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A priori skepticism, conventionalism and contextualism

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A Priori Skepticism, Conventionalism and Contextualism

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DEDICATION

To the memories of my beautiful Aunt Maha Salem who radiated so much love, and my grandfather Abdelkader Salem who always told me who I should be, and how.

ABSTRACT

In this thesis we propose a skeptical scenario about a priori knowledge. The scenario is composed of three main arguments: the a priori brain-in-a-vat argument, the problem of deduction, and the rule–following paradox. We propose a solution for a priori skepticism that is based on two philosophical schools: conventionalism and contextualism. Finally, we provide a sufficient condition – although hard to satisfy – for relieving a priori skepticism.

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1 Introduction

The idea that humans can acquire and possess knowledge *independently* of experience was rooted in philosophical thought since the times of the Ancient Greeks. In fact, two major philosophical schools emerged, in which the relationship between knowledge and experience was problematized and became the subject of conflicting epistemological outlooks. The *rationalist* school claimed that a portion of human knowledge can *only* be justified independently from experience (viz. *a priori* knowledge), while the *empiricist* school generally denied this possibility, and claimed that all knowledge can only originate in and be justified with reference to sensuous experience (viz. *a posteriori* knowledge). A third approach to knowledge is *skepticism* which comes in many forms[1]. One form is the *epistemological* form (i.e. focuses on the epistemic status of a certain proposition). Another is the *ontological* form (i.e. focuses on the state of existence of a specific entity like the 'universal'). Also, skepticism can vary in degree; it can be local or radical. *Local* skepticism is concerned with limited beliefs about specific subjects, like beliefs about mathematical objects. On the other hand, *radical* skepticism challenges all beliefs formed about all subject matters. Historically, radical epistemological skepticism can be traced back to two sources: *Pyrrhonian* skepticism and *Cartesian* skepticism. The former type counterargues any belief through some skeptical strategies that are at least as compelling as the original argument(s) offered. The latter type mainly uses the skeptical hypotheses strategy to undermine the supposed knowledge claims. In the Cartesian method, the skeptic will start by suggesting that the claimed knowledge might *possibly* be wrong, and that the skeptical hypothesis might *possibly* be true. But since we don't know whether the skeptical hypothesis is right or wrong, it follows that we also lack knowledge of the original proposition in question. A variety of skeptical hypotheses have been proposed. In Descartes's *Meditations on First Philosophy,* the *first* Meditation offered three skeptical arguments: (i) the sense illusion argument, (ii) the dream argument, and (iii) the 'Evil Genius' argument. The Evil Genius argument – which is the most radical one – might be sketched as follows: imagine that you live in a world where an Evil Genius is deceiving you about all subject matters. Ac-

[1] Our discussion of skepticism relies on Pritchard (2016).

cordingly, all your beliefs about the world are systematically false[2]. With the rise of philosophical materialism in the 20th century, some philosophers claimed that the Cartesian Evil Genius hypothesis is *logically impossible* as it is not possible for a *matterless* mind to exist[3]. Therefore, Hilary Putnam (1981) proposed a new version of the Cartesian skeptical hypothesis that is consistent with materialism: the brain-in-a-vat hypothesis[4]. Imagine that you are a disembodied brain floating in a vat, and connected to a massive computer that can produce a simulation indiscernible from external reality. Now, your experience of the external world will be *identical* with the experience of an ordinary person who is *not* a "brain-in-a-vat" (BIV[5]). Thus, the skeptical hypothesis claims our inability to show that our *basic beliefs* about the external world are *de facto* true. Denote any external world propositional claim by P, and denote the skeptical hypothesis by S_h; then the skeptical argument will take the following form[6]:

- a_1. I am unable to know the denials of S_h.
- a_2. If I am unable to know the denials of S_h, then I do not know that P is true.
- c. Thus, I do not know that P is true.

S_h is any hypothesis that is irreconcilable with commonly claimed propositional knowledge. The strength of S_h is that it imposes a scenario that is *subjectively indistinguishable* from 'normal' situations. The skeptical premise in (a) claims that it is not possible to refute S_h, and that it holds, at least in terms of logical possibility. But if (a) is correct, then any proposition P the truth of which depends on the falsehood of S_h is threatened too. Hence, the skeptic concludes in (c) that P is unknowable.

Most commonly, skeptical arguments have been applied to *a posteriori* propositions (e.g. the existence of the external world) since

[2] See Greco (2007) for an exposition of external world skepticism.
[3] See Brueckner (2016).
[4] Putnam's original formulation used the plural term "brains-in-a-vat", however most of the recent literature on external world skepticism refers to the singular "brain-in-a-vat" (maybe to stick to the Cartesian formulation).
[5] Putnam essentially used semantic externalism to argue against the brain-in-a-vat scenario.
[6] This formulation is due to Pritchard (2016)

they are falsifiable by nature, at least in principle. In this essay, however, we are interested in another underexplored type of skepticism, namely skepticism concerning the truth of a priori propositions. We call this latter type of skepticism *a priori skepticism*. Our interest in a priori skepticism stems from three different problems that are already available in philosophical literature. These problems, if combined, provide well-founded grounds for being skeptic about a priori knowledge. Generally speaking, we distinguish between two types of a priori skepticism: (i) the a priori brain-in-a-vat argument, and (ii) the problem of formal language skepticism.

The a priori brain-in-a-vat argument is obtained by twisting the external world brain-in-a-vat argument to target a priori instead of a posteriori knowledge[7]. On the other hand, the problem of formal language skepticism targets a specific type of a priori knowledge, namely our formal logical-mathematical knowledge. The problem of formal language skepticism is twofold: (a) the problem of deduction, and (b) the rule-following paradox. The former can be found in Susan Haack's (1976) critique of the justification of deductive systems. What she aims at is constructing an analogous formulation of the problem of induction for deductive inferential systems. But if we are entitled to doubt the validity of deductive arguments, then we are certainly also entitled to be skeptic about a priori knowledge. Differently, the rule-following paradox originated from Saul Kripke's (1982) interpretation of Wittgenstein's *Philosophical Investigations*. Kripke's rule-following paradox can be summarized as follows: any rule can be *restated* in a way that makes any action *conform* to it. If Kripke is correct then (almost) anything can be inferred from any rule – including logical and mathematical rules – which feeds our skepticism about a priori knowledge. Another vital aspect in the discussion of a priori skepticism revolves around who has the burden of proof? Michael Rescorla (2009) describes two general philosophical positions regarding this question. *Dialectical foundationalism* asserts that our reasoning is contingent on certain privileged propositions, and therefore we are not supposed to defend them just because they are challenged by an interlocutor. On the contrary, *Dialectical egalitarianism* argues against the claimed special status of these propositions, and that all propositions can be

[7] A priori skepticism can be found in Beebe (2011) and Willenken (2015).

philosophically challenged. In this essay, we are going to escape this debate by endorsing Dialectical egalitarianism.

At face value, it seems very difficult to be skeptic about a priori knowledge, as we have the intuition that this represents a kind of knowledge of something that holds necessarily, i.e *cannot* be otherwise. Upon further reflection, Kripke argued that this need not be the case[8]. Thus by releasing a priority from its supposedly essential relation to necessity, the task of doubting the *a priori* becomes easier. But, then, what is the source of the apparent necessity of a priori knowledge? We will argue for two main sources: conventionalism and contextualism. The rest of our thesis will mainly defend a contextualistic and a conventionalistic view of a priori knowledge, a view we label *contextual-conventionalism*. With the help of contextual-conventionalism, we will be able to track the origin of a priori skepticism to one fundamental source, namely the lack of *external* conventions. We conclude that if we want to refute a priori skepticism, we first need to resolve the problem of the absence of external conventions.

Our thesis is organized as follows: in sections 2-4 we will lay down the – aforementioned – three philosophical problems (viz. the a priori brain-in-a-vat argument, the problem of deduction, and the rule-following paradox). In section 5, we will explore the relation between a priority and modality. Necessity will be studied by relying on Kripke's notion of *rigid designators*. At the end of this investigation, it is found that rigid designators are problematic unless tied to specific contexts. By associating rigidity with specific contexts, we will end up with a *restricted* notion of rigidity, namely *contextual rigidity*. In section 6, we will move to the first pillar of our thesis by elaborating on the conventionalist tradition in philosophy from Henri Poincaré to David Lewis. We mainly defend a specific version of conventionalism that combines the views of Rudolf Carnap and David Lewis against the attacks of W.V. Quine. Then, in section 7, we will present the second pillar of our thesis, namely contextualism. In the last section, we will present two possible solutions for a priori skepticism. The first solution will be based on a reading

[8] Kripke's ideas will be discussed in detail in section 5.

of Wittgenstein's *hinge propositions* which has been used by some philosophers to counter external world skepticism[9]. Next, we show that hinge propositions might work as an anti-skeptical strategy for the a priori brain-in-a-vat hypothesis, but not for the problem of formal language skepticism. Instead, we propose a new method which combines both conventionalism and contextualism, namely *contextual-conventionalism*[10] to deal with the a priori brain-in-a-vat hypothesis *and* the problem of formal language skepticism. Finally, we provide a *sufficient* condition – call it absence of external conventions – under which contextual-conventionalism can be the remedy for a priori skepticism.

2 Formal Language Skepticism I: The Circularity Problem

2.1 Alston's Notion of Epistemic Circularity

Circular justification has always been problematic for many philosophers across history[11]. Despite being defended by a contemporary group of epistemologists including Laurence BonJour (1985) and Keith Lehrer (1974, 1990), there is still a wide resistance to this philosophical position[12]. In this section we are interested in a special type of circular justification, namely *epistemic circularity*. Generally speaking, an argument is *epistemically circular* if it argues for the *reliability* of a belief source by using premises or methods that rely on the *same* belief source in question. We owe the contemporary notion of epistemic circularity to William Alston (1986, 1989) who basically claims that there is no plausible way of having a justified belief about our basic sources of knowledge (e.g. perception, intuitive reasoning, introspection, memory, etc.) without using an epistemically circular method. The problem of such conclusion is that it allows for an 'anything-goes' argument since we can infer any proposition P by relying on a belief source that uses P itself.

[9] C.f. Crispin Wright (1985) and Peter Strawson (1985).

[10] There is another vein of semantic responses to skepticism due to Donald Davidson. See for instance Davidson (1986), and Brueckner (1992).

[11] For a general overview, see Eemeren et al (1996).

[12] See Murphy (2016)

Alston (1986) starts his investigation by posing a question (originally asked by Thomas Reid (1863)):

> For Thomas Reid the most basic epistemological issue was whether we are proceeding rationally in trusting, as we do, our basic sources of belief perception, introspection, memory, testimony, and reasoning. Less metaphorical terms than 'source of belief' would be dispositions, tendencies, or habit to form beliefs of certain kinds in certain circumstances. (1)

Alston is mainly interested in asking Reid's question regarding *perceptual beliefs*. In other words, Alston asks whether we are rationally justified in relying on perceptual experiences of the immediate physical environment to form perceptual beliefs. He uses what he calls an inductive *Track Record Argument* (TRA) to check the validity of this question. This TRA goes as follows:

1. Perceptual experiences indicate $\beta_1, \ldots, \beta_n$.
2. $\beta_1, \ldots, \beta_n$ are true.
3. Perceptual experiences indicate $\beta_1, \ldots, \beta_n$, and $\beta_1, \ldots, \beta_n$ are true.
4. Hence, perceptual experiences generate accurate truth regarding $\beta_1, \ldots, \beta_n$.
5. Therefore, perceptual beliefs formed by perceptual experiences are reliable.

The problem of the TRA appears when moving from (1) to (2). To justify that '$\beta_1, \ldots, \beta_n$ is true,' the TRA implicitly assumes that perceptual experience is reliable based on previous observations which is supposed to be the conclusion. This is what Alston labels *epistemic circularity*. In his words: "this kind of circularity involves a commitment to the conclusion as a presupposition of our supposing ourselves to be justified in holding the premises.[13]"

Lynch and Silva (2016) provide a restricted version of epistemic circularity:

[13] (1993, 14)

Definition 1. (Epistemic Circularity) An argument A for the reliability of a belief source is epistemically circular for a thinker S just in case (i) A's conclusion that X is a reliable belief source, (ii) S's belief in at least one of A's premises is a result of S employing X, and (iii) S's belief in these premises would not have been justified had S not employed X[14].

In addition to epistemic circularity, Alston (1989, 1993) is committed to a generic form of *reliabilism* which states:

Definition 2. (Alston's Reliabilism): subject S is justified to believe P if and only if P has a sufficiently reliable causal source.

Hence if sense perception, along with induction, is reliable, then we are justified in accepting the premises of the TRA, and hence the conclusion is obtained by an inductive procedure. Nevertheless, Alston still acknowledges a problem in justifying the premises of the TRA. He writes:

> Epistemic circularity does not in and of itself disqualify the argument. But even granting this point, the argument will not do its job unless we *are* justified in accepting its premises; and that is the case only if sense perception is in fact reliable. This is to offer a stone instead of bread. We can say the same of any belief-forming practice whatever, no matter how disreputable. We can just as well say of crystal ball gazing that if it is reliable, we can use a track-record argument to show that it is reliable. But when we ask whether one or another source of belief is reliable, we are interested in *discriminating* those that can be reasonably trusted from those that cannot. Hence merely showing that if a given source is reliable it can be shown by its record to be reliable, does nothing to indicate that the source belongs to the sheep rather than with the goats. (1993, 17)

In the first line of the quotation given above, Alston writes that "epistemic circularity does not in and of itself disqualify the argument." What he probably means is that if we can grant that reliabilism, sense perception and induction hold, then we are justified

[14] 4.

to accept the conclusion of the TRA. This *epistemic demonstration* is problematic since it does not discriminate between reasonably trusted sources and other non-reasonably trusted sources (e.g. between sense perceptions and crystal ball-gazing). Lynch and Silva (2016) illustrate why epistemic demonstration is not enough – since it clashes with other epistemic principles – by the following fictional scenario:

> Susan and Sally form beliefs about their immediate physical environment in surprisingly different ways. Susan forms perceptual beliefs like we do: she takes her perceptual experiences as of P and proceeds to believe P (provided she has no relevant undefeated defeaters). Sally, by contrast, forms perceptual beliefs quite differently: she takes her perceptual experiences as of P and (provided she has no relevant undefeated defeaters) she proceeds to believe (not-P&MP) where MP is the proposition that I'm in the Matrix and I'm having an experience as of P. However, she always acts as if P, and thus is able to live quite successfully despite her many false perceptual beliefs. One day Susan and Sally encounter each other, and, looking out upon a beautiful lake, Susan comments, 'That's a beautiful lake,' Sally replies, 'There is no lake. But our misleading experience as of a lake is beautiful.' Susan and Sally continue to share their beliefs about their immediate environment, and both come to realize that they have extensively inconsistent views and that this difference is owed to their very different ways of responding to their perceptual experiences. It is immediately obvious to Susan and Sally that their ways of arriving at perceptual beliefs cannot both be reliable, i.e., at least one of their ways of arriving at perceptual beliefs is unreliable. (12-13)

The moral of the fictional story is to show how epistemic disagreement can be dependent on the fundamental methods employed by different people to form their beliefs. Lynch and Silva (2015) label this disagreement as *deep epistemic disagreement* and the situation as *deep epistemic disagreement situation*. More precisely, they define the former as follows:

Definition 3. S_1 is in a deep epistemic disagreement with S_2 just in case: (i) S_1 employs an epistemic principle, EP, to arrive at true beliefs in some domain of inquiry, (ii) S_2 rejects that epistemic principle as a reliable guide to forming true beliefs in that domain, and (iii) S_1 has no further epistemic principle, EP^*, that does not rely on or presuppose the reliability of EP with which he can show that EP is in fact a reliable mean of arriving at true beliefs[15].

The problem of the deep epistemic disagreement is that it is *asymmetric* in the sense that subjects disagree on the reliability of their epistemic principles. Moreover, there is no plausible way, from the subjects' view-point, to weigh a specific epistemic principle over another.

So far, we have been discussing the problem of epistemic circularity with reference to perceptual beliefs, however, in this thesis we are interested in exploring whether a similar problem can arise regarding *a priori knowledge* and *a priori justification*. Therefore, in the next section we are going examine a different form of epistemic circularity that is related to the deductive process of justification.

2.2 The Problem of Deduction

The problem of induction has been much debated in modern philosophy since David Hume's *Treatise of Human Nature*; however, its analogous counterpart – the problem of deduction – did not attract much attention until Lewis Carroll wrote a famous paper in 1895 describing the problem. In this paper, Carroll narrated a discussion in which Achilles is trying to convince the tortoise to accept the Euclidean argument:

- P_1: Things that are equal to the same are equal to each other.
- P_2: The two sides of this Triangle are things that are equal to the same.
- C: The two sides of this Triangle are equal to each other.

