Multiaxial fatigue in drill pipes under non-proportional loading

Nahla Helmy

Follow this and additional works at: https://fount.aucegypt.edu/etds

Recommended Citation

APA Citation

MLA Citation

This Master's Thesis is brought to you for free and open access by the Student Research at AUC Knowledge Fountain. It has been accepted for inclusion in Theses and Dissertations by an authorized administrator of AUC Knowledge Fountain. For more information, please contact thesisadmin@aucegypt.edu.
Multiaxial Fatigue of Drill Pipes under Non-Proportional Loading

by

Nahla Abbas Helmy

Thesis submitted in partial fulfillment of the requirements of the degree of Master of Science in Mechanical Design

August, 2016
ACKNOWLEDGMENTS

I would like to express my gratefulness to all the people who helped me during this endeavor. I would like thank my advisor, Dr. Maher Younan for his guidance, mentoring, patience and support. In addition, I would like to acknowledge my parents who taught me to read and learn, my brother for his help, my friends for support and my dear husband who always believed in me.
ABSTRACT

The scope of this research is to estimate the fatigue life of drill pipes subjected to multiaxial state of stress. Several approaches that model and estimate fatigue life under non-proportional multiaxial loading were investigated. The critical plane approach was chosen with Fatemi and Socie numerical model. A software program (Elrond) was constructed to estimate the fatigue life for non-proportional loading. The code was validated using previous researchers experimental fatigue data sets. The developed software was then applied to a real life drilling problem. The “Build and Hold” drilling job condition was analyzed using the developed software. Results were compared with the API RP7G fatigue design guidelines. Fatigue life charts were prepared using fatigue lives estimated by Elrond at several axial loading, dogleg severities and axial torque amplitudes. Also cumulative fatigue life charts using fatigue life values from Elrond were calculated according to the method of Hansford and Lubinski developed, and used in API RP7G. Elrond results were less conservative than the API results, especially for low values of axial torque.
# TABLE OF CONTENTS

## Chapter 1 INTRODUCTION ................................................................. 1

1.1 History of Oil Industry ................................................................. 1

1.2 Drill Pipes Functions .................................................................. 3

1.3 Fatigue in Drill Pipes ................................................................... 4

1.4 The API Fatigue Criterion ........................................................... 5

## Chapter 2 Literature Review ............................................................. 6

2.1 Fatigue Analysis of the Drill String .............................................. 6

2.2 Multiaxial Fatigue Research Development ................................... 10

2.2.1 Empirical Models .................................................................. 11

2.2.2 Equivalent Stress/Strain Models ............................................. 13

2.2.3 Stress Invariants models ......................................................... 18

2.2.4 Energy Based Models ............................................................. 20

2.2.5 Critical Plane Approaches ....................................................... 21

2.3 Scope of Work ............................................................................ 27

## Chapter 3 Methodology ................................................................. 29

3.1 Building the model ....................................................................... 37

*Elrond*, Verification Version .......................................................... 37

3.2 Validating the Model .................................................................... 44

3.2.1 1045 HR Steel Data Set .......................................................... 45

-vi-
3.2.2 Inconel 718 Data Set ................................................................. 47

3.3 Elrond, Drilling Version .............................................................. 49

3.3.1 Bending Moment and Wellbore curvature .................................. 49

3.3.2 Subroutine Stresses ................................................................. 50

3.3.3 Material Selection and Mechanical Properties ............................... 53

3.3.4 Cumulative Fatigue Assessment ................................................. 56

Chapter 4 Results ............................................................................. 58

4.1 Validation Case1, Steel 1045HR ..................................................... 58

4.1.1 Fatigue Life Estimation ............................................................... 58

4.1.2 Critical Plane Direction .............................................................. 63

4.2 Validation Case2, Inconel 718 ......................................................... 69

4.2.1 Critical Plane Direction .............................................................. 71

4.3 Downhole Drilling Conditions ......................................................... 72

4.3.1 Fatigue Life Estimation ............................................................... 72

4.3.2 Critical Plane Direction .............................................................. 84

4.4 Cumulative Fatigue Assessment ..................................................... 86

Chapter 5 CONCLUSION AND FUTURE DIRECTIONS ..................... 91

5.1 Conclusions .................................................................................. 91

5.2 Recommendations ......................................................................... 92

References ......................................................................................... 94

Appendix A ......................................................................................... 98
LIST OF FIGURES

Figure 1.1 True Vertical Depth and Measured Depth in an Inclined Wellbore............2
Figure 1.2 Drill Pipe and Tool Joints Connection [4]...............................................3
Figure 2.1 Pipe segment into curved section and under several forces[10]...............6
Figure 2.2 Crack Types According to Brown and Miller......................................23
Figure 2.3 Crack Opening Mechanism[43].............................................................25
Figure 3.1 Example of Downhole Torque and Bending Loads and their Non-Proportionality ..........................................................30
Figure 3.2 Deflected DP under Loading in a Curved Wellbore Section ............32
Figure 3.3 (a) DP Segment under loading, (b) Non-Proportional Drilling Stresses ....33
Figure 3.4 Proportional Loading and Corresponding Mohr Circle.........................34
Figure 3.5 Non-Proportional Loading and Corresponding Mohr Circle ............35
Figure 3.6 Elrond Program Main Window ............................................................39
Figure 3.7 CriticalPlanCalc Subroutine ...............................................................41
Figure 3.8 Numerical Solver Subroutine ..............................................................44
Figure 3.9 Elrond Main Window, Default Version .................................................52
Figure 3.10 Subroutine Stresses .........................................................................53
Figure 4.1 Observed vs. Predicted Life, Fatemi-Socie Results [43]......................59
Figure 4.2 Observed Data [43] vs. predicted life, Elrond results .........................60
Figure 4.3 Comparison Between Predicted Life from Elrond and Predicted Life from Fatemi-Socie’s Work, Both vs. Experimental Results – $\lambda=0.5$ ......................62
Figure 4.4 Comparison Between Predicted Life from Elrond and Predicted Life from Fatemi-Socie’s Work, Both vs. Experimental Results – $\lambda=1$ ..............................................63

Figure 4.5 Estimated Critical Plane Directions, Proportional Loading ..................65

Figure 4.6 Estimated Critical Plane Directions, Non-Proportional loading ..............66

Figure 4.7 Direction of Maximum Shear and Normal Stresses and Critical Plane Direction, $\lambda=0.5$ ...............................................................................................................67

Figure 4.8 Direction of Maximum Shear and Normal Stresses and Critical Plane Direction, $\lambda=1$ ...............................................................................................................68

Figure 4.9 Direction of Maximum Shear and Normal Stresses and Critical Plane Direction, $\lambda=0.5$ ...............................................................................................................69

Figure 4.10 Fatigue Lives from Elrond vs. Fatigue Lives from Socie et al. Experimental Work [44] .................................................................................................................71

Figure 4.11 Predicted Critical Plane Directions .......................................................72

