and highly informative graph invariants.* Zbornik Radova, 2004(18): p. 5-40.

87. Fiedler, M., *Some minimax problems for graphs.* Discrete Mathematics, 1993. **121**(1): p. 65-74.

88. Kirkland, S.J., *An upper bound on algebraic connectivity of graphs with many cutpoints.* Electronic Journal of Linear Algebra, 2001. **8**(1): p. 8.

89. Maas, C., *Transportation in graphs and the admittance spectrum.* Discrete Applied Mathematics, 1987. **16**(1): p. 31-49.

90. Ghosh, A. and S. Boyd, *Upper bounds on algebraic connectivity via convex optimization.* Linear algebra and its applications, 2006. **418**(2-3): p. 693-707.

91. Merris, R., *Laplacian graph eigenvectors.* Linear algebra and its applications, 1998. **278**(1-3): p. 221-236.

92. Lefevre, J., *Fiedler vectors and elongation of graphs: a threshold phenomenon on a particular class of trees.* arXiv preprint arXiv:1302.1266, 2013.

93. Hosainy, M.E., K. Seddik, and A. Elezabi, *Fault-Tolerant PMU Placement usin Algebraic Connectivity of Graphs*, in *Innovative Smart Grid Technologies (ISGT)*. 2017: Torino, Italy.

94. Huang, L., et al., *Optimal PMU placement considering controlled islanding of power system.* IEEE Transactions on Power Systems, 2014. **29**(2): p. 742-755.