Surprisingly, the tortoise wonders: what if a person accepts P_1 and P_2, but rejects C ? Achilles responds that if one accepts P_1 and P_2, then he *must* accept C. The tortoise suggests adding Achilles' note – that if one accepts P_1 and P_2, then he must accept C – as an

[15] 13.

extra premise to the argument, and calls it P_3. Despite that, and unexpectedly, the tortoise asks about the possibility of accepting P_1, P_2 and P_3, but still denying C. Achilles tries to strengthen the Euclidean argument by adding an extra premise P_4 expressing that 'if P_1, P_2, and P_3 are true, then C must be true.' After some time, Achilles realizes that this process can continue *ad infinitum* without forcing the tortoise to accept the conclusion C. The tortoise summarizes its general position by arguing that "Whatever Logic is good enough to tell me is worth writing down[16]." In that way, the exact method of reasoning should always be stated explicitly in the form of extra premises which implies that a conclusion can never be forced. Haack (1976) notices that Achilles' failure in convincing the tortoise is analogous to the failure of adding the extra premise of the "uniformity of nature" to justify induction. One way to solve this, for Haack, is to propose a set of rules that can justify the conclusion C from premises $P_1$and P_2 in the aforementioned argument. Therefore, she suggests defining an argument by the following:

Definition 4. For $n \geq 1$, an *argument* is a sequence $P_1, P_2, ..., P_n, C$ of sentences where $P_1, P_2, ..., P_n$ are premises and C is the conclusion.

Then, she proposes a *syntactic* definition for a valid deductive argument,

Definition 5. An argument $P_1, P_2, ..., P_n \vdash C$ is *deductively* valid in a system S if the conclusion C follows from the premises $P_1, P_2, ..., P_n$, and the axioms of S, by virtue of the *rules of inference* of S.

Hence, we can justify a modus ponens argument, for instance, because of a set of *predetermined* rules of inference in S. Clearly, these rules of inference must be deductively valid too; otherwise, we cannot rely on them. The dilemma, again, is that we fall in circularity because we are justifying deduction using a deductive method. So Haack proposes another *semantic* definition for a deductively valid argument:

Definition 6. An argument $P_1, P_2, ..., P_n \models C$ is *deductively* valid if it *impossible* for the premises $P_1, P_2, ..., P_n$ to be true and for the conclusion C to be false.

The semantic definition proposes that the premises entail the conclusion. If this is the case, then any deductive argument need not

[16] 280.

be justified as it will be true by *virtue of meaning*, i.e. the meaning of the set of premises $\{P_1, P_2, ..., P_n\}$, and the meaning of the conclusion C. In addition, the semantic definition cannot rely on the *schema* (form) of the argument, rather it has to rely on some *instances* of the argument. Due to the *ambiguity* of the notion of meaning and the *non-schematic* form of the semantic definition, Hack argues against it:

> The claim that one can just *see* that the premisses justify the conclusion is implausible in the extreme in view of the fact that people can and do disagree about which arguments are valid. Second, there is an implicit generality in the claim that a particular argument is valid. For to say that an argument is valid is not just to say that its premisses and its conclusion are true – for that is neither necessary nor sufficient for (semantic) validity. Rather, it is to say that its premisses *could not* be true without its conclusion being true also, i.e. *that there is no argument of that form with true premisses and false conclusion*. But if the claim that a particular argument is valid is to be spelled out by appeal to other arguments of that form, it is hopeless to try to justify that form of argument by appeal to the validity of its instance. (118)

Hence, from Haack's perspective, there is no clear way of assigning the semantics of $\{P_1, P_2, ..., P_n\}$, $\models$, and C in a way that guarantees that it is impossible for $P_1, P_2, ..., P_n$ to be true and for C to be false[17].

To wrap up, Carroll's method in forcing a deductive conclusion suffers from *premise-circularity*, while Haack shows that a deductive justification of deduction suffers from *rule-circularity*[18]. These results can be considered as an extension of the notion of epistemic

[17] In section 6, we will argue for a conventional defense of the semantic definition of deductive validity.

[18] Rule-circularity is not necessarily a problem if one is willing to accept that deduction is not epistemologically firmer than induction. Hack writes:

> Those of us who are sceptical about the analytic/synthetic distinction will, no doubt, find these consequences [the problem of deduction] less unpalatable than will those who accept it. And those of us who take a tolerant attitude to non-standard logics – who regard logic as a theory, revisable, like other theories, in the light of experience – may even find these consequences welcome. (118)

circularity to incorporate a priori justification[19].

3 Formal Language Skepticism II: The Rule-Following Paradox

3.1 Wittgenstein: On Rules of Language

In §185 of *Philosophical Investigations* (PI), Wittgenstein refers to a story introduced earlier in §143 that investigates the notion of *rule-following* and its relation to meaning-constitution. A student was instructed by his master to write down a series of the form: $\{0, n, 2n, 3n, ...\}$ at the order of $(+n)$. Hence, if the student follows the order of $(+2)$, he will get the following series $\{0, 2, 4, 6, ...\}$. Suppose the student wrote down that series up to 1000, then he was asked to continue the series – beyond 1000 – following the same $(+2)$ rule. Strangely, the pupil started continuing the series with $\{1000, 1004, 1008, 1012, ...\}$. When the master warned the student that he is continuing the series with *wrong* numbers, the student replied that he is following the same rule since the beginning of the series-counting process, and hence he cannot be mistaken. Wittgenstein writes:

> Now we get the pupil to continue a series (say +2) beyond 1000 — and he writes 1000, 1004, 1008, 1012. We say to him: 'Look what you've done!' — He doesn't understand. We say: 'You were meant to add *two* look how you began the series!' — He answers: 'Yes, isn't it right? I thought that was how I was *meant* to do it.' —— Or suppose he pointed to the series and said: 'But I went on in the same way.' — It would now be no use to say: 'But can't you see . . . . ?' — and repeat the old examples and

[19] One proposed solution to the problem of deduction is to distinguish between *rules of inference* and *logical implications*. A rule of inference like modus ponens allows us to infer q from (p and (if p then q)), however this rule – in itself – is not a truth-bearer. Differently, a logical implication like "if 'p' and 'if p then q' implies 'q'" is a statement in the metalanguage specifying a *necessary* logical relation between the premises and the conclusion of the object language (i.e. the rule of inference). Thus, the premise-circularity problem is the mere result of treating logical implications as rules of inferences. We think this solution is vulnerable to Haack's criticisms as there is still no defense of the validity of the metalanguage. There are also other resolutions like the one proposed by Gilbert Ryle (1945, 6) who suggested that Carroll's regress problem is the result of confusing 'knowing-how' with 'knowing-that'. Ryle basically argues that knowing the rules of inference (viz. 'knowing-that') is not a *sufficient* condition for being able to implement these rules in practice (viz. 'knowing-how').

> explanations.— In such a case we might say, perhaps: It comes natural to this person to understand our order with our explanations as *we* should understand the order: 'Add 2 up to 1000, 4 up to 2000, 6 up to 3000 and so on.' (§185, *PI*)

Wittgenstein thinks that the student's behavior might be considered as a special interpretation of the rule (+2) according to which the student understands to add 2 up to 1000, add 4 up to 2000, add 6 up to 3000, etc. Therefore, the student was *conforming* to the rule when he produced 1004 as the number following 1000 in the series. This story is brought up to point out that following a rule is not *sufficient* for understanding the *meaning* of that rule. Wittgenstein writes:

> How is it decided what is the right step to take at any particular stage? — 'The right step is the one that accords with the order — as it was *meant*' — So when you gave the order +2 you meant that he was to write 1002 after 1000 — and did you also mean that he should write 1868 after 1866, and 100036 after 100034, and so on — an infinite number of such propositions? — 'No: what I meant was, that he should write the next but one number after *every* number that he wrote; and from this all those propositions follow in turn.' — But that is just what is in question: what, at any stage, does follow from that sentence. Or, again, what, at any stage we are to call 'being in accord' with that sentence (and with the *meaning* you then put into the sentence — whatever that may have consisted in). It would almost be more correct to say, not that an intuition was needed at every stage, but that a new decision was needed at every stage. (§186, *PI*)

The crux of the problem can be summarized in the following questions: what determines the *meaning* of a rule? And *who* decides *which* meaning to ascribe to a rule? Since there are *infinite* situations to which we can apply – almost – any rule, and humans have a finite capacity, then it might be difficult to determine what a rule means at each single *new* situation. That is why Wittgenstein considers that attaching a meaning to a rule is not a matter of intuition,

but a matter of decision that is needed to be made in every new situation. Hence, the generation of the number 1002 after 1000 is in accordance with (+2), but other numbers are possible as well. Saul Kripke, among others, has interpreted Wittgenstein as presenting a *skeptical paradox* regarding the meanings of rules of languages, which we are going to discuss next.

3.2 Kripke's Skeptical Paradox

In this section we introduce Saul Kripke's main thoughts about the previously discussed ideas of Wittgenstein on rule-following. Kripke presents his ideas in his influential book *Wittgenstein on Rules and Private Languages* (WRPL) which was an exposition on some of the Wittgensteinian ideas in *Philosophical Investigations*. It is worth noting that there is a wide disagreement in the literature concerning the question whether Kripke's reading of Wittgenstein's passages on rule-following as developing and aiming at a skeptical paradox really corresponds to Wittgenstein's intentions. That is why the literature refers to *WRPL* as *Kripkenstein*. In this thesis, we are interested in the rule-following paradox on its own as presented in *WRPL* – as an original problem in the philosophy of language – regardless of Wittgenstein's original intentions. Kripke writes:

> In the following, I am largely trying to present Wittgenstein's argument, or, more accurately, that set of problems and arguments which I personally have gotten out of reading Wittgenstein. With few exceptions, I am not trying to present views of my own; neither am I trying to endorse or to criticize Wittgenstein's approach ... Probably many of my formulations and recastings of the argument are done in a way Wittgenstein would not himself approve. So the present paper should be thought of as expounding neither 'Wittgenstein's' argument nor 'Kripke's': rather Wittgenstein's argument as it struck Kripke, as it presented a problem for him. (5)

Recall that the problem arose when the master considered the series (1000, 1004, 1008, 1012 ...) that the the pupil – according to what he was taking to be the proper interpretation of the rule (+2) – had constructed, to be the result of a *mistake*. Kripke interprets the story as a skeptical paradox about meaning-constitution in general,

and rule-following in particular. The problem for Kripke is that any rule can be *restated* in a way to make any act performed *conform* to it. So the student in the Wittgensteinian story has a special interpretation for (+2) that means add 2 to up to 1000, then add 4 above 1000. A straightforward response is that the student's interpretation is not possible, simply because our rule(+2) means "add 2 for *all* integers," and not just up to 1000. Unfortunately, it turns out that such response will not succeed for reasons stated later. In §201 Wittgenstein states "this was our paradox: no course of action could be determined by a rule, because every course of action can be made out to accord with the rule." Kripke considers this paradox as the central problem of *PI*. He restates the problem as follows:

> I, like almost all English speakers, use the word 'plus' and the symbol '+' to denote a well-known mathematical function, addition. The function is defined for all pairs of positive integers. By means of my external symbolic representation and my internal mental representation, I 'grasp' the rule for addition. One point is crucial to my 'grasp' of this rule. Although I myself have computed only finitely many sums in the past, the rule determines my answer for indefinitely many new sums that I have never previously considered. This is the whole point of the notion that in learning to add I grasp a rule: my past intentions regarding addition determine a unique answer for indefinitely many new cases in the future." (8)

Here, Kripke is spelling out the intuitive angst that we might have after considering the Wittgensteinian story. We all know that we applied any rule (whether mathematical or not) only to a finite number of cases in the past. But we moreover hold an implicit assumption – which is contained in our grasp of the meaning of 'rule' – namely, that the rule under consideration can be applied to an infinite number of cases in the future. What Kripke will do next is challenge that intuition by constructing an ingenious thought experiment that puts into doubt the conviction that we can ascribe a *stable meaning* for a specific sign in any language[20]. Kripke's thought experiment goes as follows: imagine a student who has never performed any addition operation for numbers greater than

[20] Kripke's argument is equally valid for natural languages as it is for formal languages.

57, and he was then asked 'what is the value of 68 plus 57?' The student responds by doing the arithmetic as he was taught before and produced the answer '125.' This is true for the student not only in the arithmetic sense, but also in the metalinguistic sense as the student is using the term 'plus' to denote a function to the effect that '57+68' *should* produce '125.' Now a skeptic comes to the student and suggests that perhaps his past usage of the term 'plus' should produce '5' instead of '125.' To justify his claim the skeptic suggests that since the student only performed additions of numbers less than 57, then this is a complete *new* application of the 'plus' function. In other words, the skeptic is doubting the legitimacy of extending our usage of 'plus' as '+' from all $n-1$ cases to the nth case. So it might be the case that the student's past usage of the terms 'plus' and '+' denotes a function called *quus* denoted by Θ:

$$\begin{cases} x \Theta y = x + y & if\ x, y < 57 \\ x \Theta y = 5 & if\ otherwise \end{cases}$$

The *quus* function fits perfectly to the student's past usage of the 'plus' term, and there is no logical reason to reject it since it is the first time for the student to experiment the addition of two numbers greater than 57. Again, the student's common sense response is to reject the skeptic's argument by claiming that when he used the term 'plus' he *meant* 'plus' and not 'quus.' It only remains for the student to *demonstrate* how his *intention* for a specific *meaning* affected his past usage of the term, and here, as we shall now see, the paradox immediately becomes manifest. Kripke writes:

> Now if the sceptic proposes his hypothesis sincerely, he is crazy; such a bizarre hypothesis as the proposal that I always meant quus is absolutely wild. Wild it indubitably is, no doubt it is false; but if it is false, there must be some fact about my past usage that can be cited to refute it. For although the hypothesis is wild, it does not seem to be *a priori* impossible. (9)

Thus, the difficulty lies in the absence of any fact concerning the student's past usage that would favour the operation 'addition' over the operation 'quaddition' (viz. the operation involving quus). But

since quaddition is not impossible a priori, and there is no fact about the student's past behavior that rules out quaddition, 'quaddition' remains an equally valid interpretation of '+' as 'plus'. Kripke considers the instruction of *counting* as a natural candidate for solving the problem. Thus when asked about the value of '$x + y$=?', the student can just count (using his fingers for instance) the value of x, then the value of y, then combine both and count the total. This is a very natural way of avoiding addition and produces the same result. Nevertheless, it is obvious that this procedure falls under the skeptic's fire too. The skeptic responds by claiming that maybe the student was using the term 'count' to mean *quount* which is a process that means 'count' if one is joining two heaps that are individually less than 57, otherwise count '5' if each heap is 57 or more. It is clear that the skeptic can apply this strategy *recursively* to any simpler terms that might be used to avoid addition or counting. To take it even further, Kripke extends this paradox to natural language, and not just to mathematical operations or logical rules.

> Of course, these problems apply throughout language and are not confined to mathematical examples, though it is with mathematical examples that they can be most smoothly brought out. I think that I have learned the term 'table' in such a way that it will apply to indefinitely many future items. So I can apply the term to a new situation, say when I enter the Eiffel Tower for the first time and see a table at the base. Can I answer a sceptic who supposes that by 'table' in the past I meant tabair, where a 'tabair' is anything that is a table not found at the base of the Eiffel Tower, or a chair found there? (19)

The central question now is: is there any way for the non-skeptic to weigh 'plus' over 'quus'? At first glance it seems that there are many answers to this question. However, Kripke thinks that there are only two possible candidates, mental facts and behavioral facts, and he quickly dismisses both of them. In an attempt to solve the problem, Kripke tries to define the scope of the paradox:

> Of course the problem can be put equivalently in terms of the sceptical query regarding my present intent: nothing in my mental history establishes whether I meant plus

> or quus. So formulated, the problem may appear to be epistemological – how can anyone know which of these I meant? Given, however, that everything in my mental history is compatible both with the conclusion that I meant plus and with the conclusion that I meant quus, it is clear that the skeptical challenge is not really an epistemological one. (21)

Although the skeptical problem seems to be an epistemological one (i.e. how do I *know* that what I meant in the past by '+' was 'plus' and not 'quus'?), it is more of a *constitutive* one. It is not about our *inability* to find the right epistemological path to generate fixed meanings for terms; rather, there might be *no* potential candidates for meaning-constitutive facts at all. The main challenge for Kripke then is to show that there are no meaning-constitutive facts by eliminating all possible candidates.

According to Boghossian (1989), Kripke rejects all candidates because they do not satisfy at least one of these two criteria: (i) meaning is *normative*; (ii) meaning is *infinitary*. Concerning the normativity criterion, if the student means 'plus' by '+', then this state of affairs indicates how the student *ought* to apply 'plus,' not how the student *will* apply 'plus.' Concerning the infinitary criterion, we can think of the meaning of any term as having an infinitary character which means that it *could* be applied to an infinite number of cases. So if the student means 'plus' by '+', then there is an infinite number of true states of affairs to which he can apply 'plus.' Kripke uses these two criteria mainly to reject the *dispositionalist* account of meaning which argues that individual dispositions determine the *factuality* of meanings. A dispositionalist position fails normativity since it simply tells us what individuals *would* do, and not what they *ought* to do. Dispositionalism will also leave no room for individual mistakes as it defines what we are *supposed* to get from following a rule by what we *actually* got from following that rule, and hence leaves no possibility for mistakes. Moreover, dispositionalism also fails the infinitary criterion; Kripke argues that since we are finite beings (in finite time), we can only hold a finite sum of dispositions which fails to capture the meaning of '+'[21].