Figure 4.12 Normal strains according to Coffin-Manson Relation .........................73

Figure 4.13 Estimated Fatigue Life for Different Well Curvatures and Axial Forces, $T_x=10-20$ kib-ft (13,558-27,116 N-m) ..............................................................................75

Figure 4.14 Estimated Fatigue Life for Different Well Curvatures and Axial Forces, $T_x=15-30$ kib-ft (20,337-40,675 N-m) ..............................................................................76

Figure 4.15 Estimated Fatigue Life for Different Well Curvatures and Axial Forces, $T_x=10-30$ kib-ft (13,558-40,675 N-m) ..............................................................................77

Figure 4.16 Fatigue failure Surface at Different Axial Torque Amplitudes ............78

Figure 4.17 Fatigue Lives vs. Dogleg Severities at Three Different Torque Amplitudes .........................................................................................................................79

Figure 4.18 Fatigue Life, Hours to failure, $T_x=10-20$ kib-ft (13,558-27,116 N-m) ...81
Figure 4.19 Fatigue Life, Hours to failure, Tx=15-30 klb-ft (20,337-40,675 N-m)....82

Figure 4.20 Fatigue Life, Hours to failure, .................................................................83

Figure 4.21 Fatigue Life under Axial Force and Torque Load Only.........................84

Figure 4.22 Critical Plane Direction, Tx= 10-20 klb-ft (13,558-27,116 N-m).........85

Figure 4.23 Critical Plane Direction, Tx= 15-30 klb-ft (20,337-40,675 N-m).........86

Figure 4.24 Cumulative Fatigue Life, Tx= 10-20 klb-ft (13,558-27,116 N-m).......88

Figure 4.25 Cumulative Fatigue Life, Tx= 15-30 klb-ft (20,337-40,675 N-m).......89

Figure 4.26 Cumulative Fatigue Life, Tx= 10-30 klb-ft (13,558-40,675 N-m).......90
LIST OF TABLES

Table 3-1 Mechanical monotonic and cyclic properties of the 1045 HR steel [42].... 45
Table 3-2 monotonic and cyclic properties of Inconel 718 [43].......................... 48
Table 3-3 Mechanical Monotonic and Cyclic Properties Used in the Drilling Case Study
.................................................................................................................................................. 54
NOMENCLATURE

$A$ ........ cross section area

$E$ ........ modulus of elasticity

$F$ ........ axial force

$G$ ........ shear modulus

$I$ ........ area moment of inertia

$K'$ ........ cyclic strength coefficient

$K'_o$ ........ cyclic shear strength coefficient

$L$ ........ half length of the pipe body

$M$ ........ bending moment

$M_b$ ........ Lubinski’s bending moment

$N_f$ ........ fatigue cycles to failure

$T$ ........ torque

$b$ ........ fatigue strength exponent

$b_0$ ........ shear fatigue strength exponent

$c$ ........ fatigue ductility exponent

$c_0$ ........ shear fatigue ductility exponent

$c_p$ ........ pipe body curvature

$c_w$ ........ wellbore curvature

$k$ ........ Fatemi-Socie material constant

$n'$ ........ cyclic strain hardening exponent

$n'_o$ ........ cyclic shear strain hardening exponent

$x$, $y$, $z$ .... reference axes, $x$ is along pipe longitudinal axis
$\Delta \varepsilon_e$ ....... elastic strain range

$\Delta \varepsilon_p$ ....... plastic strain range

$\Delta \gamma_e$ ....... shear elastic strain range

$\Delta \gamma_p$ ....... shear plastic strain range

$\theta$ ........... angle of rotation

$\varepsilon$ ........... normal strain

$\varepsilon'_f$ ........... fatigue ductility coefficient

$\varepsilon_{max}$ ..... maximum normal strain amplitude

$\gamma$ ........... shear strain

$\gamma_a$ ........... shear strain amplitude

$\gamma'_f$ ........... shear fatigue ductility coefficient

$\gamma_{max}$ ..... maximum shear strain amplitude

$\gamma_{\theta}$ ........... shear strain on plane $\theta$

$\lambda$ ........... biaxial strain ratio

$\nu$ ........... Poisson’s ratio

$\sigma$ ........... normal stress

$\sigma'_f$ ........... fatigue strength coefficient

$\sigma_{\theta}$ ........... normal stress on plane $\theta$

$\sigma_{n,\text{max}}$ ... maximum normal stress amplitude

$\sigma_y$ ........... yield strength

$\sigma_u$ ........... ultimate strength

$\tau$ ........... shear stress

$\tau'_f$ ........... shear fatigue strength coefficient

$\omega$ ........... angular frequency
Chapter 1

INTRODUCTION

1.1 History of Oil Industry

Although petroleum is a major factor in modern civilization and technological advancement in the twentieth century; drilling for oil and oil wells is not recent. Evidences of prehistoric drilling for hydrocarbons were found in China and Japan A.D. 347 and A.D. 600 respectively [1]. Also, several wells were found drilled using simple hand tools in Europe and North America around 1498, and in Baku in 1848 [1]. The first modern wells drilled using the Cable Tool technology were in the United States in the 19th century. Cable tool rigs hammer through soil and rocks using a preliminary iron bit attached to a simple form of drill string. Although rotary drilling was known in ancient Egypt as early as 3000 B.C. [1], percussion drilling was dominant in early oil drilling due to its low cost and simple technology. In 1930’s, rotary drilling started to spread in North America [1], and with that the use and development of drill pipes started to spread.

With the large demand on oil and gas, and depletion of the shallow reservoirs, industry had to go deeper and into more challenging resources, and that required for the drilling technology to rapidly advance to respond to these demands. In 1950, the average depth for oil wells in the United States was less than 4700 ft. (1433m) [2]. In 1990 that depth increased to 6000 ft. (1829m), and by 2000 it reached 7000 ft. (2134m) [2]. The increase was not only on the depth of drilling, but was also in the types of challenges faced in drilled wells. In 1990 only less than 10% of wells drilled in USA were horizontal wells,
by 2011 50% of drilled wells were horizontals [2]. In 1975, a well with measured depth (MD) to true vertical depth (TVD) ratio of 2 was classified as Extended Reach Drilling (ERD) [2]. Measured depth is the actual length of the wellbore path; true vertical depth is the vertical projection of the wellbore path. In vertical wells TVD and MD are the same, while in highly inclined or horizontal wells there is a significant difference between the two. Figure 1.1 presents the difference between MD and TVD in an inclined well. Recently, Maersk set a new world record in drilling El-Shahin well -Qatar- with MD/TVD of 11.13 [3]. This high pace development in drilling technology required for better designed, and highly enduring drilling tools, including the drill pipes.

Figure 1.1 True Vertical Depth and Measured Depth in an Inclined Wellbore
1.2 Drill Pipes Functions

Drill pipes (DPs) are main components in most of drilling operations. They consist of a steel tube connected to tool joints at both ends using friction welding as presented in Figure 1.1.