[21] A conventionalist theory of meaning can satisfy Kripke's twofold criteria. We defend such position in sections 6 and 8.

4 The Brain-in-a-Vat and the A Priori

Historically, Descartes was aware of the possibility of doubting our a priori knowledge. In *Mediation I* of his *Meditations on First Philosophy*, he wrote:

> And, besides, as I sometimes imagine that others deceive themselves in the things which they think they know best, how do I know that I am not deceived every time that I add two and three, or count the sides of a square, or judge of things yet simpler, if anything simpler can be imagined? (8)

He even considered, in *Meditation III*, the possibility that God has created him in a way to be deceived regarding a priori knowledge:

> But when I took anything very simple and easy in the sphere of arithmetic or geometry into consideration, e.g. that two and three together made five, and other things of the sort, were not these present to my mind so clearly as to enable me to affirm that they were true? Certainly if I judged that since such matters could be doubted, this would not have been so for any other reason than that it came into my mind that perhaps a God might have endowed me with such a nature that I may have been deceived even concerning things which seemed to me most manifest. (13)

The core of Descartes' argument is to show the *plausibility* of producing knowledge – out of a skeptical scenario – that is subjectively indistinguishable from his knowledge of a priori propositions while actually being false; as they are the product of deception[22]. In this

[22] Descartes attempted to refute skepticism through his notable *cogito ergo sum* argument. He wrote:

> Accordingly, this piece of knowledge—*I am thinking, therefore I exist*—is the first and most certain of all to occur to anyone who philosophizes in an orderly way. (*Principles of Philosophy*, 195)

And,

> So that after having reflected well and carefully examined all things, we must come to the definite conclusion that this proposition: I am, I exist, is necessarily true each time that I pronounce it, or that I mentally conceive it. (*Meditation II*, 9)

section we are interested in a different skeptical scenario regarding a priori knowledge. The goal is to propose a modified version of the *external world* brain-in-a-vat (Ex-BIV) skeptical scenario to generate a similar skeptical scenario about a priori knowledge[23]. Generally speaking, we can think of two types of skepticism regarding the a priori. The first can be taken to be an empiricist commitment to knowledge (viz. that all knowledge is *a posteriori*)[24]. The proponents of this kind of skepticism regards 'self-evident' propositions as knowledge, but disagree with the rationalists about the *warrant* of these propositions. The second type of skepticism, which we are interested in, is more radical as it questions whether we can have a priori knowledge in the first place. The a priori brain-in-a-vat (Ap-BIV) is a thought experiment about a brain-in-a-vat that is being tricked regarding a priori knowledge. The main difficulty that faces a priori skepticism – in its second sense – is related to metaphysical modality concerns. Many philosophers can accept the plausibility of Ex-BIV skeptical hypothesis since any proposition about the external world seems to be a *contingent* proposition. On the other hand, the Ap-BIV skeptical scenario seems to be prima facie *logically impossible* since it deals with *necessary* propositions. Another difficulty is that Ap-BIV seems to be self-refuting as the skeptic has to use a form of a priori justification – based on a priori knowledge – to construct a valid Ap-BIV scenario. The last objection is best articulated in the words of Matthias Steup:

> It is generally agreed that PAPS[25] are knowable. There is skepticism about knowledge of the external world, other minds, and the past. Skepticism about PAPS, however, is rarely pursued. Indeed, considering that knowledge of PAPS includes knowledge of the laws of logic, and more specifically, knowledge of an argument's validity, it is hard to see how a skeptical argument for anything could get off the ground without the prior assumption that knowledge of PAPS is indeed possible. So the knowability of PAPS is not at issue. (2005, 10-11)

[23] The brain-in-a-vat argument might be regarded as the modern formulation of the Cartesian Evil Demon. It was first introduced in Hilary Putnam's *Reason, Truth and History* (1981) in which he used semantic externalism to argue against a version of the brain-in-a-vat hypothesis.

[24] See, for example, Kitcher (1992), Kornblith (2000) and Devitt (2005).

[25] Viz. 'putatively a priori propositions'.

Nevertheless, James Beebe (2011) provides a response to the self-defeating objection raised by Steup:

> Ordinarily, when we put forward philosophical arguments, we want to be able to know that our premises are true and that they support our conclusions. If the conclusion of an argument undermines our ability to know the premises, one of these common epistemic goals will be thwarted. However, in the case of skeptical arguments, showing that we lack knowledge of a certain kind is the central goal. Thus, if it turns out that a skeptical argument shows that we cannot even have knowledge of the premises of that argument, that may be no objection to the argument. If leading us by a plausible *train of reasoning* to a point where it appears that we cannot have knowledge of the premises we started with was part of the very goal of the skeptical argument, our resulting inability to know them is a sign the argument has succeeded— not a sign that it has failed. (599-600, emphasis added)

An extensive discussion of a priori skepticism can be found in Beebe (2011) and Willenken (2015) upon which we are going to rely in our exposition.

Definition 7. (Ap-BIV Skeptical Hypothesis): Let Ap be any a priori proposition, and let the skeptical hypothesis S'_h be: 'I am Ap-BIV'.

- P_1: I am unable to know the denials of S'_h.
- P_2: If I am unable to know the denials of S'_h, then I do not know that Ap is true.
- C: Hence, I do not know that Ap is true.

The first premise can be supported using the notion of *Dretskean sensitivity*[26].

Definition 8. (Dretskean Sensitivity): If a proposition P was not true, then subject S would not believe P[27].

[26] See Dretske (1971) and Nozick (1981).
[27] This formulation is due to Pritchard (2016).

The Dretskean sensitivity condition can be thought of as a modal criterion for assessing the truth of beliefs. It requires the subject to recognize her false beliefs by appealing to the *nearest possible worlds*. That means that if S believes P, and P is false, then there exists a near possible world where S would not believe P. Put differently, if P is false in a world W, and there exists a near possible world W' where S believes P to be false, then P is a Dretskean sensitive propositional knowledge. For instance, assume that S lives in our ordinary world W where water is formed of H_2O. Now we can conceive of another close possible world W' where everything else is almost the same except that water is formed of H_5O. In W', S would easily know that her belief of water being H_2O – say after consulting a chemistry textbook – is false. Therefore, Dretskean sensitivity is a scheme to sort out false beliefs from true ones by appealing to the modal notion of possible worlds. Clearly, the proposition "I know that I am not Ap-BIV" is *not* Dretskean sensitive since in the nearest possible world W', I will not be able to know its falsehood because I am already being deceived by Ap-BIV. Therefore, the skeptic can defend P_1 in the Ap-BIV skeptical hypothesis by arguing that P_1 is *not* Dretskean sensitive, and hence *cannot* be *falsified*[28]. Also, P_2 of the Ap-BIV skeptical hypothesis can be justified based on the assumption that knowledge is *closed* under entailment.

Definition 9. (Closure Principle): If subject S knows P, and knows that P entails Q, then S knows that Q.

The closure principle only demands the subject to know the *logical consequences* of proposition P. For example, knowing that "Adam is a wearing a shirt" is a logical consequence of knowing that "Adam is wearing a black shirt." Similarly, "I know that I am not Ap-BIV" is a logical consequence of the proposition "I know that 2+5=7 is true," as knowing the latter implies knowing the former. Hence the skeptic can endorse P_2 by endorsing the closure principle. These two justifications (viz. Dretskean sensitivity and the closure principle) can be objected on several grounds; however, an extensive discussion

[28] In a similar fashion to Hempel's (1945) paradox of the ravens, an objection can be raised against P_1 by undermining the notion of Dretskean sensitivity. Let us think of the following truth-condition: if S believes P, then P is true. Prima facie, this truth-condition is very weak as it allows almost any proposition to be true. But, this truth-condition is *logically equivalent* to Dretskean sensitivity. Thus, if we do not accept this truth condition, we ought to reject the Dretskean sensitivity criterion.

of them is beyond the scope of this essay[29]. As discussed above, the Ap-BIV scenario is presented as a paradox: two acceptable premises and an unacceptable conclusion.

Commonly, we can think of two categories of paradoxes: *contradictory* and *non-contradictory*. The former type comes with the format of the *sorites* paradoxes where the conclusion takes the form of a contradiction (e.g. showing P and not P), and hence cannot be accepted. The liar's paradox is an example of this. If a liar says "I am lying," it means that he is not lying. Similarly, if he says "I am not lying," it means he is lying. On the other hand, in non-contradictory paradoxes we reach a conclusion that is not contradictory, but simply 'absurd' in some sense. An example would be the conclusion of the Ap-BIV skeptical hypothesis[30].

5 Apriority and Modality

5.1 Apriority

In his celebrated *Critique of Pure Reason* (CPR), Kant makes a fundamental distinction between three conceptually-related oppositions: the first is the distinction between a priori and a posteriori knowledge[31], the second is the distinction between necessary and contingent propositions, and the third is the distinction between analytic and synthetic propositions[32]. In the introduction of the first edition of *CPR,* Kant defines *a priori* knowledge as being "independent of experience" as opposed to *a posteriori* knowledge which is "borrowed solely from experience." He writes:

> Experience tells us, indeed, what is, but not that it must necessarily be so, and not otherwise. It therefore gives us no true universality; and reason, which is so insistent upon this kind of knowledge, is therefore more stimulated by it than satisfied. Such universal modes of knowledge,

[29] See Pritchard (2008) for a discussion on sensitivity-based theories in contemporary epistemology, and see Kvanvig (2006) for a general discussion of the different versions of the closure principle. Also, refer to Leite (2004) for a critical refutation of the closure principle as a foundational concept in the skeptical argument.

[30] If someone subscribes to the view that Ap-BIV skeptical hypothesis is self-refuting, then indeed it will be a contradictory paradox.

[31] Cf. CPR, A 2/B 2.

[32] Cf. CPR, A 6/B 10 – A 10/B 14.

> which at the same time possess the character of inner necessity, must in themselves, independently of experience, be clear and certain. They are therefore entitled knowledge *a priori*; whereas, on the other hand, that which is borrowed solely from experience is, as we say, known only *a posteriori*, or empirically. (A2)[33]

When claiming that a priori knowledge is a type of knowledge that is independent of experience[34], Kant is not claiming that we would have reached this knowledge if we had no experience at all. On the contrary, Kant takes experience to be a prerequisite for all knowledge acquisition, including a priori knowledge. Rather, what he means is that experience is not a *constitutive* part of our a priori knowledge. He writes:

> There can be no doubt that all our knowledge begins with experience. For how should our faculty of knowledge be awakened into action did not objects affecting our senses partly of themselves produce representations partly arouse the activity of our understanding to compare these representations, and, by combining or separating them, work up the raw material of the sensible impressions into that knowledge of objects which is entitled experience? In the order of time, therefore, we have no knowledge antecedent to experience, and with experience all our knowledge begins. But though all our knowledge begins with experience, it does not follow that it all arises out of experience. (B1)

The two criteria for a priori knowledge, according to Kant, are: *necessity* and strict *universality*[35]. Concerning necessity, he writes:

[33] All translations of *CPR* are from Smith (1929).

[34] The Kantian view can also be summed up in Frege's words in *The Foundations of Arithmetic*:

> [When we classify a proposition as *a priori*,] this is not a judgment about the conditions, psychological, physiological, and physical, which have made it possible for us to form the content of the proposition in our consciousness; nor is it a judgment about the way in which some other man has come . . . to believe it to be true; rather, it is a judgment about the ultimate ground upon which rests the justification for holding it to be true." (3)

[35] It is worth noting that Kant distinguishes between *analytic* judgments a priori and *synthetic* judgments a priori. The criterium for synthetic judgments a priori is that these function

> Experience teaches us that a thing is so and so, but not that it cannot be otherwise. First, then, if we have a proposition which in being thought is thought as *necessary*, it is an *a priori* judgment; and if, besides, it is not derived from any proposition except one which also has the validity of a necessary judgment, it is an absolutely *a priori* judgment. (B3)

Apparently then, experience can only inform us about contingent propositions. However, thinking about a proposition as necessary is a *sufficient* condition for being a priori. In addition, Kant distinguishes between a priori judgment(s) which can be derived from other *unnecessary* proposition(s), and *absolutely* a priori judgment(s) which is derived *only* from other necessary proposition(s). The second criterion for a priori knowledge is *strict universality*,

> If, then, a judgment is thought with strict universality, that is, in such manner that no exception is allowed as possible, it is not derived from experience, but is valid absolutely *a priori* ... Necessity and strict universality are thus sure criteria of *a priori* knowledge, and are inseparable from one another. (B4)

In recent years – probably under the influence of Saul Kripke's work – there have been some attempts to disentangle apriority from necessity. Hence, apriority was defined in other terms[36]:

Definition 10. (PK1): S's belief P is justified a priori if and only if S's belief P is non-experientially justified[37] and cannot be defeated

as conditions of the possibility of experience,

> The highest principle of all synthetic judgments is therefore this: every object stands under the necessary conditions of synthetic unity of the manifold of intuition in a possible experience. Synthetic a priori judgments are thus possible when we relate the formal conditions of a priori intuition, the synthesis of imagination and the necessary unity of this synthesis in a transcendental apperception, to a possible empirical knowledge in general. We then assert that the conditions of the possibility of experience in general are likewise conditions of the possibility of the objects of experience, and that for this reason they have objective validity in a synthetic a priori judgment. (A 158/B197)

General pure logic (cf. CPR A 50/B 74 ff.) constitutes an analytic theory of the form of thought which completely abstracts from its contents, whereas transcendental logic does not completely abstract from the objective validity of thought, but rather inquires into its conditions, for its main task see A 154/B 193. Thus, whereas the law of non-contradiction is the "highest principle" of all judgments (cf. B189), transcendental logic explains the possibility of synthetic judgments a priori. (A 154/ B193)

[36] See for instance, Putnam (1983) and Kitcher (1983).

[37] I.e. P is justified by a source that is non-experiential.

by experience.

Alberto Casullo (2012) objects to PK1 based on a series of arguments, and proposes instead a weaker version of PK1:

Definition 11. (C1) S's belief P is justified a priori if and only if S's belief P is non-experientially justified.

Clearly, C1 is weaker than PK1 (since PK1 implies C1, and not the opposite). Henceforth we are going to adopt C1 as our definition of a priori knowledge[38].

5.2 Apriority and Necessity

As mentioned above, Kant distinguishes between necessity and contingency. So although it is true that 'I am writing my thesis right now'; it is also true (and thus possible) that I could have been doing something else. Nevertheless, it seems difficult to imagine that '2+2 =5' could have been *true* in any plausible way. The former type can be labeled as *contingent* propositions while the latter can be labeled as *necessary* propositions. Kant writes that experience alone cannot tell us about necessity and strict universality: "Experience tells us, indeed, what is, but not that it must necessarily be so, and not otherwise. It therefore gives us no true universality.[39]" From such statements we can infer that Kant thinks:

- (KP1) If P is a priori true then P is a necessary proposition.

And similarly,

- (KP2) If P is a posteriori true then P is a contingent proposition[40].

The sufficient condition of necessity expressed in KP1 follows trivially since Kant describes necessity and strict universality as the two fundamental properties of the a priori. Nonetheless, it turns out that *both* KP1 and the other direction of KP1 (viz. if P is a necessary proposition then P is true a priori) could be *falsified*. Kripke (1971, 1980) took the lead in breaking the Kantian equivalence between necessity and a priority by claiming that there might be (i) contingent

[38] Note that we can make a leap from a priori justification to a priori knowledge by preserving the other knowledge-conditions.

[39] CPR, A2.

[40] See Van Cleve (1999) for example.

a priori propositions, and (ii) necessary a posteriori propositions[41]. He writes in lecture 1 of *Naming and Necessity*:

> Philosophers have talked ... [about] various categories of truth, which are called '*a priori*,' 'analytic,' 'necessary'... The terms are often used as if *whether* there are things answering to these concepts is an interesting question, but we might as well regard them all as meaning the same thing ... Consider what the traditional characterizations of such terms as '*a priori*' and 'necessary' are. First the notion of a prioricity is a concept of epistemology. I guess the traditional characterization from Kant goes something like: *a priori* truths are those which can be known independently of any experience ... I will say that some philosophers somehow change the modality in this characterization from *can* to *must*. They think that if something belongs to the realm of a priori knowledge, it couldn't possibly be known empirically. This is just a mistake. Something may belong in the realm of such statements that *can* be known *a priori* but still may be known by particular people on the basis of experience. (34-35)

Then he adds concerning necessity,

> The second concept which is in question is that of necessity. Sometimes this is used in an epistemological way and might then just mean *a priori* ... But what I am concerned with here is a notion which is not a notion of epistemology but of metaphysics ... We ask whether something might have been true, or might have been false. Well, if something is false, it's obviously not necessarily true. If it is true, might it have been otherwise? Is it possible that, in this respect, the world should have been different from the way it is? If the answer is 'no,' then this fact about the world is a necessary one. If the answer is 'yes,' then this fact about the world is a contingent one. This in and of itself has nothing to do with anyone's knowledge of anything. It's certainly a philosophical thesis, and not a matter of obvious definitional equivalence, either that everything a priori is necessary or that everything necessary

[41] It goes without saying that Kripke's thesis is controversial, see (Laporte 2016).

> is a priori ... But at any rate they are dealing with two different domains, two different areas, the epistemological and the metaphysical. (35-36)

As seen above, Kripke strikes harshly against confusing the epistemological with the metaphysical. Following this division, Kripke is ready to present his thesis: (i) there are contingent a priori propositions, and (ii) there are necessary a posteriori propositions. We start by investigating (ii).

5.2.1 Necessary A Posteriori Propositions

In the preface of *Naming and Necessity*, Kripke argues for the *necessity* of identities using Leibniz's law of indiscernibility of identicals[42]:

> Already when I worked on modal logic it had seemed to me, as Wiggins has said, that the Leibnitzian principle of the indiscernibility of identicals was as self-evident as the law of contradiction ... it was clear from $(x)\,\Box\,(x = x)$ and Leibnitz's law that identity is an 'internal' relation: $(x)(y)(x = y \supset \Box x = y)$ (3)

So Kripke's task now is to show that we can have *a posteriori* necessary identities. In order to do this, he coined the notion of *rigid designator* (although the concept had previously been used by some philosophers).

- RD (Rigid Designator): A rigid designator is a term that *designates* the *same* object in *all possible worlds*[43].

Think of the following statement 'Hesperus = Phosphorus' which is a *true* identity statement. 'Hesperus' is the name given, in Greek mythology, to an evening star; while 'Phosphorus' is the term given to a morning star. During ancient Greek times, these two heavenly bodies were thought to be different, but later astronomers discovered that both terms refer to the same object, namely Venus. Clearly the identity 'Hesperus = Phosphorus' was only known to be true, a posteriori.

[42] For property F and objects x and y, we have $x = y \rightarrow \forall F(Fx \leftrightarrow Fy)$

[43] There is no consensus on the nature of these possible worlds (c.f. Menzel 2016). Nevertheless, here we can think of them, in the most general sense, as 'plausible counterfactual scenarios.'

> So two things are true: first, that we do not know a priori that Hesperus is Phosphorus, and are in no position to find out the answer except empirically. Second, this is so because we could have evidence qualitatively indistinguishable from the evidence we have and determine the reference of the two names by the positions of two planets in the sky, without the planets being the same. (104)

So does that mean that 'Hesperus = Phosphorus' is contingently true? Kripke's answer is 'no.' The identity statement 'Hesperus = Phosphorus' is necessarily true (in the metaphysical sense) as it is an identity of two *rigid designators*. To see this, note that the first proper name 'Hesperus' designates one object in all possible worlds (viz. Venus). By the same token, the second proper name 'Phosphorus' designates also the same object in all possible worlds (viz. Venus). Therefore, if the identity 'Hesperus = Phosphorus' is true then it is *necessarily* true, since 'Venus=Venus' is necessarily true in all possible worlds. Now consider the following identity statement 'Hesperus = the brightest non-lunar heavenly body in the evening sky.' This statement is true but *not* necessarily true. This is because the term 'the brightest non-lunar heavenly body in the evening sky' is not a rigid designator as we can perfectly conceive another possible world – different from our actual world – where 'the brightest non-lunar heavenly body in the evening sky' designates 'Mars' for instance. Hence, this identity is true regarding our actual world, but not necessarily true regarding all possible worlds. In addition to identity statements with proper names, Kripke shows other categories of statements to be necessary a posteriori like (i) identities involving natural kinds[44], and (ii) attributions of essential properties to objects[45].

There are many objections to Kripke's conclusions. A solid critique comes from direct reference theorists (e.g. Soames (2002)[46], and Salmon (1986)[47]). According to them, we are allowed to think of directly-referring terms as terms that lack any *descriptive* characterization of the referent. Based on this, statements like 'Hesperus = Phosphorus' and 'Hesperus = Hesperus' are non-differentiable since

[44] C.f. pp. 116-126.
[45] C.f. pp. 110-115.
[46] Pp. 240-243.
[47] Pp. 133–142.

they pick out the same referent. By stripping away the descriptive characterization of 'Hesperus = Phosphorus' and of 'Hesperus = Hesperus', both statements turn out to be the same. But since 'Hesperus = Hesperus' can be known a priori, then 'Hesperus = Phosphorus' can also be known a priori[48].

5.2.2 Contingent A Priori Propositions

Kripke presents his thoughts about this category of propositions using the 'standard meter' example. He hypothesizes that 'one meter' is the length of S, where S is an *arbitrary* stick in Paris. To avoid any discrepancies in length because of time, Kripke fixes the length at time t_0. The question now is: 'Is it a necessary truth that S is one meter long at t_0?' Kripke answers as follows:

> Someone who thinks that everything one knows a priori is necessary might think: 'This is the definition of a meter. By definition, stick S is one meter long at t_0. That's a necessary truth.' But there seems to me to be no reason so to conclude, even for a man who uses the stated definition of 'one meter.' For he's using this definition not to give the meaning of what he called the 'meter,' but to fix the reference. (54-55)

Hence, the role of the definition here is not meaning-ascription, but reference-fixing. Moreover, the object we used for reference-fixing is just *accidental*. In other words, it just happened that S had the exact length we wanted to describe at t_0; but it could have happened that another object S' was the correct reference-fixing tool.

> ...There is a certain length which he wants to mark out. He marks it out by an accidental property, namely that there is a stick of that length. Someone else might mark out the same reference by another accidental property.

[48] In a similar fashion to Kripke, Quine argued against direct reference theorists. Consider the following example from Quine (1970):

- (i) All cordates are cordates.
- (ii) All cordates are renates

Assume that both statements are true. Then for a reference theorist, (i) and (ii) are identical statements as they have the same reference. Nevertheless, there seems to be an intuitive semantic difference between (i) and (ii). Statement (i) expresses that all creatures with hearts are creatures with hearts, while statement (ii) expresses that all creatures with hearts are creatures with kidneys.

> But in any case, even though he uses this to fix the reference of his standard of length, a meter, he can still say, 'if heat had been applied to this stick S at t_0, then at t_0 stick S would not have been one meter long.' (55)

Kripke's thought can be analyzed using rigid designators. The identity was defined as: 'the length of S at t_0= one meter'. The 'one meter' term is a rigid designator as it points out to the same length in all possible worlds. On the other hand, 'the length of S at t_0' is not a rigid designator as it might designate different lengths at different worlds depending on S and t_0.

> But a simple answer to the question is this: Even if this is the only standard of length that he uses, there is an intuitive difference between the phrase 'one meter' and the phrase 'the length of S at t_0.' The first phrase is meant to designate rigidly a certain length in all possible worlds, which in the actual world happens to be the length of the stick S at t_0. On the other hand 'the length of S at t_0' does not designate anything rigidly. In some counterfactual situations the stick might have been longer and in some shorter, if various stresses and strains had been applied to it. (55)

From this discussion, Kripke concludes that the statement 'S is one meter long at t_0' is *contingently a priori*.

> What then, is the epistemological status of the statement 'Stick S is one meter long at t_0,' for someone who has fixed the metric system by reference to stick S ? It would seem that he knows it a priori. For if he used stick S to fix the reference of the term 'one meter,' then as a result of this kind of 'definition' ... he knows automatically, without further investigation, that S is one meter long. (56)

It goes without saying that the a priority of 'S is one meter long at t_0' is only known to the subject who proposed this definition. However, it seems difficult for anyone else who was not involved in the reference-fixing procedure to know a priori that 'S is one meter long at t_0.' But this differentiability in the epistemic status of Kripke's definition is not worrying since all what he needs to show is

that the class of contingent a priori statements is *non-empty*. After Kripke, some philosophers extended the category of contingent a priori statements to include other categories like indexicals[49].

5.3 More on Rigidity

In this section we are going to investigate Kripke's notion of rigidity more extensively and discuss some of its possible implications. We are going to focus mainly on proper names, nevertheless, our results are *extendable* to a large category of rigid designators. At first, Kripke distinguishes between two types of rigidity: rigidity *de jure*, and rigidity *de facto*[50]. The former type occurs when the designated reference is identified to be a single object in all possible worlds by virtue of the semantic rules of language. An example of this would be the identity statements, for instance 'Nixon=Nixon.' The proper name 'Nixon' will always designate the person *Nixon* in all possible worlds, if this person *Nixon* exists. On the other hand, the latter type indicates what rigidity *actually* happens to be in some cases where the designated object x just happens to uniquely have the property $F(x)$ in all possible worlds. An example of a de facto rigid designator is 'the smallest prime' which just happens to indicate a unique entity in all possible worlds, namely number 'two.' It is not by virtue of the semantic rules of language that the 'smallest prime' denotes 'two' in all possible worlds, but because mathematical facts are unique in all possible worlds[51]. In *Naming and Necessity*, Kripke intends to defend the thesis that proper names are rigid de jure to refute *descriptivism*. Intuition plays an important role in Kripke's formulation of rigidity: "In these lectures, I will argue, intuitively, that proper names are rigid designators, for although the man (Nixon) might not have been the President, it is not the case that he might not have been Nixon (though he might not have been called 'Nixon')[52]"

According to Kripke, the intuitivity of rigidity can be gained from simple statements (i.e. non-modal statements). He gives an example to show this: assume that the meaning of 'Aristotle' is 'the last great philosopher of antiquity'. Then we can have the following:

[49] C.f. Braun (2015) for an extensive discussion on indexicals.
[50] P. 21, ft. 21.
[51] Here Kripke assumes a realist commitment to mathematics.
[52] 49.

- A1. Aristotle was fond of dogs.
- A2. The last great philosopher of antiquity was fond of dogs.

These two sentences are simple ones (with no modal content). To show the rigidity of 'Aristotle' and the non-rigidity of 'the last great philosopher of antiquity' we can appeal to intuition about the truth-value of A1 and A2 in counterfactual scenarios. The truth-value of the A1 and A2 might converge or diverge in different possible worlds. However, in all possible worlds, the proper name 'Aristotle' will always denote the *same* person. On the other hand, the term 'the last great philosopher of antiquity' might refer to different persons (Plato, for example) as we move across possible worlds. Closely related to rigidity is the concept of *necessity of identity*. Kripke notices that there are at least three different senses of talking about 'necessity of identity:'

> We must distinguish three distinct theses: (i) that identical objects are necessarily identical; (ii) that true identity statements between rigid designators are necessary; (iii) that identity statements between what we call 'names' in actual language are necessary. (4)

The first thesis is that everything is, necessarily, identical to itself.

- K1.$(x)\,\Box\,(x = x)$.

The second thesis is from modal logic; it says for every x and for every y, if $x = y$, then necessarily $x = y$.

- K2. $(x)(y)[(x = y) \rightarrow \Box(x = y)]$.

The third thesis is about identity statements involving proper names.

- K3. If $F = N$, then necessarily $F = N$, where F and N are proper names[53].

Building on this, we can define rigidity and rigid names more properly as:

[53] There is some disagreement in the literature about whether (K3) follows from (K2) or not; see Burgess (2013).

Definition 12. (Rigidity) A designator D of an object o is said to be *rigid* if it is the case that for any possible world w: (i) if o exists in w, then D designates o in w; and (ii) if o does not exist in w, then D does not designate any other object o' in w.

Stanley (1997) provides three possible ways according to which D *fails* to be a rigid designator of o.

- Rd1. If o exits in w, but o is not designated by D.
- Rd2. If o exists in w, but D designates another object o'.
- Rd3. If o does not exists in w, and D designates another object o'.

Therefore,

Definition 13. (Name Rigidity thesis) A proper name N is *rigid* only under the condition that if N designates object o, then N designates o *rigidly*.

Stanley's task now is to show that the name rigidity thesis holds indeed. In other words, he needs to demonstrate that all proper names are rigid designators. For that purpose he uses *reductio ad absurdum* by assuming that if Rd1-Rd3 are *true* in case of proper names, then we have a *contradiction*. Hence we have to deny that Rd1-Rd3 hold in case of proper names, and hence conclude that proper names are rigid designators. First, Let o be the designated object, and let D be the designator. Now assume that D designates o and that Rd1 is true, then:

- Rd1'. $\exists o[o = D \wedge \Diamond(o \text{ exists} \wedge o \neq D)]$

Replacing D with a proper name will result in the following statement "there exists someone who is Plato, and he possibly exists without being Plato" which is intuitively false. The second case, Rd2, also does not hold using a similar argument since it follows directly from (Rd1'). Finally, assume that D designates o and that Rd3 is true, then:

- Rd3'. $\exists o[o = D \wedge \Diamond(D \text{ exists} \wedge D \neq o)]$

Likewise, substituting D with a proper name results in a counterintuitive statement like: "there exists someone who is Plato, and Plato could possibly exist without being him." Therefore, we can conclude that if D is a proper name, it designates one and only one object o.

5.4 The Problem of the Rigidity Thesis

In this section, we are going to point out several problems within the rigidity thesis. Let us take the case of proper names[54] as rigid designators. One problem is that a proper name can have *more* than one bearer. Think of the name 'Socrates' which points to 'the master of Plato' and to 'Sócrates Brasileiro' the Brazilian football player. When abstracted from a specific *context*, the proper name 'Socrates' neither picks any nor both of them at the same time. Hence, when Alessandro says "I like Socrates," it is not vivid which 'Socrates' does his proposition pick. The absence of an inclusion/exclusion mechanism for the designated object(s) by proper name(s) results in the *multiple bearers' problem*. Nevertheless, Kripke rejects this line of reasoning, and insists that names can be *individuated* by the bearer.

> Some have thought that the simple fact that two people can have the same name refutes the rigidity thesis. It is true that in the present monograph I spoke for simplicity as if each name had a unique bearer. I do not in fact think, as far as the issue of rigidity is concerned, that this is a major oversimplification. I believe that many important theoretical issues about the semantics of names (probably not all) would be largely unaffected had our conventions required that no two things shall be given the same name. In particular, as I shall explain, the issue of rigidity would be unaffected. For language as we have it, we could speak of names as having a unique referent if we adopted a terminology, analogous to the practice of calling homonyms distinct 'words,' according to which uses of phonetically the same sounds to name distinct objects count as distinct names. This terminology certainly does not agree with the most common usage, but I think it may have a great deal to recommend it for theoretical purposes. (7-8)

Here Kripke is suggesting an analogy between homonyms and proper names. So if we can think of the word 'bank' (viz. the side of a river) as distinct from the word 'bank' (viz. a modern financial

[54] Here proper names can be considered as types and not tokens.

institution), then we can also think of the name 'Socrates' (viz. the master of Plato) as distinct from the name 'Socrates' (viz. Sócrates Brasileiro). Hence, according to Kripke, these two names are better thought of as completely distinct rather than being thought of as the same word with a differentiated semantic component. So how can we then differentiate their meaning? Kripke answers:

> As a speaker of my idiolect, I call only one object 'Aristotle,' though I am aware that other people, including the man I call 'Onassis' or perhaps 'Aristotle Onassis,' had the same given name ... In practice it is usual to suppose that what is meant in a particular use of a sentence is understood from the *context*. (9, emphasis added)

There are two main Kripkean claims here that we want to deal with. The first claim is that every proper name has a distinct bearer (i.e. is designating a distinct object) in all possible worlds. The second claim is that the semantics of proper names can be differentiated by referring to a specific context. There are various problems with these two claims, which we are going to discuss in order. Jerrold Katz (2001) criticizes the Kripkean analogy – seen to hold – between homonyms and proper names because it leads to some implausible results[55]:

- a. Two individuals with the name 'John Smith' will be mistaken to say, on Kripke's account, some natural statements like "we have the same name" or 'John Smith' is my name, too.'
- b. The definite article 'the' in 'The Albert Einstein lived in Princeton' will have no interpretation unless the name 'Albert Einstein' has more than one bearer.
- c. An adjective like 'Junior' associated with the son's name will be redundant.
- d. If 'Monica Lewinsky' marries 'Kenneth Starr,' then she will not be able to change her name to 'Monica Starr.'

All of these counterintuitive scenarios – that contradict the common use of natural language – stem directly from applying the Kripkean

[55] For the full critique, refer to Katz (2001), pp.150-152.

thesis that "distinctiveness of the referents will be a sufficient condition for distinctiveness of the names[56]."