![Drill Pipe and Tool Joints Connection](image)

**Figure 1.2 Drill Pipe and Tool Joints Connection [4]**

The drill pipes functions include:

- Pumping drilling fluids downhole to the bit
- Transferring torque and rotation to the bit and BHA (Bottom Hole Assembly)
- Lowering and raising the bit and BHA
For standardization purposes; the American Petroleum Institute (API) in its Report for drill pipes specification [5] classified DPs according to the following four characteristics:

- Outside diameter (e.g., 2 3/8 in, 5 in, 6 5/8 in) and wall thickness
- Nominal unit weight (e.g., 6.65, 19.5, 34.2 lb./ft.)
- Steel grade, which determines the strength. DP can have one of five grades (E, X, G, S, Z and V) with E is the lowest and V is the highest grade
- Class, which determine the condition of the DP based on its remaining wall thickness. Class New has never used before, class Premium is used but has more than 80% remaining wall thickness, class II has more than 70% remaining wall thickness and class III has less than 70%

1.3 Fatigue in Drill Pipes

During operation, drill string components, including drill pipes are subjected to different multiaxial loading conditions for extended periods of time, and this can lead to failure of drill string components. Failure in petroleum industry is a very expensive event, and can lead to catastrophic incidents. In addition to the cost of the failed component itself, sudden failure of the DP during drilling results in several additional costs, including:

- NPT (nonproductive time) cost due to extra hours or days of drilling stoppage,
- Fishing cost of the tools below the failed pipe,
- Sometimes it is difficult to retrieve(fish) the lost tools and they are considered “lost in hole”,
- “Lost in hole” tools lead to cementing the original wellbore and drilling a side track well, usually with higher cost.
According to Josteen [6], 14% of drill rigs had failures, and each single downtime event costs around $106,000 in average. Hill [7], as reported by Vaisberg [8], after collecting drilling data between 2001 and 2003 stated that 68% of drill string failures were due to fatigue. Same results were concluded by Pitts [9], also reported by Vaisberg [8], where fatigue cracks were responsible for 73% of defective drill pipes.

1.4 The API Fatigue Criterion

The API have recommendations and criterion concerning loading and wellbore curvature to avoid drill string fatigue failure. This criterion is based on Lubinski’s [10] work using uniaxial fatigue failure rules. Although Lubinski’s work was published in the 1960’s, his method remains the basis for the majority of research made currently in drill string fatigue. Lubinski’s charts are the ones adopted by API for fatigue failure criteria [11]. While Lubinski’s work is fundamental, the drilling industry and technology made several advances since then. Types like 3-D wells, highly deviated and ERD (extended reach wells) are common now. While the API fatigue charts have a maximum Dog-Leg Severity (DLS) of 10 deg./100 ft, Bryan, Cox, Blackwell, Slayden, & Naganathan [12] stated that more than 50% of horizontal wells drilled in 2009 had dogleg severity between 10-15 deg./100ft. These new technologies come with higher loads and friction, and that makes uniaxial fatigue analysis inadequate for present drilling operations.
Chapter 2

Literature Review

2.1 Fatigue Analysis of the Drill String

Probably the most important contribution in the study of drill string fatigue was that performed by Lubinski [10]. He studied the effect of wellbore curvature (dogleg severity) on drill pipes and drill collars, relating wellbore curvature to fatigue life and axial force as illustrated in Figure 2.1.

He used a modified Goodman diagram to estimate bending stress limits under certain axial loading conditions. Lubinski lowered the endurance limit in Goodman diagram and assumed a cutoff on the mean stress at 67,000 psi (462 kPa) to account for slip marks and wear on the pipe body. Then by analyzing pipe curvature and bending...
moment; he produced his famous plots setting safe/un-safe fatigue zones for drill pipes and collars. His equation –Equation 2-1- for calculating bending moment at any point along the pipe was.

\[ M = EIc_p + S_0X + F_sY - \int_0^X (Q\sin\beta) XdX \] (2-1)

Where:

- \( E \) denotes the modulus of elasticity,
- \( I \) denotes the pipe cross-section moment of inertia,
- \( c_p \) denotes the pipe curvature,
- \( S_0 \) denotes the shear force acting on pipe body,
- \( F_s \) denotes the tension through the length of the pipe segment,
- \( Q \sin\beta \) denotes the lateral load per unit length,
- \( c_w \) denotes wellbore curvature,
- \( \alpha \) denotes the angle by which the hole turns over the length \( L \), \( \alpha = L \times c_w \)

The pipe curvature can be calculated from the wellbore curvature \( c_w \) (dogleg severity) as in Equation 2-2.

\[ c_w = c_p \frac{\tanh K_1L}{K_1L} \] (2-2)

Where:
In the model above, Lubinski related the bending moment in the DP body to the wellbore curvature, tension on both ends of the DP and gravity forces, neglecting the effect of the drilling torque on the pipe. Also, he considered no-contact or point-contact between the deflected pipe and the wellbore. Which may not be the case if the pipe is subjected to high deflection.

The API, in its related Standard RP7G [11], and based on Lubinski’s work [10], formulated the relation between bending stress, wellbore curvature and tension in drill pipes as in Equation 2-4.

\[
\sigma_b = \frac{432,000 \delta_b \tanh K_t L}{\pi E D K_t L} \tag{2-4}
\]

Where:

\(c_w\) denotes wellbore curvature (dogleg severity), degrees per 100 feet,

\(\sigma_b\) denotes bending stress, psi

\(E\) denotes modulus of elasticity, psi

\(D\) denotes drill pipe OD, Inches,

\(L\) denotes half the length of DP, inches,
\( F_x \) denotes the tension through the length of the pipe segment, pounds.

Equation 2-4 is valid only for range 2 drill pipes, i.e. the ones with the length of 30ft (9.14 m), so \( L \) equals 15ft (4.57 m). Then bending moment can be easily calculated using Equation 2-5.

\[
M_y = EIc_w
\]  

Grondin and Kulak [13], discussed, in the light of their experimental work, the API fatigue criteria and Lubinski’s fatigue analysis. They stated that Lubinski’s fatigue curve considered lower bound of test results performed by Bachman [14], and that, in addition to the pipe-wellbore contact model, resulted in overestimating of the bending stress in Lubinski’s fatigue model.

Other researchers like Rollins [15] worked to investigate the most vulnerable part of the drill pipe body. His work resulted that this part is nearly 4 feet away from the tool joint. Also that a highly corrosive environment will result in the absence of endurance limit in steel.

Wilson & Shepard [16] found that a short Minimum Internal Upset (MIU) in the drill pipe resulted in rapid fatigue damage and poor quality of internal plastic coating. Miller [17] investigated the importance of placing HWDP (heavy weight drill pipes) between drill pipes and drill collars. He stated that adding HWDP facilitates the stiffness transition between low stiffness DPs and highly stiff drill collars; which reduces fatigue failure incidents in drill pipes.
Grondin and Kulak [13] investigated the work done by Lubinski [10] and the current API fatigue limitations [11]. They concluded that Lubinski’s work over estimated the bending stress of the curved pipe, due to the fact that it ignored the nonlinear contact between the pipe and the wellbore. This lead to very conservative fatigue life estimation in his work and in the API standards.

Other researchers like Dale [18], investigated fatigue life and crack propagation using fracture mechanics methods. He used Paris’s law to relate fatigue life to stress intensity factor and bending load.

All the above researches, despite their importance, did not consider the load multiaxility or phase differences. With the high pace of advancement in drilling technology, more challenging drilling jobs were introduced with more challenging loading conditions, which calls for more detailed investigation of fatigue phenomena in drill string components.

### 2.2 Multiaxial Fatigue Research Development

The research in fatigue field started in the 19th century. In 1840, European researchers noticed that railway shoulders fail in a way different than the usual rupture associated with monotonic loading [19]. In the 1840s and 1850s, the term “fatigue” was coined to describe failures accompanying repeated loads. In 1860s, the German researcher August Wohler performed the first systematic fatigue tests under repeated stresses; thus he was called the father of fatigue testing [19].
Multiaxial fatigue problems have proven to be more sophisticated than uniaxial fatigue for a number of reasons. First, the presence of two or more stresses with different phase angles and frequencies make fatigue life estimation more difficult than the simple approach for the uniaxial case. Second, multiaxial fatigue testing is complicated and expensive, especially in case of strain controlled tests, variable amplitude loading or rotating equipment such as rotating shafts or drilling tools. Parts subjected to non-proportional multiaxial cyclic loads in the low-cycle fatigue range, are usually subjected to change in their traditional cyclic stress-strain behavior like additional or extra hardening and mean stress relaxation. These challenges made it difficult for a universal multiaxial fatigue model to be established, and research in that area has taken many directions. Multiaxial fatigue models can be classified into five main categories.

- Empirical Models
- Equivalent Stress/Strain Models
- Stress Invariants Models
- Energy Based Models
- Critical Plane Approach

Models for each of the above categories are briefly presented below showing the advantages and disadvantages of each.

**2.2.1 Empirical Models**

Empirical approaches were the first attempts to tackle multiaxial fatigue problem. In 1950, Gough [20] conducted investigations on biaxial in-phase fatigue using in-phase
reversed bending-torsion loading system. The loading ratio \( \left( \frac{f}{q} \right) \) values were 0, 0.5, 1.5, 3.5 and \( \infty \). He used both, solid and hollow specimen, with several steel grades and heat treatments. He related the multiaxial failure to an elliptical surface using two empirical relations for ductile and brittle materials, Equations 2-6 and 2-7 respectively

\[
\begin{align*}
(q/t)^2 + \left( \frac{b}{t} - 1 \right) \left( \frac{f}{b} \right)^2 + \left( 2 - \frac{b}{t} \right) \left( \frac{f}{b} \right) &= 1 \\
(q/t)^2 + \left( \frac{f}{b} \right)^2 &= 1
\end{align*}
\]  

where:

\( f \) = applied bending stress

\( q \) = applied torsion stress

\( b \) = endurance limit in tension

\( t \) = endurance limit in torsion

In 1953, Findley [21] carried several tests on 76S-T61 aluminum alloy using a test machine designed specially to allow for applying bending, torsion or a combination of bending and torsion with different values of mean stress. Test specimens were solid cylinders machined from circular bars, then their surfaces were polished to minimize imperfections. Although his work was limited to proportional loading only; it led to introduction of many important concepts. He suggested material anisotropy as a clarification to the variation between endurance limit in tension and torsion, and recommended a correction factor to account for it.
Findley [18] defined the “state of stress vector” as the vector sum of the three principal stresses at a point \((\sigma_1>\sigma_2>\sigma_3)\). The magnitude of this vector was suggested as an empirical criterion for the multiaxial stress fatigue as presented in Equation 2-8.

\[
S = \sqrt{\sigma^2 + 2\tau^2} \leq \frac{b}{t} \tag{2-8}
\]

Where

- \(S\) is the state of stress vector
- \(\sigma\) is the applied bending stress
- \(\tau\) is the applied shear stress
- \(b\) is the fatigue strength in bending
- \(t\) is the fatigue strength in torsion

The main problem with this criterion lies in its inability to differentiate between static and alternating stresses. That led to predicting failure under hydrostatic loading only [22] which is unacceptable. The main drawback of the proposed empirical criteria is that their accuracy and applicability are mainly related to how close the applications is to the stress/strain system it was constructed from and its data quality.

2.2.2 Equivalent Stress/Strain Models

The main scheme of these models is reducing the multiaxial state of stress to an equivalent uniaxial state, then use a uniaxial fatigue model to calculate fatigue life. Due to their simplicity and conservatism; these models are popular in multiaxial fatigue
applications [19]. The most frequently used approaches in this category are: the maximum principal stress theory, the maximum shear stress theory (Tresca theory) and the octahedral shear stress theory (Von Mises theory). The equivalent stress - as reported in Stephens, Fatemi, Stephens, & Fuchs [19]- is calculated according to each of the mentioned approaches as in Equations 2-9, 2-10 and 2-11 respectively.

\[
S_{qa} = S_{a1} \quad \text{(2-9)}
\]

\[
S_{qa} = S_{a1} - S_{a3} \quad \text{(2-10)}
\]

\[
S_{qa} = \frac{1}{\sqrt{2}} \sqrt{(S_{a1} - S_{a2})^2 + (S_{a2} - S_{a3})^2 + (S_{a3} - S_{a1})^2} \quad \text{(2-11)}
\]

Where \(S_{qa}\) is the equivalent nominal stress amplitude and \(S_{a1}, S_{a2}\) and \(S_{a3}\) are the principal alternating nominal stresses. While both Tresca and von-Mises theories are relatively suitable for ductile materials, maximum principal stress theory gives better results with brittle ones.

Fatigue, either uniaxial or multiaxial can be divided into two main types according to fatigue life. High Cycle Fatigue (HCF) and Low Cycle Fatigue (LCF). High cycle fatigue is usually related to applications where all loads remain in the elastic domain, where low cycle fatigue failure theories are applied where significant plasticity is expected and should be accounted for through the analysis [19]. The above stress based analyses are usually related to HCF applications. In LCF applications, the analogues strain versions of those theories are formulated as in Equation 2-12, 2-13 and 2-14 as reported in Stephens, Fatemi, Stephens, & Fuchs [19].

\[
\varepsilon_{eq} = \varepsilon_{a1} \quad \text{(2-12)}
\]
\[
\varepsilon_{eq} = \frac{\varepsilon_{a1} - \varepsilon_{a3}}{1 + \nu}
\]

(2-13)

\[
\varepsilon_{eq} = \sqrt{\frac{(\varepsilon_{a1} - \varepsilon_{a3})^2 + (\varepsilon_{a2} - \varepsilon_{a3})^2 + (\varepsilon_{a3} - \varepsilon_{a1})^2}{2(1 + \nu)}}
\]

(2-14)

Where \( \varepsilon_{qa} \) is the equivalent nominal strain amplitude and \( \varepsilon_{a1}, \varepsilon_{a2} \) and \( \varepsilon_{a3} \) are the principal alternating strains.