Now we will explore the second Kripkean claim concerning the rigidity of proper names. Here he is basically claiming that when Alice speaks about 'Socrates' in a philosophy class, then we can comfortably suppose that Alice intends 'Socrates-master of Plato' and *not* 'Socrates-Brazilian footballer.' The context imposes this understanding, and hence the proper name 'Socrates' is rigid as it picks out a unique referent in all possible worlds, namely 'Socrates-master of Plato.' Nonetheless, by pushing this contextualist approach further, it turns out that the whole notion of rigidity is facing serious pitfalls. To see this consider the following story: World w_1 is the actual world as we know it, and hence Socrates is indeed the master of Plato, call him $Socrates_1$. Now, postulate another world w_2 where Socrates was the master of Plato up to a point in time t_1 where he had an unfortunate accident and lost his memory. After t_1, Socrates became hostile to philosophy. Hence in w_2, we have 'Socrates-master of Plato' before t_1, call him $Socrates_2$, and similarly we have another 'Socrates-*not* master of Plato' after t_1, call him $Socrates_3$. Thus, if we are back to our philosophy class and Alice says: "I like Socrates" then the word 'Socrates' in "I like Socrates" takes three possible truth-values:

- a) In w_1, for all t 'Socrates'='$Socrates_1$' and hence Alice's proposition is *true*.
- b) In w_2, before t_1 we have 'Socrates'='$Socrates_2$' and hence Alice's proposition is *true*.
- c) In w_3, after t_1 we have 'Socrates'='$Socrates_3$' and hence Alice's proposition is *false*[57].

To make her statement true in all possible worlds, Alice needs to rephrase it – by adding a temporal restriction – to something like "I like Socrates at all t in w_1, and before t_1 in w_2 ." Only by doing so, Alice makes sure that the proper name 'Socrates' refers to the person she intends: 'Socrates-master of Plato,' and not to the other

[56] 8, ft. 9.

[57] Here Alice's intended 'Socrates-master of Plato' is absent, and we assume that she will not like someone who is hostile to philosophy.

Socrates who is hostile to philosophy. But now we can see where the argument is going, as we can construct a different possible world w_4 which proceeds exactly as w_3 with the exception that '$Socrates_3$' regained his memory at time t_2 and continued to tutor Plato, call him '$Socrates_4$.' So now we have the following four possibilities:

- a) In w_1, for all t 'Socrates'= '$Socrates_1$' and hence Alice's proposition is *true*.
- b) In w_2, before t_1 we have 'Socrates'= '$Socrates_2$' and hence Alice's proposition is *true*.
- c) In w_3, after t_1 we have 'Socrates'= '$Socrates_3$' and hence Alice's proposition is *false*.
- d) In w_4, after t_2, we have 'Socrates'= '$Socrates_4$' and hence Alice's proposition is *true*.

Here again, Alice's general statement "I like Socrates" is false because of the (c) case where possibly 'Socrates= $Socrates_3$.' To fix it, she needs to *contextualize* her statement: "I like Socrates at all t in w_1, before t_1 in w_2, and after t_2 in w_4." Alas, we still can construct more possible worlds with more temporal conditions *ad infinitum*. This means that the proper name 'Socrates' in "I like Socrates" will always *fail* to designate the intended 'Socrates-master of Plato' because we need to place an infinite number of restrictions (i.e. contexts) to match all possible worlds. Stated differently, recall that Kripke responds to the multiple bearer's problem by arguing that the exact sense of the proper name can be understood from the given context, and hence there is no room for supposing that the same proper name can designate multiple referents. Nonetheless, we showed that there can be an infinity of contexts to be placed to guarantee the uniqueness of the designated referent of the proper name, and hence *proper names might not be rigid at all*.

5.5 Contextual Rigidity

To fix the *non*-rigidity of proper names resulting from the possible infinity of contexts imposed, we will propose a restricted version of rigidity, namely *contextual rigidity*.

Definition 14. (Contextual Rigidity): A contextually rigid designator D_c is a term that designates the same object o given a specified context c in all possible worlds.

And it follows that,

Definition 15. (Contextual Name Rigidity): A proper name N is contextually rigid only under the condition that if N designates object o, then N designates o in a *contextually rigid* manner.

Also, we can redefine the necessity of identity statements involving proper names,

Definition 16. (Contextual Necessity): Given a specified context c, If $F = N$, then it is *contextually necessary* that $F = N$, where F and N are contextually rigid names.

In the next part, we will investigate the effect of the notion of contextual rigidity on Kripke's necessary a posteriori propositions, and contingent a priori propositions.

5.5.1 Contextually Necessary A Posteriori Propositions

Recall that Kripke argues that a proper name identity statement like 'Hesperus=Phosphorus' is necessary a posteriori. Nonetheless, by implementing the contextualist line of thought explained above, we can argue that such statement is indeed a posteriori, but *not* necessary. To do this we need to argue against the rigidity of the proper names 'Hesperus' and 'Phosphorus.' Imagine the following story: world w_1 is the actual world where 'Hesperus=Phosphorus' is true. Now consider a counterfactual world w_2 where 'Hesperus=Phosphorus' is true at all time before t_1. But it just happened that from t_1, there was a political decree to limit the use of the name 'Hesperus' to the 'son of Eos'; and hence the statement 'Hesperus= Phosphorus' after t_1 becomes clearly false. So for 'Hesperus' to become a rigid designator in worlds w_1 and w_2, we need to specify a context in w_2 that verifies our original statement. The modified statement will be something like 'Hesperus=Phosphorus at all t in w_1, and before t_1 in w_2,' which is *contextually necessary* a posteriori. This contextualist method can be used systematically to create counterfactual worlds that violate the non-restricted necessity of proper name identities.

5.5.2 Contextually Necessary A Priori Propositions

The natural extension of the previous train of thought is to ask: Is a priori knowledge necessary à la Kripke? In other words, can we argue for the necessity of a priori knowledge based on the Kripkean notion of rigid designators? We will claim a *negative* answer to this question. More specifically, we will propose that a priori knowledge is *contextually necessary* but not necessary. First let us consider the following scenario regarding a priori standard arithmetic:

- In world w_1 the domain of integers is defined for all t, and hence the equation 'x+5=2' has a necessary solution of '-3'.
- In world w_2, before time t_1, the only defined arithmetic domain is the set of natural numbers, and hence the equation 'x+5=2' has no solution. After t_1, negative integers were introduced in the arithmetic system, and hence '-3' becomes a necessary solution to 'x+5=2'

Thus, the statement '-3 +5=2' is not necessary, but the statement '-3+5=2 after t_1' is contextually necessary. Similarly, we can argue for the contextual necessity of a priori knowledge using Aristotelian logic. One of the famous syllogisms in Aristotelian logic that was considered by medievals to be valid is the *Darapti*:

- *P1*. All As are Cs.
- *P2*. All As are Bs.
- *C*. Some Cs are Bs.

Nevertheless, the Darapti syllogism is invalid in first-order logic mainly due to the absence of the existential import in first-order logic[58]. Now consider the following scenario:

- In world w_1, the only used logical system is the Aristotelian logic for all t. Hence the Darapti is valid for all t.
- In world w_2, the only used logical system before t_1 is the Aristotelian logic. However, after t_1, first-order logic replaced Aristotelian logic as the used system. Therefore, the Darapti is valid before t_1, but invalid after t_1.

[58] For a discussion about this, see Priest (2006), section 10.8.

Here also the a priori statement 'The Darapti syllogism is valid' is not necessary as the Darapti is *not* a rigid designator. To see this, note that the name 'Darapti' picks a valid syllogism in w_1, but an invalid one in w_2 after t_1. So in a sense the 'Darapti' is not a rigid designator as it does not pick out the same object in the two worlds for all t. Still, the restricted statement 'The Darapti syllogism is valid at all t in w_1, and before t_1 in w_2' is *contextually necessary* as the Darapti is a *contextual rigid designator*.

6 Conventionalism, Apriority and Necessity

6.1 Revising the A Priori

In his *Critique of Pure Reason*, Kant often refers to three great a priori sciences: geometry, arithmetic, and logic. For him the objective reference of our sensuous experience is constituted by certain cognitive a priori forms – of intuition and thought – which order the matter of intuition and thus constitute its relation to objective correlates. These formal structures of experience (viz. the pure intuitions of space and time and the categories) are exemplified in *and* function as conditions of the possibility of the three aforementioned sciences. Space, in this sense, corresponds to geometry, time corresponds to arithmetic, and the categories correspond to logic. For Kant, geometry and arithmetic[59] are *synthetic* a priori systems of propositions[60], whereas formal logic – in contradistinction to transcendental logic – is a body of *analytic* propositions, which, as such, are a priori true[61]. This gives us the fields of Euclidean geometry, standard arithmetic, and Aristotelian logic[62]. Traditionally, and even intuitively, a priori knowledge was taken to be *non-revisable* knowledge, mainly due to its association with analyticity and necessity.

In the previous section, we argued that a priori knowledge might not be necessary in the Kripkean sense; rather, it is contextually necessary. Contextual necessity aims to restrict Kripkean necessity to limited contexts; and therefore it might be the case that some statements are necessary in some contexts, but not necessary in others. In what follows, we are going to investigate whether the

[59] The dominant twentieth century view, mainly due to Frege, is that arithmetic is analytic a priori, and not synthetic a priori as claimed by Kant (c.f. Hinckfuss (1996)).
[60] B15-16, *CPR*.
[61] B190 and B193, *CPR*.
[62] See Priest (2004).

context of utterance plays a role in shaping our ideas about necessity by looking at the most prestigious a priori science, namely logic. In an attempt to resolve the dilemma of the revisability of logic, Graham Priest (2014) distinguishes between three *senses* of the word 'logic:'

- Logica *docens*
- Logica *utens*
- Logica *ens*

First, *logica docens* is logic as claimed by logicians (i.e. what logicians teach about logic in their textbooks). Second, *logica utens* is logic as practiced, for reasoning, by people. Finally, *logica ens* is logic *in-itself* (i.e. what is the *real* notion of validity, and so on). Priest argues that it is not only possible for logica docens to be revised, but it was *de facto* revised.

> At any rate, one needs only a passing acquaintance with logic texts in the history of Western logic to see that the *logica docens* was quite different in the various periods. The differences between the contents of Aristotle's *Analytics*, Paul of Venice's *Logica Magna*, the Port Royale *Logic*, or *the Art of Thinking*, Kant's Jäsche *Logik*, and Hilbert and Ackermann's *Principle's of Mathematical Logic* would strike even the most casual observer. It is sometimes suggested that, periods of oblivion aside, the development of logic was cumulative. That is: something once accepted, was never rejected. Like the corresponding view in science, this is just plain false. (213)

Here, Priest is stressing that the process of revising *logica docens* was not a mere extension of logical systems over a continuum. For example, as seen before, the Darapti – which is a valid Aristotelian syllogism – cannot be validated within first-order logic. Moreover, it cannot be validated within first-order logic without invalidating other principals of Aristotelian logic. He writes,

> But it might well be suggested that the adoption of classical logic did not revise Aristotelian logic in any interesting sense: Aristotelian logic was perfectly correct as far as it

> went; it was just incomplete. Classical logic simply extended it to a more complete theory. Such a suggestion would be false. It is a well-known fact, often ignored by philosophers (though not, perhaps, historians of philosophy) that Aristotelian logic is incompatible with classical logic in just the same way that non-Euclidean geometries are incompatible with Euclidean geometry." (2006, 164 - 165)

Therefore, our *logica docens* is revisable to a great extent. It is notable that Priest's argument is not novel by any mean as W.V. Quine argued in his seminal paper *Two Dogmas of Empiricism*[63] that it is very hard to draw the dividing line between revisable and non-revisable theories; and hence all a priori sciences are just as revisable as a posteriori ones. A decade later, Hilary Putnam (1962) argued for the revisability of logic too. In 1969, Putnam suggests that classical logic (viz. First-order logic) can be replaced by quantum logic when reasoning about the micro-scale world. Putnam even takes his argument further by claiming the possibility of revising the law of non-contradiction based on *empirical* findings.

> But we should be clear about what the centrality argument[64] does not show. It does not show that a putative law of logic, for instance the Principle of Contradiction, could not be overthrown by direct observation. Presumably I would give up the Principle of Contradiction if I ever had a sense datum which was both red and not red, for example. And the centrality argument sheds no light on how we know that this could never happen. (1983, 110)

The more difficult question is whether logic *qua* Logica ens can be revised. Priest thinks that this question is reducible to another simpler question, namely: what is *validity*? If the notion of validity changes, Logica ens can change.

> Whether the truth of validity-claims can change will depend on what, exactly, constitutes validity. Let me illustrate. Suppose that one held a "divine command" theory

[63] (1951, 20-43).

[64] The centrality argument, for Putnam, is that the laws of logic are "presupposed by so much of the activity of argument itself that it is no wonder that we cannot envisage their being overthrown, or all of them being overthrown, by rational argument" (1983, 110).

> of validity: something is valid just if God says so. Then God being constant and immutable, what is valid could not change. On the other hand, suppose that one were to subscribe to the "dentist endorsement" view of validity: what is valid is what 90% of dentists endorse. Clearly, that can change (11)

He then argues that l*ogica ens* cannot be revised given the available notions of validity: the model-theoretic and proof-theoretic notions. A full discussion of this point, however, does not fall within the scope of this essay.

6.2 On Conventionalism: Wittgenstein

The story of conventionalism can be traced back to Henri Poincaré who was working within a Kantian framework, however, he was still not completely satisfied with it. The source of his dissatisfaction is that the theorems of geometry did not fit in any Kantian typology of knowledge. For example, the state of *Euclidean space* is not analytic a priori, nor synthetic a priori, nor a posteriori. It is not a priori since it is *conceivable* to imagine a *non-Euclidean space* with no contradictions[65]; hence it is not a necessary truth. In addition, it is not a posteriori too since it is not possible for the empirical world to falsify that 'space is Euclidean.' Put differently, there are many consistent 'geometries' that are incompatible with each other, but still cannot be proved or disproved using experience, so it is not an empirical truth either. Poincaré solves this geometric-typological problem by introducing the new epistemic class of *conventional knowledge*. Hence a statement like 'space is Euclidean' becomes just conventional knowledge that does not express a necessary truth, nor an empirical truth. He writes,

> Are they [the axioms] synthetic a priori intuitions, as Kant affirmed? They would then be imposed upon us with such a force that we could not conceive of the contrary proposition, nor could we build upon it a theoretical edifice. There would be no non-Euclidean geometry. ([1902] 1952, 48)

[65] Recall that if a proposition is a priori then it is necessary, according to Kant.

With the rise of logical positivism, the conventionalist program was to reduce all necessary truth(s) to linguistic conventions. A major contribution of Wittgenstein in the debate on the nature of necessity is to emphasize the role of *language-games* in perceiving internal relations among objects. Alberto Coffa (1991) compares the shift made toward the *constitutive role of language* by Wittgenstein to the shift toward the *constitutive role of reason* made by Kant[66]. The later Wittgenstein shared with the logical positivists their dissatisfaction with the available philosophical treatments of necessary truth, but he diverged from them on many other points[67]. Generally speaking, conventionalists – along with Wittgenstein – reject the positions of Platonism, empiricism and psychologism regarding the nature of necessary truth. Wittgenstein expresses this in his *Remarks on the Foundations of Mathematics* (RFM).