Despite their simplicity and convenience of use, these methods- either stress or strain based- lack the capability to predict fatigue life correctly under the case of principal axes rotation (non-proportional loading), and the ability to account for mean stress effect and the prediction of crack direction.

In 1979, Dietmann and Lempp [23], as reported Garud [22], conducted a series of HCF tests under conditions of on-proportional, tension-torsion loading, in an attempt to investigate the effect of phase angle on fatigue life. They concluded that the effect is maximized when not only the phase angle equals 90°, but also in the case of tension to torsion load ratio of 0.5. They also claimed that the effect is related to the tension-torsion endurance limits ratio, which is connected to the material ductility. Thus the non-proportionality has a more damaging effect on harder than softer metals. These conclusions make the classical yield theories like Tresca or Von Mises less conservative when applied to non-proportional multiaxial fatigue situations [22].

Sines [24] observed that although theories of maximum principal stress and the Von Mises gave reasonably good correlation with some brittle and ductile materials respectively, the fatigue failure is not caused by normal or shear stresses individually. After
a series of experiments with biaxial alternating stresses and combinations of static and alternating stresses, Sines [24] noticed that a crack may have started from alternating shear stress at some inclusion in the material. As the failure starts by the alternating shear stresses; it seems convenient that the stress normal to the planes of greatest alternating shear has a major effect on fatigue life. Thus, Sines [24] proposed the following Equation 2-15.

$$\frac{1}{2}\sqrt{(\sigma_{a1} - \sigma_{a3})^2 + (\sigma_{a2} - \sigma_{a3})^2 + (\sigma_{a3} - \sigma_{a1})^2 - \alpha \sigma_h} = A$$  \hspace{1cm} (2-15)

Where:

- $\alpha$ is a material constant
- $\sigma_h$ is the hydrostatic (mean) stress
- $A$ is a material constant proportional to uniaxial fatigue strength

Sines [24] showed that static tension was more damaging than static torsion. Sines results showed that crack length is mitigated by normal compressive stress. Sines method shows good correlation with experimental fatigue life in the case of proportional loading. However, it cannot be applied to cases of non-proportional loading as it assumes fixed positions of the principal axes [19].

Davis and Connelly [25] indicated that the multiaxial stress state has an effect on material ductility. They showed that ductility decreases due to triaxial tension, and
proposed a tri-axiality factor (TF) to relate the multiaxial effective ductility to the uniaxial tensile ductility (elongation), Equation 2-16 presents the relation.

\[
TF = \frac{1}{\sqrt{2}} \left[ (\sigma_1^2 + \sigma_2^2 + \sigma_3^2)^{1/2} / \sigma_{eq} \right] = \frac{I_1}{\sigma_{eq}}
\]

(2-16)

Note that TF = 2 for biaxial loading, TF = 1 for uniaxial case, and TF = 0 for pure torsion.

In 1972, Zamrik [26] proposed a modification to Von Mises equivalent strain approach by suggesting the total strain criterion \( \epsilon_{TOT} \) as in Equation 2-17.

\[
\epsilon_{TOT} = (2/3)^{1/2} \left( \epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2 \right)_{max}^{1/2}
\]

(2-17)

Where \( \epsilon_1, \epsilon_2, \epsilon_3 \) are the three principal strains. Although this approach was valid for both in-phase and out of phase loading, it had the limitation of requiring that mean components of strain to be zero. In 1993, Zamrik and Mirdamadi [27] proposed the “Z-Parameter”, which is defined by fatigue properties under uniaxial and pure torsion tests, and expressed as in Equation 2-14.

\[
Z = \frac{3}{2(1+\nu^e)} \frac{\tau_f'/G}{\sigma_f'/E} \frac{\epsilon_f'}{\gamma_f'}
\]

(2-18)

They suggested extending the well-known Coffin-Manson uniaxial fatigue life relation to in-phase multiaxial case by relating it to the von-Mises equivalent strain life.
criteria after adjustment by means of TF factor together with their proposed Z-Parameter as in Equation 2-19.

\[
(Z^{TF-1}) \frac{1}{2} \Delta \varepsilon_{\text{eq}} + (A^{TF-1}) \frac{1}{2} \Delta \varepsilon_{\text{eq}}^p = \frac{\sigma'(2N_f)^b}{E} + \varepsilon'(2N_f)^c
\]  

(2-19)

Where \( \Delta \varepsilon_{\text{eq}}, \Delta \varepsilon_{\text{eq}}^p \) are von-Mises equivalent elastic and plastic strain ranges respectively, and \( \Lambda \) is a ductility parameter determined experimentally. But again this model was valid only for proportional loading with no identification to crack direction or initiation plane.

2.2.3 Stress Invariants models

These models relate the fatigue life to the second invariant of the deviatoric stress tensor \((I_2)\). According to these models, fatigue failure will occur on planes of maximum octahedral shear stress, and they define a damage parameter that consists from combination of the second stress invariant and the hydrostatic stress [22].

In 1959, Sines [28] proposed a multiaxial fatigue model based on the second stress invariant. This model, till today, is considered the most popular high cycle multiaxial fatigue criterion [29]. Sines model was formulated in Equation 2-20.

\[
\sqrt{J_{2,a}} + k\sigma_{H,M} \leq \lambda
\]  

(2-20)

Where
$J_{2,a}$ is the second stress invariant

$\sigma_{H,M}$ is the mean value of the hydrostatic stress

$k$ and $\lambda$ can be determined from fully reversed torsion and pulsating bending tests as in Equations 2-21 and 2-22.

\[
\begin{align*}
    k &= \left(\frac{3t_{-1}}{f_0}\right) - \sqrt{3} \\
    \lambda &= t_{-1}
\end{align*}
\]  

(2-21)  

(2-22)

Where,

$t_{-1}$ is the fatigue limit for fully reversed torsion test

$f_0$ is fatigue limit for pulsating bending test

According to Papadopoulos et al. [29], application of this model to reversed bending leads to:

\[
\sigma_{H,M} = 0 \quad \text{and} \quad \sqrt{J_{2,a}} = \frac{f_{-1}}{\sqrt{3}}
\]

Which leads to:

\[
t_{-1}/f_{-1} = 1/\sqrt{3}
\]  

(2-23)
Equation 2-23 means that fatigue limits in torsion and fully reversed bending have a constant ratio for all metals. Experimental results show that this ratio ranges from 0.5 for mild metals to 1 for brittle ones [29].

While Sines used the mean value of the hydrostatic stress, Crossland [30] according to Papuga [31], suggested using the maximum value $\sigma_{H,\text{max}}$. This change gave better results compared to Sines when matching experimental results, although both were non-conservative when comes to non-proportional loading.

### 2.2.4 Energy Based Models

These models relate fatigue damage in the material to an energy component, usually plastic work per cycle. Garud [22] integrated the product of each stress component and plastic strain increment, then summed the six integrals to calculate the plastic work per cycle. Ellyin and Golos [32] stated that under cyclic loading, share of the energy absorbed by the material is converted into heat, the remaining energy is what causes damage. They suggested that component durability can be described using the quantity of total strain energy the material could contain.

Ellyin & Kujawski [33] suggested adding a new factor (the multiaxial constraint function) to the distortion strain energy term Ellyin and Golos [32] proposed before. The proposed factor function was to account for load multiaxiality. They related the damage function to the distortion strain energy, multiaxial constraint function and mean value of the normal stress. This resulted in good correlation with proportional data, but the approach was not tested against non-proportional loading data [33].
Although energy based approaches claim to have the advantage of handling both proportional and non-proportional loading, they have some limitations and considerations that make them less favorable in analyzing multiaxial fatigue problems. These considerations include:

- Energy is a scalar quantity; thus the approaches have no capability of describing crack initiation and growth on specific directions [19].
- Terms like plastic strains and plastic work are significantly small in HCF problems, and are difficult to quantify with confidence [19].
- Plastic work is not a material property, and plastic work change per component life has to be formulated during any loading scheme [34].

2.2.5 Critical Plane Approaches

Based on several experimental investigations, cracks tend to nucleate and grow on specific planes, called the “critical planes”. According to Socie [35], the critical planes orientation depends on material and loading conditions. They are either planes of maximum shear stress/strain, planes of maximum normal stress/strain, or planes having maximum damage term. The main advantage of critical plane criteria is their ability to predict crack propagation direction after nucleation. Karolczuk & Macha [36] proposed a general expression for fatigue failure in Equation 2-24.

\[
F[\sigma_n(t), \tau_{ns}(t), \varepsilon_n(t), \varepsilon_{ns}(t), K] > Q
\]  

(2-24)

Where:
\( \sigma_n, \tau_{ns} \) are the normal and shear stress components on the critical plane

\( \varepsilon_n, \varepsilon_{ns} \) are the normal and shear strain components on the critical plane

\( K \) is a material coefficient

\( Q \) is the fatigue limit

Findley [37] defined the critical plane as the plane having the maximum combination of shear stress amplitude and the maximum normal stress acting on the plane of maximum shear stress. He related the two stress components linearly as in Equation 2-25.

\[
\tau_{ns,a} = f - k\sigma_{n,\text{max}}
\]  (2-25)

While Findley did not define the coefficient \( f \), he defined \( k \) as a material constant related to bending and torsion fatigue strengths. He noticed that ductile materials have small \( k \) value; and their critical plane is close to the direction of the maximum shear stress plane. For brittle materials it was the opposite; with large value of \( k \), the critical plane direction approached the maximum principal stress direction [38].

Brown and Miller [39] proposed two types of crack initiation and propagation. Type A cracks which initiate and propagate on shallow surface layers with minimum depth and type B cracks which initiate also on the surface, but they propagate inward perpendicular to it, and are considered the more damaging type. An example of type A is component under torsional load, where the shear stress acts on a direction parallel to the length of the
Type B cracks are usually associated with biaxial tension loading [40] where the shear stress is normal to the surface and driving the crack inward. Figure 2.2 shows both types of the cracks in an elementary cube. Brown and Miller [39] concluded that the governing components of crack propagation are the maximum shear strain, and the maximum normal strain acting on maximum shear strain plane; considering it to be the critical plane.

![Type A Crack and Type B Crack](image)

**Figure 2.2 Crack Types According to Brown and Miller**

Kanazawa, Miller, and Brown [41] presented an analytical derivation to the plane of maximum shear and normal strains, assuming it to be critical plane direction. They also performed several experimental tests using thin walled cylinders to investigate the effect of phase angle in the case of non-proportional loading. They concluded from their
analytical and experimental work that phase angle of 90° has the most damaging fatigue effect.

McDiarmid [42] in his later work formulated a linear relation between alternating shear stress and maximum normal stress. He defined the critical plane as the plane experiencing the maximum “Damage” term combining both. He also related fatigue life to crack type as defined by Brown and Miller [39]. His experimental work also indicated that non-proportional loading resulted in shorter fatigue life than proportional loading, even for the same stress ranges.

Fatemi and Socie [43] proposed a critical plane model relating fatigue life to maximum shear strain and maximum normal stress acting on maximum shear strain plane. The adopted model has several advantages. First it can predict fatigue life under proportional and non-proportional loading conditions. Second, it accounts for both, material hardening due to non-proportional loading, and mean normal stress effect. Many researchers noticed that the non-proportionality of loading leads to additional hardening in the material and shorter fatigue life, see for example the works of: Dietmann and Lempp [23], McDiarmid [42], and Kanazawa [41]. Kanazawa [41] stated, that due to the rotation of principal axes; more crystallographic slip planes are stimulated and intersect compared to proportional loading.

Socie, Waill, & Dittmer [44], Fatemi and Stephens [45] discussed the effect of mean stress on fatigue life. They concluded that while shear stress mean value will not contribute to the damage. Mean value of the normal stress is a main driving factor for crack opening. As illustrated in Figure 2.3, and at the microscopic level, cracks usually have
irregular surface; and shear loading results in friction forces and interlocking which reduce crack tip driving forces. A tensile stress perpendicular to the crack surface reduces the friction forces and increases the crack tip driving forces, which allows the crack to propagate.

![Torsion and Tension Diagram](image)

Figure 2.3 Crack Opening Mechanism[43]

The critical plane model proposed by Fatemi-Socie [43] accounts for both non-proportionality hardening, and mean stress effect. It establishes a fatigue life model as per Equation 2-26.

\[
\gamma_{\text{max}} \left( 1 + \frac{k \sigma_{\text{n,max}}}{\sigma_y} \right) = \frac{r_f}{c} (2N_f)^{b_0} + \gamma_f (2N_f)^{c_0}
\]  

(2-26)
The material constant $K$ is determined by fitting data from simple uniaxial test and data from simple torsion test. Maximum shear strain, and regardless of the sign convention occurs on two orthogonal planes, each has a different value of maximum normal stress. This happen due to the load non-proportionality which distorts the hysteresis loops. The critical plane is then the plane with the largest shear strain amplitude and the maximum normal stress acting on that maximum shear strain amplitude plane ($\gamma_{max}/2, \sigma_{n,max}$).