> It is as if this expressed the essence of form.—I say, however: if you talk about essence—, you are merely noting a convention. But here one would like to retort: there is no greater difference than that between a proposition about the depth of the essence and one about— a mere convention. But what if I reply: to the depth that we see in the essence there corresponds the deep need for the convention. (I:74, [1956] 1967)

But how can we, then, link the notion of the 'conventional' to the notion of the 'necessary'? After all, it seems that the conventional is contingent while the necessary is non-contingent. There are two general responses by conventionalists to this question. Firstly, the (assumed) necessary truths and conventional rules share a *normative* feature. So, the rejection of their violations is due to their normativity. Wittgenstein writes: "If 2 and 2 apples add up to only 3 apples, i.e. if there are 3 apples there after I have put down two

[66] Coffa writes:

> "Wittgenstein's and Carnap's insights on the a priori belong in the same family as Kant's. One could, in fact, mimic Kant's famous "Copernican" pronouncement to state their point: If our a priori knowledge must conform to the constitution of meanings, I do not see how we could know anything of them a priori; but if meanings must conform to the a priori, I have no difficulty in conceiving such a possibility." (263)

[67] For an elaborate account of the history of conventionalism from Poincaré to Quine, see Ben-Menahem (2006).

and again two, I don't say: 'So after all 2 +2 are not always 4; but "Somehow one must have gone[68]'. " Secondly, and more importantly, conventional rules and necessary truths are both *constitutive*. For the conventionalist, the common sense understanding that necessary truths are descriptive (i.e. describing an external reality) is flawed. Instead, these laws of thought constitute our mode of reasoning, call this the *priority argument*:

> The steps which are not brought in question are logical inferences. But the reason why they are not brought in question is not that they 'certainly correspond to the truth'—or something of the sort,—no, it is just this that is called 'thinking,' 'speaking,' 'inferring,' 'arguing.' There is not any question at all here of some correspondence between what is said and reality; rather is logic *antecedent* to any such correspondence; in the same sense, that is, as that in which the establishment of a method of measurement is *antecedent* to the correctness or incorrectness of a statement of length. (*RFM*, I:155)

But if the laws of thought are conventional, then we face the famous problem of *arbitrariness*. In other words, if the so-called necessary laws are being postulated by convention, then they could have been otherwise, which is counterintuitive. Alan Sidelle (2009) articulates this objection as follows:

> One of the more familiar arguments against modal Conventionalism goes something like this: According to Conventionalism, what is necessary, or essential, is so because of our conventions, our ways of conceiving and/or talking about the world. But our conventions, whatever they are, might have been different. If so, the Conventionalist must admit that what is necessary or essential might not have been so. But, then, it is not really necessary or essential then! So conventionalism is false. (224)

Wittgenstein was quite aware of this problem, as a possible 'response' in *Philosophical Grammar* (1974) might show: "Grammatical rules determine a meaning and are not answerable to any meaning they could contradict ... The rules of grammar are arbitrary in

[68] *RFM*, I:156.

the same sense as the choice of a unit of measurement[69]." The quote suggests the Wittgensteinian endorsement of the arbitrariness view of conventionalism. Ben-Menahem (2006) distinguishes between two types of arbitrariness of conventions. In one type, arbitrariness is interpreted as *unjustifiability*. So in that sense, a convention is arbitrary if it cannot be justified by exogenous factors. In the other type, a convention is arbitrary if we can change it without changing its meaning or nature. Now we can see that the arbitrariness of constitutive conventions regarding a priori knowledge comes only in the first sense. In that way, David Lewis (1969) considers that 'driving on one side of the road' is not a convention as it can be justified[70]; while 'driving on which side of the road[71]' is a convention as it cannot be justified. Applying this to our modes of inference expressed in logic, Wittgenstein probably thinks of it as arbitrary à la first type of arbitrariness but not à la second type of arbitrariness. The priority argument, mentioned above, shows that the rules of inference used in reasoning must be there in order to equip the notion of reason itself with a meaning, and hence these rules of inference are arbitrary in the first sense. But if these rules of inference change, our mode of reasoning will change too, and hence they are not arbitrary in the second sense. For the conventionalist, labeling 'necessities' as 'conventional' does not undermine their compelling force in our thought by any mean. This form of conventionalism is mainly a refutation of the *meta*-necessity of necessary statements rather than a refutation of their necessity[72]. Ben-Menahem (2006) comments on this point:

> ... Traditional necessary truths lose nothing when seen as constitutive conventions rather than super-truths, for as far as our actual life and thought is concerned, what is constitutive of our basic activities is every bit as unassailable as traditional necessary truth. (266)

Despite resolving the prima facie tension between conventionalism and necessity, there remains a more subtle paradox in the Wittgensteinian version of conventionalism. Recall that Wittgenstein thinks

[69] I: 133.

[70] To reduce the number of accidents, for instance.

[71] I.e. right side or left side.

[72] Sidelle (2009) criticizes this form of conventionalism (viz. that accepts the necessity of statements but only rejects their meta-necessity).

that the notion of necessity arises from some linguistic conventions, and cannot be related to any *deeper truth* about the world. This is obvious from his treatment of mathematics as an *invention* rather than as a *discovery*. He writes in *Lectures on the Foundations of Mathematics* ([1939] 1976): "One talks of mathematical discoveries. I shall try again and again to show that what is called a mathematical discovery had much better be called a mathematical invention[73]." So what is traditionally thought of as necessary is a mere reflection of our linguistic practices as opposed to a fundamental truth: "The connexion which is not supposed to be a causal, experiential one, but much stricter and harder, so rigid even, that the one thing somehow already is the other, is always a connexion in grammar." (I: 128)

But, paradoxically, this conventionalist view is challenged by Wittgenstein's rule-following paradox discussed before in section 3. The dilemma of the rule-following paradox is that *every* action can be interpreted according to a given rule; "no course of action could be determined by a rule, because every course of action can be made to accord with the rule[74]." Nevertheless, and paradoxically, conventionalism was exactly supposed to answer why we follow this specific rule and not another one, namely because of a specific convention. But the rule-following paradox questions that by allowing the same rule to be interpreted in different courses of action, and allowing the same action to be interpreted by many rules. Wittgenstein attempts to solve this by showing that "...there is a way of grasping a rule which is not an interpretation, but which is exhibited in what we call 'obeying the rule' and 'going against it' in actual cases[75]." It is not clear here if this solution is going to work since conventionalism – although itself being unjustified – is supposed to give us a justification of *why* we apply specific rules in specific ways.

Michael Dummett (1978) attempted to save the Wittgensteinian position from inconsistency by distinguishing two types of conventionalism. The first type of conventionalism is that widely held by the logical positivists which can be summarized in A.J. Ayer's ([1936] 1946) words:

[73] P. 22
[74] *PI*, I: 201.
[75] *PI*, I: 201.

> Just as it is a mistake to identify a priori propositions with empirical propositions about language, so I now think that it is a mistake to say that they are themselves linguistic rules. For apart from the fact that they can properly said to be true, which linguistic rules cannot, they are distinguished also by being necessary, whereas linguistic rules are arbitrary. At the same time, if they are necessary, it is only because the relevant linguistic rules are presupposed. Thus, it is a contingent, empirical fact that the word "earlier" is used in English to mean earlier, and it is an arbitrary, though convenient, rule of language that words that stand for temporal relations are to be used transitively; but given this rule, the proposition that, if A is earlier than B and B earlier than C, A is earlier than C becomes a necessary truth. (17)

This type is what Dummett labels *modified conventionalism* which distinguishes between *trivial* conventionalism like using the word 'earlier' to mean earlier, and *non-trivial* conventionalism like the transitivity rule. Hence there is a difference between what a specific community adopts as convention (e.g. modus ponens as a mode of inference), and the consequences of this convention (e.g. the truth-value of the application of modus ponens). Dummett suggests that Wittgenstein rejects modified conventionalism in favor of *full-blown conventionalism*. In the latter form of conventionalism, each individual application of a rule is a *new* convention[76]. Thus, the formation of convention becomes an *individual choice* in full-blown conventionalism, while being a *communal choice* in modified conventionalism[77].

6.3 More on Conventionalism: Carnap, Quine and Lewis

Generally speaking, we can think of two ways of *defining* terms within any language. The first way is known as *implicit definition*[78]. The idea is that if we can *fix* the meaning of an expression by imposing a set of constraints on its usage in longer expressions (e.g.

[76] Wittgenstein writes: "It would almost be more correct to say, not that an intuition was needed at every stage, but that a new decision was needed at every stage" (*PI*, I: 158).

[77] The Dummettian solution received multiple criticisms from many philosophers like Stroud (1965) and Diamond (1991), among others.

[78] See Hale and Wright (2000) for an exposition on implicit definitions.

sentences), then these *stipulated* constraints will define the truth of those sentences based on previously understood vocabulary. Oppositely, we have the *explicit definition* which aims to supply a *semantic equivalent* expression of the definiendum. Conventions can be regarded as a series of stipulated *implicit* definitions that constitute the *meaning* – and *truth* – of the defined expressions. Consequently, the necessity of all a priori statements is a result of our linguistic conventions, rather than representing any metaphysical truth. By extending these ideas to logical systems, Carnap[79] established his *linguistic doctrine of logical truth*. For any formal system, Carnap's project proceeds by stipulating a set of axioms and a set of rules of inference, both of which are taken to implicitly define the logical constants of the formal system in consideration. These transformation rules (viz. the rules of inferences) which specify the legitimacy of our inferences in a formal system are merely conventional[80].

Definition 17. *Carnap's truth-by-convention*: The logical truths of a language system LS are all and only those sentences of LS that are true-in-LS solely in virtue of the linguistic conventions for LS[81].

By changing the linguistic conventions of the language system LS, the logical truth of sentences in LS might vary accordingly. Carnap thinks that we can move freely between conventions, and hence language systems. More particularly, Carnap (1937, 51) proposes a *principle of tolerance* which states that "it is not our business to set up prohibitions, but to arrive at conventions." This induces an element of contingency and arbitrariness in the adopted convention as there are no general specifications for any system's rules[82]. Carnap (1937) writes:

> *In logic there are no morals*. Everyone is at liberty to build up his own logic, i.e., his own form of language, as

[79] See Carnap (1937, 1947, 1950, 1955).

[80] The epistemic problem of how we can be justified in accepting the truth logical of statements (and rule of inferences) based on schematic explicit definitions is a hard problem. Gillian Russell (2008) wrote about this:

> I am inclined to think it is one of the biggest unsolved problems facing philosophy today. If I were to construct a 'Hilbert List' of unsolved problems in philosophy, it would certainly make the top three—but perhaps this is just because I need to solve it and I can't. (164, ft. 5)

[81] This formulation is due to Ebbs (2011, 194).

[82] For an extensive discussion of Carnap's thoughts, see Coffa (1991).

> he wishes. All that is required of him is that if he wishes to discuss it, he must state his methods clearly, and give syntactical rules instead of philosophical arguments. (52)

Historically, Carnap's notion of truth-by-convention lost its vigor[83] mainly due to Quine's objections[84]. The central argument against Carnap appears in Quine's influential article *Truth by Convention*:

> In the adoption of the very conventions ... whereby logic itself is set up, however, a difficulty remains to be faced. Each of these conventions is general, announcing the truth of every one of an infinity of statements conforming to a certain description; derivation of the truth of any specific statement from the general convention thus requires a logical inference, and this involves us in an infinite regress. (270)

The problem of the truth-by-convention program, according to Quine, is that if logical truth is postulated by linguistic conventions, then we need as many conventions as the number of logical truths. Nevertheless, this seems very counterintuitive as there can be an infinite number of logical truths which require an infinite number of conventions. The only maneuver available for the conventionalist is to assume a *general* convention (i.e. a metatheoretic convention) from which she can logically infer the other more specific conventions. But the difficulty, again, is that the rules of logical inferences themselves have to be based on other conventions which in turn need other metaconventions to justify, and hence running into *infinite regress*. Here, Quine uses the same intuition of Lewis Carroll for the unjustifiability of deduction to argue for the unjustifiability of truth-by-convention. He writes:

> The linguistic doctrine of logical truth is sometimes expressed by saying that such truths are true by linguistic

[83] Boghossian (1996) wrote:

> In his classic early writings on analyticity – in particular, in "Truth by Convention," "Two Dogmas of Empiricism," and "Carnap and Logical Truth" – Quine showed that there can be no distinction between sentences that are true purely by virtue of their meaning and those that are not. In so doing, Quine devastated the philosophical programs that depend upon a notion of analyticity – specifically, the linguistic theory of necessary truth. (360)

[84] For instance, Quine (1936, 1951a, 1951b, 1960).

> convention. Now if this be so, certainly the conventions are not in general explicit ... For it is impossible in principle, even in an ideal state, to get even the most elementary part of logic exclusively by the explicit application of conventions stated in advance. (The difficulty is the vicious regress, familiar from Lewis Carroll which I have elaborated elsewhere). Briefly the point is that the logical truths, being infinite in number, must be given by general conventions rather than singly; and logic is needed then to begin with, in the meta-theory, in order to apply the general conventions to individual cases. (Quine 1960, 357)

One way to avoid the infinite regress of conventions is by relaxing the *explicit* stipulation of conventions, and adopting a different kind of conventionalism – conventionalism through behavioral practice, for instance. Likewise, Quine rejects this alternative since it is opaque how a convention can be formerly adopted without spelling out its *explicit* formulation. Also, the newly formulated language will lose its deliberateness, and hence lose some essential characteristics we think it should possess.

> When we first agree to understand 'Cambridge' as referring to Cambridge in England failing a suffix to the contrary, and then discourse accordingly, the role of linguistic convention is intelligible; but when a convention is incapable of being communicated until after its adoption, its role is not so clear. In dropping the attributes of deliberateness and explicitness from the notion of linguistic convention we risk depriving the latter of any explanatory force and reducing it to an idle label. We may wonder what one adds to the bare statement that the truths of logic and mathematics are a priori, or to the still barer behavioristic statement that they are firmly accepted, when he characterizes them as true by convention in such a sense. (Quine 1936, 273)

Non-explicit adoption of linguistic conventions, according to Quine, is not well motivated then. If these conventions merely appear as mere *regularities of behavior*, it will be hard to justify the tight relation between the truth of a logical sentence and its meaning. It

will turn out that the formulated language is just a description of a previously adopted behavior, and hence it will lose its supposed truth. In order to respond to the Quinean objections we need, first, to differentiate between *formal language conventionalism* (viz. conventionalism pertaining to formal languages) and *natural language conventionalism* (viz. conventionalism pertaining to natural languages). Quine's objections do not apply to the former type of conventionalism as the syntax and semantics of any formal language are simply *stipulated* with the aid of natural language, and there is no need for other prior conventions as Quine suggests. In their unpublished correspondence[85], Carnap seems to notice this regarding formal languages:

> The difference between analytic and synthetic is a difference internal to two kinds of statements inside a given language structure; it has nothing to do with the transition from one language to another. 'Analytic' means rather much the same as true in virtue of meaning. Since in changing the logical structure of language everything can be changed, even the meaning assigned to the '.' sign, naturally the same sentence (i.e., the same sequence of words or symbols) can be analytic in one system and synthetic in another, which replaces the first at some time. Since the truth of an analytic sentence depends on the meaning, and is determined by the language rules and not the observed facts, then an analytic sentence is indeed 'unrevisable' in another sense: it remains true and analytic as long as the language rules are not changed. (431-432)

Let us think of a natural language (say English) whose syntax and semantics are held fixed for a period of time. Now the question is: can we generate multiple formal languages from the same natural language without manipulating the latter? If changing the logical structure of a formal language changes everything regarding that language (e.g. an analytic statement in one formal language might be a synthetic statement in another formal language), then it appears that we have a positive answer to our question. For instance we can start by generating classical logic, and then make

[85] Quine, W. V., and Rudolf Carnap. *Dear Carnap, Dear Van: the Quine-Carnap correspondence and related work*. Richard Creath (ed.) CA: University of California Press, 1990.

some amendments (i.e. impose different stipulations) on its syntax and semantics to create three-valued logic. This can be done using the same linguistic conventions of English without running into infinite regress. Interestingly, our argument uses the reverse methodology of two important programs in linguistics, namely the program of formal syntax (Chomsky 1957)[86] and the program of formal semantics (Montague 1974)[87]. We think our argument is more 'natural' as it takes natural languages to be more *primitive* than the formal language of logic[88]. Nevertheless, Quine's objections are perfectly applicable to natural language conventionalism. Fortunately, there is a possible way out of this predicament via David Lewis' (1969) account on conventionalism. For Lewis, the whole problem of linguistic conventions boils down to *coordination problems*[89]. For Lewis, conventions arise as *non-explicit* solutions to coordination problems, and hence have no clear codification to be spelled out. This move will avoid the Quinean objection against Carnap. Lewis (1969) defines a *convention* as follows:

Definition 18. (Convention[90]): A regularity R in the behavior of members of a population P when they are agents in a recurrent situation S is a convention if and only if it is true that, and it is

[86] The Chomskian program is to define the set of all well-formed sentences of a natural language.

[87] This program aims to find an association between logical semantics and meaningful expressions of natural languages. On the relation between formal languages and natural languages, Montague (1970) writes:

> There is in my opinion no important theoretical difference between natural languages and the artificial languages of logicians; indeed, I consider it possible to comprehend the syntax and semantics of both kinds of language within a single natural and mathematically precise theory. On this point I differ from a number of philosophers, but agree, I believe, with Chomsky and his associates. (373)

[88] The relation between natural languages and logical systems has been investigated since few decades. For instance, Van Benthem (1986, 1987) observed that natural languages have a form of 'natural logic', which is a system of universal forms of reasoning that operate on the surface form of natural languages. Likewise, Lakoff (1970) argued for the deep correspondence between logical rules of natural languages and the grammatical structures of natural languages:

> ... most of the reasoning that is done in the world is done in natural language. And correspondingly, most uses of natural language involve reasoning of some sort. Thus it should not be too surprising to find that the logical structure that is necessary for natural language to be used as a tool for reasoning should correspond in some deep way to the grammatical structure of natural language. (151)

[89] In fact, all social interactions are dominated by coordination problems; from economic coordinations on media of exchange, to the meanings of hand gestures.

[90] 78.

common knowledge in P that, in almost any instance of S among members of P,

1. almost everyone conforms to R;
2. almost everyone expects everyone else to conform to R;
3. almost everyone has approximately the same preferences regarding all possible combinations of actions;
4. almost everyone prefers that any one more conform to R, on condition that almost everyone conform to R;
5. almost everyone would prefer that any one more conform to R', on condition that almost everyone conform to R',

where R' is some possible regularity in the behavior of members of P in S, such that almost no one in almost any instance of S among members of P could conform to both R' and to R.

As noticed, Lewis' conventionalism is constituted by a series of expectations and preferences that reinforce a behavior-regularity. Moreover, this R is arbitrary and contingent as it could be substituted by any other R'' as long as conditions (1)-(5) are satisfied[91]. Now we have a more consistent narrative that avoids the Quinean objections. The story goes as follows: firstly, natural languages evolved via *implicit* conventions à la Lewis. Then, formal languages developed à la Carnap from natural languages. This implies that formal languages themselves are also conventional. To see this, Lewis identifies *consequences* of conventions as follows:

Definition 19. A consequence $R*$ depends on R only if there is a regularity R' that is alternative to R, and not-$R*$ is a logical consequence of R'[92].