For brittle materials, where crack propagation is mainly along the maximum normal stress plane, similar approach to Fatemi-Socie approach was developed by Smith, Watson, and Topper [46]. They [46], as reported in hens, Fatemi, Stephens, & Fuchs [19], defined the critical plane as the plane having the maximum tensile strain amplitude, and the maximum normal stress acting on the plane of maximum tensile strain. The model was formulated as in Equation 2-27.

$$\sigma_{n,max} \frac{\varepsilon_{max}}{2} = \frac{\sigma_f^2}{E} \left(2N_f\right)^{2b} + \sigma_f' \varepsilon_f'(2N_f)^{b+c}$$  \hspace{1cm} (2-27)

This model gives better results with brittle materials where crack propagation is mainly along the maximum normal stress plane.

Carpinteri, Ronchei, Spagnoli and Vantadori [47] inspected the problem of principal axis rotation due to load non-proportionality. They tried to overcome the problem by calculating average directions of the principal stresses according to Equation 2-28.

$$\delta = \frac{3\pi}{8} \left[1 - \left(\frac{\tau_{af-1}}{\sigma_{af-1}}\right)^2\right]$$  \hspace{1cm} (2-28)
Where:

\( \delta \) denotes angle between the averaged direction of \( \sigma_{1,max} \) and the normal \( \mathbf{w} \) to the critical plane.

\( \sigma_{af,-1}, \tau_{af,-1} \) denotes the fatigue limit for fully reversed normal stress and for fully reversed shear stress, respectively. According to their criteria [47] the multiaxial fatigue limit was expressed as in Equation 2-29.

\[
\left( \frac{N_{a,eq}}{\sigma_{af,-1}} \right)^2 + \left( \frac{C_a}{\tau_{af,-1}} \right)^2 = 1
\]  

Equation 2-29

Where \( N_{a,eq} \) is equivalent normal stress amplitude calculated from Equation 2-30, and \( C_a \) is the shear stress vector amplitude calculated using the Prismatic Hull Method.

\[
N_{a,eq} = N_a + \sigma_{af,-1} \left( \frac{N_m}{\sigma_u} \right)
\]  

Equation 2-30

Where \( N_a \) and \( N_m \) are normal stress amplitude and mean stress value respectively, and \( \sigma_u \) is the ultimate tensile strength of the material.

2.3 Scope of Work

The scope of this research is to use the critical plane approach with Fatemi-Socie model [43] and apply it to a drill pipe segment under multiaxial, non-proportional loading. The model is modified to accommodate for industry standards, and was customized to downhole condition of build-hold in a curved section using Lubinski’s bending equation. A software program has been developed to calculate fatigue life of the pipe under certain
loading and wellbore conditions. The model and software have been validated using experimental data from previous researches. The output of the software program predicts the remaining fatigue life and critical plane direction.
Chapter 3

Methodology

While drilling, drill string elements are subjected to significant forces and moments. Cyclic variation of these forces and moments are rarely in-phase with each other, and their non-proportionality is common in real life operations. Figure 3.1 shows an example of general downhole bending and torque loads vs. number of revolutions. This kind of time-history data is usually recorded by Measurements While Drilling tools (MWD tools), or using simulations to the drill string during drilling job, like the case in Figure 3.1.
Figure 3.1 Example of Downhole Torque and Bending Loads and their Non-Proportionality
Drilling is a sophisticated process, which includes many modes and jobs. Some of the drilling operation modes are:

- Build and Hold
- Vertical Rotating drilling (on-bottom)
- Sliding drilling
- Rotating off-bottom
- Hole reaming and back-reaming

Each of the previous modes has different loads and stress history. The current study focuses on multiaxial fatigue while directional rotating drilling (build and hold). This drilling mode is characterized by constant wellbore curvature, and therefore nearly constant bending moment. Minor time variations in bending moment and axial tension force will be disregarded. While surface torque is usually constant; directional drilling typically encounters torsional vibrations (stick-slip). In downhole stick slip, the rotating drill string starts to slow down due to high counteractive torque coming from formation and friction effects. The drill string slows down till it stops momentarily, then it is released after surface torque builds up. The drill string torque in this case fluctuates between maximum and minimum values, and can be approximated to sinusoidal loading over time. With drill pipe rotation; the constant bending moment generates sinusoidal axial stress (due to rotation). Figure 3-2 presents a deflected DP body in a curved wellbore section, under axial, torsional and bending loads. Figure 3-3 shows the DP segment under drilling stresses and an example of normal and shear stresses variation with time.
Figure 3.2 Deflected DP under Loading in a Curved Wellbore Section
Both torsional and axial stresses amplitude were calculated using Equations 3-1, 3-2, where \( \theta \) is the rotation angle measured from positive X.

\[
\sigma_x = \frac{F_x}{A} + \frac{M_y r \sin \theta}{I_y} \quad (3-1)
\]
\[
\tau_x = \frac{T_x r}{I_x} \quad (3-2)
\]

Figure 3-4, 3-5 present an example of proportional and non-proportional loading, with the corresponding Mohr circle at several times. As seen in Figure 3.4, when loading has no phase difference, and load minimum and maximum values occur simultaneously; principal axes change values during each loading cycle, but no change in principal axes direction, and Mohr circle varies only in size (proportional loading). On the other hand,
when loading components are out of phase (non-proportional) as in Figure 3.5.; principal axes do not only change values during a loading cycle, but also continuously rotate with respect to loading axes.

![Figure 3.4 Proportional Loading and Corresponding Mohr Circle](image)

Figure 3.4 Proportional Loading and Corresponding Mohr Circle
The objective of this research is to calculate drill pipes fatigue life under non-proportional multiaxial loading. As previewed in the previous chapter, the critical plane approach is one of the most appropriate approaches when analyzing non-proportional multiaxial fatigue loading. Among the several models that were proposed based on the critical plane approach, Fatemi-Socie model was chosen for its advantages of accounting for both: material hardening due to loading non-proportionality, and mean normal stress

Figure 3.5 Non-Proportional Loading and Corresponding Mohr Circle

-35-
effect. Fatemi and Socie [43] formulated their critical plane multiaxial fatigue model as per Equation 3-3.

\[
\frac{\gamma_{\text{max}}}{2} \left( 1 + k \frac{\sigma_{n,\text{max}}}{\sigma_y} \right) = \frac{\tau_f'}{\sigma_y} (2N_f)^{b_0} + \gamma_f' (2N_f)^{c_0} \tag{3-3}
\]

The Left Hand Side (LHS) of the Equation 3-3 is the “Damage” term, as the crack tends to initiate on the planes of maximum shear strain, and its driving force is the normal stress acting on these planes; they formulated the damage as a linear relation between these two quantities, then they related the damage quantity to the fatigue life using Coffin-Manson uniaxial fatigue relation. The normal stress in the LHS was divided by the yield strength to normalize the equation. The critical plane then is the plane with the largest shear strain amplitude and the maximum normal stress acting on that maximum shear strain amplitude plane \((\gamma_{\text{max}}/2, \sigma_{n,\text{max}})\). The material constant \(K\) in the Equation 3-3 is determined by fitting data from simple uniaxial test and data from simple torsion test.