But since formal languages are arbitrary, it seems that we are stuck with the problem of how to give an account of our *intuition*

[91] Although there have been multiple criticisms for Lewis' conventionalism (c.f. Binmore (2008) and Moore (2013)), we nonetheless think that the essence of his thesis survives these attacks.

[92] 80.

of the prima facie *necessity* of formal languages[93]. Upon deeper reflection, it turns out that this *common sense objection* is hard to motivate for multiple reasons. First, note that Carnap, in the aforementioned quotation, mentions that the same symbols can acquire different meanings by changing the logical structure of the object language. Subsequently, a logical rule LR for a language inherits its meaning(s) from its relation to other logical rules, not-LR, in the same language. Therefore, if the meaning of not-LR changes then the meaning of LR will change consequently. The *common sense necessity* of LR, then, is just a *relational necessity*, nothing more. Second, we have an empirical evidence about the, de facto, existence of multiple formal languages (e.g. intuitionistic logic, paraconsistent logic, three-valued logic, etc.) according to which we can reason. Moreover, these logical systems are incompatible with each other, implying that necessary truths are incompatible with each other. This begs the question of which logical system is the *true* one? Conventionalism suggests that all of them are *equally* true, and saves the whole logical enterprise.

To wrap up, we argued for a narrative about how formal languages can be rooted in natural linguistic conventions in a way that avoids Quine's original criticisms of Carnap. In addition, the emergence of natural languages can be formulated using Lewis' non-explicit conventionalism.

7 Contextualism

One of the recent developments in analytic philosophy is the rising

[93] There is a tradition in philosophy that undermines the role of *common sense* in philosophical argumentation. For instance, in his *Prolegomena to any Future Metaphysics*; Kant writes:

> "To appeal to ordinary common sense ... is one of the subtle discoveries of recent times, whereby the dullest windbag can confidently take on the most profound thinker and hold his own with him ... this appeal is nothing other than a call to the judgment of the multitude; applause at which the philosopher blushes, but at which the popular wag becomes triumphant and defiant." [4: 259].

Attacks on common sense can also be found in Hegel's *Phenomenology of Spirit*,

> "Since the man of common sense makes his appeal to feeling, to an oracle within his breast, he is finished and done with anyone who does not agree; he has only to explain that he has nothing more to say to anyone who does not find and feel the same in himself. In other words, he tramples underfoot the roots of humanity." (43)

popularity of *contextualism* as an influential player in many branches of philosophy (e.g. philosophy of language, philosophy of logic, epistemology, metaphysics, and others). The most obvious manifestation of contextuality is in the usage of *indexicals*. Imagine person A who says to person B "*My* opinion is right and *your* opinion is mistaken," and person B responds by saying the same statement to person A. Intuitively, we are eligible to think that the two statements are *not* compatible although, *prima facie,* they are expressed with the same utterance. In this sense, the designation of indexicals changes from one context to another depending on the speaker, time, place, among other determinants. David Kaplan (1989) provided a list of such indexicals like: personal nouns (e.g. "she," "he," "it"), demonstrative pronouns (e.g. "this," "that"), adjectives (e.g. "present," "actual"), adverbs ("here," "later," "tomorrow"), and others. In addition, contextualism can be motivated by the usage of *modal* terms.

Example 1. Suppose that both Amira and Dina utter the following statement: "Ashraf might have been a philosopher[94]." Here Amira means that – according to her knowledge – Ashraf *was possibly* a philosopher. Dina, on the contrary – who knows that Ashraf is not a philosopher – means that it has been *possible* for Ashraf to be a philosopher.

Hence, Amira's usage of the word 'might' is an *epistemic* one, while Dina's usage of the word 'might' is a *metaphysical* one. Changing the context of usage generated two different meanings for the word 'might.' But even if we focus only on the epistemic use of 'might,' different meanings can still be generated by changing the context.

Example 2. Imagine Jack and Jim who are both looking for Topa on campus. After some time, they both give up on finding him, and utter the following:

- (2) "Topa might be on campus."

Utterance (2) can be rephrased as:

- (2a) According to Jack's knowledge, Topa *is* on campus.

[94] Assume here that it is the case that Ashraf is *not* a philosopher.

- (2b) According to Jim's knowledge, Topa is *not* on campus.

The modal term 'might' varied from the epistemic context of Jack to the epistemic context of Jim, thus generated (2a) and (2b). Furthermore, contextualism can also be extended to the class of *gradable adjectives*[95] (e.g. tall, heavy, valuable, etc). Consider the following example from Richard (2004):

> Suppose, to take an example, that Mary wins a million dollar lottery. Didi is impressed, and remarks to a friend 'Mary's rich.' Naomi, for whom a million dollars is not really all that much, remarks in a conversation disjoint from Didi's, 'Mary is not rich at all'... It seems to most of us that Naomi is contradicting Didi. But, especially if each remark is part of a longer conversation ... it is very plausible that the truth of their claims about wealth turns on whatever standards prevail within their conversations. This is, in any case, part and parcel of a contextualist view of the semantics of 'rich.' But then Naomi and Didi *don't* disagree, in the sense that one asserts something which is inconsistent with what the other asserts. (218)

In the same vein, the contextualist intuition can be applied to epistemic terms like 'know' which are context-sensitive. Generally speaking, Duncan Pritchard (2002) classifies *epistemic contextual theories* (i.e. contextualist theories regrading epistemic terms) into two forms: The first is *semantic contextualism*[96] in which *conversational contexts* determine epistemic contexts. The second is *inferential contextualism*[97] in which *inferential structures* are responsible for epistemic contexts. In semantic contextualism, the word 'context' means the *attributer's* conversational context. So if a person Q says "S knows P" then the verb 'knows' expresses different truth-values according the attributer's context of utterance (viz. the context of Q in this case). In DeRose's (1999) words:

> 'Contextualism' refers to the position that the truth-conditions knowledge-ascribing and knowledge-denying sentences (sentences of the form "S knows that P" and "S doesn't know

[95] See Bridges (2008) for more details.
[96] As found in the work of David Lewis (1996), and Keith DeRose (1995).
[97] As found in the work of Michael Williams (1991).

> that P" and related variants of such sentences) vary in certain ways according to the context in which they are uttered. (1)

By changing the context of utterance of any proposition, we are changing the *epistemic standards* that must be met by subject S in order to claim knowledge of P. DeRose (1999) continues:

> What so varies is the epistemic standards that S must meet (or, in the case of a denial of knowledge, fail to meet) in order for such a statement to be true. In some contexts, "S knows that P" requires for its truth that S have a true belief that P and also be in a very strong epistemic position with respect to P, while in other contexts, the very same sentence may require for its truth, in addition to S's having a true belief that P, only that S meet some lower epistemic standards. Thus, the contextualist will allow that one speaker can truthfully say "S knows that P," while another speaker, in a different context where higher standards are in place, can truthfully say "S doesn't know that P," though both speakers are talking about the same S and the same P at the same time. (1-2)

Based on this, a high school student can claim the *truth* of the following "S knows that the electron's charge is 0.00048." On the other hand, a physics professor can *consistently* claim the *falsehood* of the *same* statement"S knows that the electron's charge is 0.00048[98]." This is because the epistemic standard – regarding scientific rigor – of high school students is lower than those of professional physicists. By allowing knowledge to be *context-sensitive*, semantic contextualism can solve many hardcore philosophical problems like external-world skepticism and the possibility of free will. Concerning skepticism for example, the semantic contextualist will argue that the problem mainly arises because the skeptic is fixing a high level of epistemic standards in all conversational contexts, and the way out is to allow these standards to vary according to contexts. For the semantic contextualist, the epistemic standards employed in a philosophy class are higher than those employed in quotidian conversational contexts. Consequently, it will be a coherent position for

[98] The electron charge is $4.80320451 \times 10^{-10}$ electrostatic unit (Encyclopaedia Britannica).

a philosophy student, during classtime, to believe that he does not know the denial(s) of the skeptical hypothesis. But once he walks outside of class, he can form the belief – without self-contradiction – that he knows the denial(s) of the skeptical hypothesis.

Another prominent form of contextualism is presented by Williams (1995) and labeled by Pritchard (2002) as *inferential contextualism*. This form is distinguished from semantic contextualism by three main features. First, it is a *subject*'s contextualist thesis and *not* an attributer's contextualist thesis. Second, it has an *anti-realist* epistemic commitment. Third, contexts are determined by their inferential structures, and not by their conversational modes. Concerning the first difference, Williams' crucial factor in determining if "S knows P" (or S does not know P) is S herself, and not a third person attributer. Concerning the second difference, recall that semantic contextualism allowed for some contexts to be described as epistemically *superior* or *inferior* to other contexts. This move *presupposes* a commitment to *epistemological realism*, which is an extra theoretical burden according to Williams. He writes:

> What is relevant will depend on both the content of the claim in question and the context in which the claim is entered. If all evidence is relevant evidence, then, abstracting from such contextual details, there will be no fact of the matter as to what sort of evidence could or should be brought to bear on a given proposition ... No proposition, considered in abstraction, will have an epistemic status it can call its own. To suppose that it must is precisely to fall in with what I call "epistemological realism." (113)

And

> The sceptic must be an epistemological realist. Only epistemological realism can validate his inference from results obtained in his very special context of philosophical reflection to the general impossibility of worldly knowledge. (130)

By rejecting epistemological realism, Williams can reject any *hierarchical* classification of epistemic contexts which allows him to grant epistemic self-sufficiency for each context independently.

> To adopt contextualism, however, is not just to hold that the epistemic status of a given proposition is liable to shift with situational, disciplinary and other contextually variable factors: it is to hold that, independently of all such influences, a proposition has no epistemic status whatsoever. (P.119)

Capitalizing on this, Williams can justify the inferential nature of contextualism. The epistemic status of each proposition is decided based on how schematic inferences are made within the relative context. Therefore, for Williams, a philosophy classroom's skeptical-context does not induce higher epistemological standards, but only a different inferential structure (viz. the standards of philosophical reflection in that case). Consequently, the skeptic *cannot* force her epistemic standards on daily contexts since they simply do not represent superior standards – as there are no universal standards – that we should adopt in all contexts. For these three main differences between semantic and inferential contextualism, Pritchard argues for the superiority of the latter over the former. For our purpose, we will make use of inferential contextualism later on to argue against a priori skepticism.

8 Revisiting A Priori Skepticism

8.1 Hinge Propositions

The 'Common Sense' defense of epistemic justification has been promoted by many philosophers like Thomas Reid, G.E. Moore and Roderick Chisholm[99]. Reid once wrote: "Philosophy . . . has no other root but the principles of Common Sense; it grows out of them, and draws its nourishment from them. Severed from this root, its honours wither, its sap is dried up, it dies and rots[100]." Moore in the same vein wrote:

> There is no reason why we should not, in this respect, make our philosophical opinions agree with what we necessarily believe at other times. There is no reason why I should not confidently assert that I do really know some

[99] For a contemporary defense of the Common Sense tradition, see Lemos (2004).
[100] (*Inquiry and Essays*, 7)

> external facts, although I cannot prove the assertion except by simply assuming that I do. I am, in fact, as certain of this as of anything; and as reasonably certain of it. (*Philosophical Studies*, 163)

And in response to skeptical claims, Moore famously reacted:

> But it seems to me a sufficient refutation of such views as these, simply to point to cases in which we do know such things. This, after all, you know, really is a finger; there is no doubt about it: I know it, and you all know it. And I think we may safely challenge any philosopher to bring forward any argument in favour either of the proposition that we do not know it, or of the proposition that it is not true, which does not at some point rest upon some premises which is beyond comparison, less certain, than the proposition which it is designed to attack (*Philosophical Studies*, 228)

The Moorean *common sense refutation of skepticism* is then based on 'knowing' that some propositions (e.g. what I see is a finger) are just beyond doubt, and that we are reasonably justified in believing so. Moore here seems to weigh empirical observations over any other form of philosophical-skeptical argumentation since the latter is less 'certain' than the former. Despite the compelling Moorean reasoning, Wittgenstein – in his final philosophical work *On Certainty* (OC) – finds it flawed. Wittgenstein notes that any evidence produced to support a specific belief must be at least as *certain* as that belief. Hence, for Wittgenstein, the belief "I know that I have two hands" demands more basic grounds to justify it. But how can we find a grounding-belief, in normal circumstances, which is more certain than "knowing that I have two hands"?

> ...The certainty is subjective, but not the knowledge. So if I say "I know that I have two hands," and that is not supposed to express just my subjective certainty, I must be able to satisfy myself that I am right. But I can't do that, for my having two hands is not less certain before I have looked at them than afterwards. But I could say: "That I have two hands is an irreversible belief." That would express the fact that I am not ready to let anything count as a disproof of this proposition. (OC § 245)

Consequently, the common sense argument will not work as "having two hands" is as certain as any other evidence that can be produced to support it[101]. One resolution, for Wittgenstein, is made by removing these *Moorean truisms* from the domain of knowledge. So if "having two hands" is no longer a proposition that can be known, then it can be saved from rational doubt. Wittgenstein gives the analogy of a pupil who wants to interrupt his history lesson to doubt the existence of Earth. Wittgenstein objects this by saying: "This doubt isn't one of the doubts in our game[102]." So what Wittgenstein does, to rebut skepticism, is twisting the Moorean common sense propositions to what he labels *hinges*. These hinges are contingent claims that form the foundations of any epistemic inquiry:

> [§ 341] That is to say, the questions that we raise and our doubts depend upon the fact that some propositions are exempt from doubt, are as it were like hinges on which those turn.
>
> [§ 342] That is to say, it belongs to the logic of our scientific investigations that certain things are indeed not doubted.
>
> [§ 343] But it isn't that the situation is like this: We just can't investigate everything, and for that reason we are forced to rest content with assumption. If I want the door to turn, the hinges must stay put ...
>
> [§ 345] If I ask someone "what colour do you see at the moment ?" in order, that is, to learn what colour is there at the moment, I cannot at the same time question whether the person I ask understands English, whether he wants to take me in, whether my own memory is not leaving me in the lurch as to the names of colours, and so on. (OC)

Hinge propositions, H, are different from the Moorean common sense propositions as they are *not* meant to be *justified* from the beginning. Any possible epistemic investigation is *contingent* on them, and hence they cannot be epistemically justified. If you need to justify H, then you need to produce an evidence E that is more certain than H itself. For Wittgenstein this seems self-contradictory as H is the most basic belief regarding the epistemic issue in consideration.

[101] OC § 250.
[102] OC § 317.

> ...When one says that such and such a proposition can't be proved, of course that does not mean that it can't be derived from other propositions; any proposition can be derived from other ones. But they may be no more certain than it is itself. (OC § 1)

Consequently, any epistemic inquiry has to *presuppose* certain hinge proposition(s) before proceeding. Even philosophical skepticism has to assume an undoubtable *Archimedean point* before starting the game of doubt itself. Wittgenstein explains "if you tried to doubt everything you would not get as far as doubting anything. The game of doubting itself presupposes certainty[103]." Thus, for Wittgenstein, it will be a mistake to claim that we are *justified* in *knowing* H; rather we have to assume it *pretheoretically*.

There are multiple interpretations for the Wittgensteinian notion of hinge propositions[104]. But due to our limited scope, we are only going to focus on a particular view of hinge propositions that was advanced by Michael Williams[105]. What Wittgenstein is attempting in *OC*, according to Williams, is to provide a theoretical analysis of the *unrecognized presuppositions* of skeptical arguments. Particularly, Williams (2005) gives a general description of these hinges:

1. They are basic certainties, judgments, propositions which do not constitute part of our knowledge, but also cannot be doubted.
2. They are a kind of 'framework judgments' that pave the path for inquiring, asking, justifying, arguing, and so on.
3. They are characterized by heterogeneity[106]. For example, these hinges include: simple cognitive judgments (e.g. I have two hands), general world-claims (e.g. the earth existed more than five seconds ago), elementary mathematical claims (e.g. 12*12=144), and so on.
4. The certainty of hinges is not due to their *intrinsic* credibility, but rather due to their meaning in the *language games*. Hence by doubting them we would have to suspend making judgments.

[103] OC § 115.

[104] See for example, Putnam (1992) and Wright (1985, 1991, 2003). Also see Pritchard (2001) for a critique of these interpretations.

[105] See Williams (2001, 2004a, 2004b, 2005).

[106] This can be contrasted with the classical foundational account of knowledge where the class of basic beliefs is quite homogeneous.

5. Since these hinges are not part of our knowledge, then they are outside of the scope of justification. They are *non-epistemic* frameworks.

Recall that Williams endorsed an inferential version of contextualism. Accordingly, in each context there must be some hinge propositions (or "methodological necessities" in Williams' terms) upon which we can build our modes of inferences. In that sense, a hinge proposition cannot be evaluated from *outside* its context as in that way we will be mixing the inferential structures of different contexts.

> The skeptic takes himself to have discovered, under the condition of philosophical reflection, that knowledge of the world is impossible. But in fact, the most he has discovered is that knowledge of the world is impossible under the conditions of philosophical reflection. (Williams 1991, 130).

Hinge propositions can be thought of as *tacit* pretheoretical tools that lack any evidential support from without the epistemic context they are employed in. Consequently, it will be illegitimate for the skeptic to doubt the existence of the outside world in quotidian contexts. On a daily basis, a statement like "there is an outside world" is a hinge proposition for the vast majority of human activities. Similarly, in classical logic, the Law of Non-Contradiction (LNC) can be regarded as a hinge proposition, while this is not the case in *Dialetheism*[107]. In that vein, Williams offers a strong rebuttal for skepticism by arguing that skepticism confuses the hinge propositions of skeptical contexts (e.g. philosophical contexts) with those of non-skeptical contexts (e.g. quotidian contexts). Moreover, it is important to note that Williams – *contra* Moore – is not claiming a general rebuttal of skepticism, but rather emphasizing the *contextual nature* of skeptical hypotheses. In his words,

> I didn't say that I could claim to know that I am not a brain in a vat. In fact, I didn't even say that I could claim not to be a brain in a vat. All I said was that I do know all sorts of mundane facts; and that for as long as I know them, I know that various sceptical possibilities do not obtain. Claiming is another matter. (352)

[107] A logical system where the LNC does not hold, see for example Priest and Francesco (2013).

Nevertheless, Pritchard (2001) points a problem in Williams' account of hinge propositions.

> By focusing on the very feature of our epistemic practices that Williams highlights – that sometimes *claiming* knowledge can be improper even though we have it ... The problem with hinge propositions is thus not that they are sometimes unknown (in certain contexts), but that one can never coherently *claim* to know them. (13)

The *impossibility* of claiming knowledge of hinge propositions is a serious challenge to Williams' account of Wittgenstein[108]. It can be compared to the problem of *arbitrariness* in the foundationalist theory of epistemic justification[109]. Nevertheless, Pritchard (2011) thinks that hinge propositions have an upper hand over traditional foundational beliefs as the former are not arbitrary.

> In short, the suggestion is that the very possibility that one belief can count as a reason for or against another belief presupposes that there are some beliefs which play the role of being exempt from needing epistemic support, and thus that it is not arbitrary that one believes hinge propositions. (9)

Concerning our original problem of a priori skepticism, the notion of hinge propositions might work fine with a view on the Ap-BIV skeptical hypothesis. Alas, it seems difficult for us to conceive how it can solve the problem of formal language skepticism in a fully-fledged manner. More specifically, hinge propositions can be used against the syntactic – but not the semantic – version of the problem of deduction in a fashion similar to Putnam's centrality argument. Nevertheless, hinge propositions will fall short of solving the rule-following paradox and the semantic version of the problem of deduction. In the end, the roots of the latter problems are closely associated to the problem of *meaning-normativity*[110]. In other words,

[108] Pritchard (2001) attempts to solve this problem by developing a different interpretation of Wittgenstein.

[109] But note first that hinge propositions cannot be regarded as foundational epistemic beliefs (i.e. non inferential self-evident beliefs) as hinge propositions might not be self-evident or undeniable.

[110] The thesis about the normativity of meaning is mainly due to Kripke's interpretation of Wittgenstein:

if we can find a good explanation for the apparent normativity of meaning of formal languages, we can then solve the problem of formal language skepticism[111].

8.2 Contextual-Conventionalism

In this part we are going to compile conventionalism à la Carnap-Lewis[112] and inferential contextualism as two complementary theses. We start by defining our main notations: let the set of knowing subjects[113] be $S : s_1, s_2, \ldots, s_n$, and the set of known propositions for subject s_i be $P : p_1, p_2, \ldots, p_m$. Similarly, we can define the set of all possible times as $T : t_1, t_2 \ldots, t_l$, and the set of all possible worlds as $W : w_1, w_2, \ldots, w_i$. Also, the set of all possible contexts is denoted by $C = c_1, c_2, \ldots, c_v$; note that C denotes only the contexts which are *epistemically relevant* to the knowing subject. Finally, we define the set of all possible conventions by $Co = co_1, co_2, \ldots, co_y$. Now take the ordered n-tuple $K(s_n, p_m, w_i, t_l, c_v, co_y)$ to denote that subject s_n knows proposition p_m in world w_i at time t_l given a specified context c_v and a specified convention co_y. [114]. Given our framework, the last two parameters – c_v and co_y – are the most decisive variables in judging the epistemic status of a proposition p_m. By changing the context c_v, the convention formed co_y changes, and in accordance the epistemic status of p_m varies too. To see this, contrast the utterance of the following two statements:

- $K(s_1, p_1, w_1, t_1, c_1, co_1)$.

> Suppose that I do mean addition by '+'. What is the relation of this supposition to the question of how I will respond to the problem '68 + 57'? The dispositionalist gives a descriptive account of this relation: if '+' means addition, then I will answer '125'. But his is not the proper account of the relation, which is normative, not descriptive. The point is not that, if I meant addition by '+', I *will* answer '125', but that, if I intended to accord with my past meaning of '+', I *should* answer '125'. Computation error, finiteness of my capacity, and other disturbing factors may lead me not to be *disposed* to respond as I should, but if so, I have not acted in accordance with my intention. The relation of meaning and intention to future action is *normative*, not *descriptive*. (1982, 37)

[111] Recently, the notion of normativity of meaning became more influential. See for example Crispin Wright (1984), Simon Blackburn (1984), and John McDowell (1984).

[112] In the sense of the position defended in section 6.

[113] Knowing subjects are treated in first person, and not through third person attributions.

[114] Similar descriptions can be used for other epistemic notions like 'justification'. In that case, take the ordered n-tuple $J(s_n, p_m, w_i, t_l, c_v, co_y)$ to denote that subject s_n is justified to belief proposition p_m in world w_i at time t_l given a specified context c_v and a specified convention co_y.

- $\neg K(s_1, p_1, w_1, t_1, c_2, co_2)$.

Given w_1, t_1, c_1 and co_1, subject s_1 *knows* p_1. While given w_1, t_1 , c_2 and co_2, the same subject s_1 does *not* know the same proposition p_1. In our aforesaid defense of the conventionalist view of logical systems, we essentially argued that any constructed formal language is grounded in conventional natural language[115]. We also argued that since formal languages *follow* from conventional natural language(s), formal languages are conventional too. If this defense holds, we hopefully have a rebuttal of the problem of formal language skepticism.

8.3 Revisiting The Problem of Formal Language Skepticism

Notably in his 1950 seminal article *Empiricism, Semantics, and Ontology,* Carnap distinguished between two classes of ontological questions:

> ... it is above all necessary to recognize a fundamental distinction between two kinds of questions concerning the existence or reality of entities ... first, questions of the existence of certain entities of the new kind *within the framework*; we call them *internal questions*; and second, questions concerning the existence or reality *of the system of entities as a whole*, called *external questions*. Internal questions and possible answers to them are formulated with the help of the new forms of expressions. The answers may be found either by purely logical methods or by empirical methods, depending upon whether the framework is a logical or a factual one. (21-22)

Given a specific framework (e.g. a framework of propositions, a framework of real numbers, etc.), there are two types of – commonly confused – questions: *internal* and *external* questions. Internal questions are those questions asked concerning the ontological status of some entities *within* a framework. For instance, a number theorist can ask: are there infinite twin prime numbers? The answer to this question is analytically determined by the framework's rules.

[115] An interesting question here would be: how do different natural languages relate to different formal systems in the history of logic?

Similarly, a physicist can ask a question about the existence of the Higgs-Boson particle. In that case, the truth-value of the claim will be settled by the rules of the adopted framework – which is the standard-model in this case – with the assistance of some empirical confirmation. Differently, external questions are those asked concerning the ontological status of a framework as a *whole*. For example, a philosopher of mathematics can ask whether numbers exist at all; or an epistemologist can ask about the existence of physical objects. For Carnap, external questions *cannot* be answered. They are *pseudo-questions* that lack any cognitive content. Carnap thinks that all sorts of ontological debates arise by confusing these two questions. While it is legitimate to make internal claims since they can be assigned a truth-value from within the framework, it is illegitimate to make external claims as they cannot be assigned a truth-value at all. Then Carnap argues that the choice between frameworks can only be made based on *practical* considerations.

> The acceptance [of a new framework] cannot be judged as being either true or false because it is not an assertion. It can only be judged as being more or less expedient, fruitful, conducive to the aim for which the language is intended. Judgments of this kind supply the motivation for the decision of accepting or rejecting the kind of entities. (31-32)

He also writes: "Shall we introduce such and such forms into our language?" In this case it is not a theoretical but a practical question, a matter of decision rather than assertion[116]." Analogously, contextual-conventionalism can be divided into two general categories: *internal* and *external* conventions. Internal conventions are those *existing* conventions formed – whether syntactic or semantic – about a specific set of rules *within* a specific context[117]. For instance, a group of number theorists can form an epistemic convention about the plausibility of a set of axioms for natural numbers (e.g. Peano's axioms). So the convention is formed from *within* the context of number theory. On the other hand, external conventions are those conventions about the *totality* of a set of rules from *without* a specific context. An example can be the convention between

[116] P. 29

[117] These conventions are meant to be epistemological ones in our domain of discourse.

some mathematicians to accept the system of natural numbers as a system of representation. In that case, they are forming a convention about the whole context of natural numbers (viz. an external convention), and not about a specific element within the context of natural numbers (viz. an internal convention). Now let us go back our original problem of a priori skepticism.

Recall that the problem of deduction is twofold: (i) the premise-circularly problem, and (ii) the rule-circularity problem. In (i), we have an infinite regress of premises where each additional rule of inference is defined as a new premise, and hence a conclusion is never reached[118]. In (ii), Susan Haack proposed two possible definitions for deduction: semantic and syntactic definitions. Nonetheless, she argued that there is no coherent defense for either of these definitions. Also concerning the rule-following paradox, Kripke argued that when using ' + ' to mean 'plus' we cannot cite any fact about our past or current usage of the term 'plus' that guarantees that we mean ' + ', and not 'Θ.' We called this twofold dilemma the problem of *formal language skepticism* as it casts our concern over the epistemic validity of the whole deductive enterprise. All being well, contextual-conventionalism might offer a route out of our epistemic angst. Interestingly enough, the core insight of the conventionalist solution was mentioned at the beginning of the conversation between Achilles and the tortoise,

> -"So you've got to the end of our race-course?" said the Tortoise. "Even though it does consist of an infinite series of distances? I thought some wiseacre or other had proved that the thing couldn't be done?"
>
> -"It can be done," said Achilles. "It has been done! Solvitur ambulando. You see the distances were constantly diminishing; and so—" (278)

Here, Achilles seems to stumble upon an *easy* rebuttal of the tortoise's theoretical reasoning: *Solvitur ambulando* (i.e. the problem of finishing the race is *solved by walking*.). If both Achilles and the tortoise *agree* that the former succeeded in ending the race-course, then Achilles surpassed the challenge. Nevertheless, if the tortoise

[118] This is since "Whatever Logic is good enough to tell me is worth writing down" (280), as stated by the tortoise to Achilles.

has a good reason to be convinced that Achilles did not finish the race, then Achilles' solvitur-ambulando-strategy will not work. To see this, we have to think of the conversation from a contextual-conventionalistic perspective. Given the conversational context of Achilles and the tortoise, the disagreement stems from the absence of any convention between Achilles and the tortoise on what exactly it means to "finish the race". On one hand, the tortoise is seeking a *theory-based* solution, while Achilles accepts an *action-based* solution. We call this: a situation of *absence of external convention*. It is not difficult to draw a close analogy between the infinite-series race paradox discussed above and the problem of formal language skepticism. In the premise circularity case, the tortoise insists that every rule of reasoning should be stated explicitly as a premise in the argument which results in an infinite regress of premises. Achilles was very suspicious about this move, and even considered that: "such obtuseness would certainly be phenomenal[119]." Hence, the regress problem can be attributed to some absence of external convention between Achilles and the tortoise about which language they should use to communicate. Achilles is appealing to an intuitive – practical-based – conception of logical entailment, while the tortoise is appealing to a philosophical conception of logical entailment. Accordingly they do not only speak different languages, but they also have an implicit disagreement about which language they should use in conversing. A similar argument can be presented in Kripke's rule-following paradox. The core problem was that any normative action can be a subject of multiple, other, normative interpretations. In that way, there was a disagreement between the teacher and his pupil about: what guarantees that the student means 'addition,' and not 'quaddition,' by the '+' operation? Yet again, the quarrel can never settle down because of the absence of any external convention about the meaning of 'addition.' The paradox will only be solved when the pupil and the master agree on whether 'addition' means ' $+$,' or 'Θ' – or any other notion.

In this sense, formal language skepticism is a genuine *prior* disagreement about the choice of a conventional language in a specified context. This is very different from a more subtle disagreement about which rules or axioms we should adopt within any specific

[119] See p. 27

framework. An illustrative example will be the question of epistemic justification. Here we also have an external disagreement about the source(s) of our epistemic justification. A skeptic who discredits the idea of epistemic justification will never be able to communicate with a non-skeptic who accepts such epistemic justification. In that case, there is no common ground to stand on. Differently, there is an *internal* disagreement among the non-skeptics of epistemic justification as they accept the notion of epistemic justification but differ about the best conceivable method of it (viz. foundationalism, coherentism, infinitism).

In short, the way to remove formal language skepticism is by removing the absence of *external* convention(s). Whether this is a *feasible* goal or not is a different question that we are not going to tackle here.

8.4 Revisiting Ap-BIV

Now we turn to the second part of a priori skepticism, namely the Ap-BIV problem. Recall that the skeptical hypothesis was: let AP be any a priori proposition, and let the skeptical hypothesis S'_h be: 'I am Ap-BIV.'

- P_1. I am unable to know the denials of S'_h.
- P_2. If I am unable to know the denials of S'_h, then I do not know that Ap is true.
- C. Hence, I do not know that Ap is true.

We will counterargue the aforementioned skeptical hypothesis based on contextual-conventionalism as follows:

Claim 1. (*Conventional argument*) Given AP and S'_h , then:

- P_1^*. If I am unable to know the denials of S'_h, then I do not know that AP is true.
- P_2^*. I know that AP is true.
- C^*. Therefore, I know the denials of S'_h.

P_1^* shall be accepted by the skeptic since it is identical to P_2 in the original skeptical argument. To save the conventional argument, we need to defend P_2^*. Our defense is based on contextual-conventionalism. If a priori knowledge is formed by virtue of convention in a specific context as we argued before, then P_2^* holds trivially. After all, the convention about what defines a priori knowledge – in this context – is established by a *sole* subject, namely *myself*. Hence, C^*holds trivially as a *private* convention. Interestingly, our defense of P_2^* overcomes Pritchard's critique of Williams' interpretation of Wittgenstein's hinge propositions. This is because the conventional argument – if correct – establishes *my knowledge* that AP is true. Unfortunately, the conventional argument does not overcome the problem of arbitrariness of Wittgenstein's hinge propositions as it allows for – almost – anything to be a priori for the convention-holder (viz. me). Moreover, other problems start to arise if we propose a third-person formulation of the conventional argument.

Claim 2. (*Generalized conventional argument*) Let AP be any a priori proposition, let S be any subject *differen*t from myself, and let S'_h be a skeptical hypothesis of the form: "I (subject S) am Ap-BIV"

- $\hat{P}_1$. If S is unable to know the denials of S'_h, then she does not know that AP is true.
- $\hat{P}_2$. S knows that AP is true.
- $\hat{C}$. Therefore, S knows the denials of S'_h.

How can I (or any other subject) know that $\hat{P}_2$ is true? After all, I have no access to subject S's *private* conventions about AP. This is a very similar position to the problem of private experience discussed by Wittgenstein:

> The essential thing about private experience is really not that each person possesses his own exemplar, but that nobody knows whether other people also have this or something else. The assumption would thus be possible—though unverifiable—that one section of mankind had one sensation of red and another section another. (PI, § 272)

One possible way to know about S's private conventions about AP is through linguistic communication. S can just convey her private conventions to me (or any other subject), and if there is an agreement about these conventions, then we know that AP is true. Nevertheless, if it happens that there is a disagreement about these conventions, then $\hat{P}_2$ will not hold and the generalized conventional argument will fail. In that case, the Ap-BIV dilemma persists because of the absence of external convention(s) among subjects about what constitutes a priori knowledge.

In the end, our contextual-conventional strategy to counter skepticism will only succeed if the skeptical subjects managed to form a convention – within a specific context – about what defines a priori knowledge. Alas, this sufficient condition is difficult to satisfy as philosophers have implicitly agreed to disagree. After all, disagreement is a central defining feature of the history of philosophy and is not expected to disappear anytime soon. This persistence of philosophical disagreement is better described in the words of Peter van Inwagen:

> Disagreement in philosophy is pervasive and irresoluble. There is almost no thesis in philosophy about which philosophers agree. If there is any philosophical thesis that all or most philosophers affirm, it is a negative thesis: that formalism is not the right philosophy of mathematics, for example, or that knowledge is not (simply) justified, true belief. (2004, 332)

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