This research procedure was divided into three major steps, first to design and develop a software computer program to predict the critical plane and fatigue life of the drill pipe. Second, validate that program using previous research and experimental results [43], [44]. Finally, use the developed software to calculate fatigue life relating to real and industry loading conditions.

-36-
3.1 Building the model

The computer program (called here) EBrandon was developed to calculate the fatigue life using the Fatemi-Socie model described above. The code was developed using the mathematical programming environment MATLAB®. While fatigue experimental input is usually strain or stress values, and using metric units, drilling industry and API prefer using British units for forces, moments and wellbore curvature degrees. For that; Elrond had two versions: the first one was developed for verifying the code using strain-controlled multiaxial fatigue tests. This version of the program consists of the main program, presented in Figure 3.6 and two subroutines as presented in Figures 3.7 & 3.8. The second version -the drilling version- which is used to estimate fatigue life in realistic drilling problems is different only in two aspects:

- It uses the British system of units for input and output as an industry standard
- The input is axial force, axial torque and wellbore curvature (instead of normal and shear strains) since this is the type of information recorded in a drilling operation.

The verification version of the program is presented in the following section. While the drilling version of Elrond is discussed in details when modelling realistic drilling problems later in this chapter.

Elrond, Verification Version

A flowchart of the main program is shown in Figure 3.6. All the input and output of the program are in this window (the main program window), and all other subroutines -
functions- are called from it. Input for the verification version of *Elrond* are material mechanical and cyclic properties and strain amplitudes from strain controlled fatigue tests. The program starts by calculating the shear strain and normal stress on all the possible planes using 2-D transformations laws; viz. Mohr’s Strain and stress Circles equations) as presented in Equations 3-4 and 3-5.

\[
\frac{\gamma_\theta}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta \tag{3-4}
\]

\[
\sigma_\theta = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \tag{3-5}
\]
Critical plane is the plane experiencing the maximum shear strain and maximum normal stress. But regardless of the sign convention, shear strain value is maximum on two orthogonal planes, and the critical plane among them is the one holding the larger value of normal stress. To automate such selection, the subroutine *CriticalPlanCalc* was developed. This subroutine uses two nested loops to search for the larger shear strain planes-regardless
of the sign convention- and the corresponding maximum normal stress acting on these planes. The subroutine choses the maximum value of the two corresponding normal stresses values, and its plane would be the critical plane. Flow chart of the subroutine CriticalPlanCalc is presented in Figure 3.7.
At this point the LHS or the damage term of Equation 3-3 is ready. While the LHS is loading, the Right Hand Side (RHS) of Equation 3-3 is mainly material dependent. The numerical model of Fatemi and Socie [40] adopted a modified Coffin-Manson equation to
calculate the RHS of their proposed multiaxial fatigue life equation. Tavernelli & Coffin [48] and Manson [49], as both reported in Stephens, Fatemi, Stephens, and Fuchs [19], proposed an equation in early 60’s to describe the relation between uniaxial strain and fatigue life. The original Coffin-Manson Equations are presented in Equations 3-6 and 3-7 for normal and torsional loading respectively.

\[
\frac{\Delta \varepsilon_e}{2} + \frac{\Delta \varepsilon_p}{2} = \frac{\sigma'_f}{E} (2N_f)^b + \varepsilon'_f (2N_f)^c \quad (3-6)
\]

and for torsional loading

\[
\frac{\Delta \gamma_e}{2} + \frac{\Delta \gamma_p}{2} = \frac{\tau'_f}{G} (2N_f)^{b_0} + \gamma'_f (2N_f)^{c_0} \quad (3-7)
\]

Fatemi and Socie [43], in the RHS of Equation 3-3 modified the above Coffin-Manson equations – Equations 3-6, 3-7- to cater for multiaxial fatigue problems. To calculate the RHS, some mechanical and cyclic properties of the material are required. to be input or calculated to complete the RHS of Equation 3-7. The RHS of Fatemi-Socie’s equation is shown below.

\[
\text{RHS} = \frac{\tau'_f}{G} (2N_f)^{b_0} + \gamma'_f (2N_f)^{c_0}
\]
RHS calls for the cyclic shear properties of the material along with its modulus of rigidity. These values are seldom available and usually calculated from uniaxial cyclic properties and modulus of elasticity respectively [43], as in Equations 3-8 to 3-12.

\[
\begin{align*}
\tau'_f &= \frac{\sigma'_f}{\sqrt{3}} \\
\gamma'_f &= \sqrt{3} \epsilon'_f \\
b_o &= b \\
c_o &= c \\
G &= \frac{E}{2(1+v)}
\end{align*}
\]  

(3-8)  
(3-9)  
(3-10)  
(3-11)  
(3-12)

Now as all the terms needed for the Fatemi-Socie equation are ready; the code assembles the equation and calls for the subroutine Solver to solve it numerically. Solver is an automatic numerical solver. It searches automatically for the root (Nf) vicinity by checking for function change of sign, then applies the Bisection numerical method to find the root. Flow chart for the Solver is presented in Figure 3.8.
Figure 3.8 Numerical Solver Subroutine

3.2 Validating the Model

As Kanazawa, Miller, and Brown [41] proved that loading with a phase difference of 90° is the most damaging phase difference with respect to fatigue life. This particular phase difference was used in most of the research studying multiaxial non-proportional loading and hence, it is adopted in the current research. Elrond program was verified using
two experimental data sets. The first data set used 1045 hot rolled steel [43] and the second was based on the Inconel alloy [44].

3.2.1 1045 HR Steel Data Set

These results were based on the experimental work conducted by Fatemi and Socie [43]. They conducted experimental multiaxial fatigue tests using normalized hot rolled 1045 steel. Experimental fatigue lives and crack direction results were compared to predicted numerical results calculated using critical plane approach. Mechanical monotonic and cyclic properties of the 1045 HR steel, as provided in [43] and presented in Table 3-1.

Table 3-1 Mechanical monotonic and cyclic properties of the 1045 HR steel [43]

<table>
<thead>
<tr>
<th>Monotonic Properties</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of elasticity (MPa), $E$</td>
<td>202,375</td>
</tr>
<tr>
<td>Yield Strength (MPa), $Y$</td>
<td>382</td>
</tr>
<tr>
<td>Modulus of rigidity (MPa), $G$</td>
<td>79,100</td>
</tr>
<tr>
<td>Poisson’s ratio, $\nu$</td>
<td>0.29</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cyclic Properties</th>
</tr>
</thead>
</table>

-45-
Test specimens were thin-walled tubes, with 25.4 mm inside diameter and thickness of 2.54 or 3.18 mm. They performed their tests on strain-controlled tension-torsion machine. Two types of tests were conducted: as follows:

- Uniaxial tension or torsion tests, which they used to determine specimen cyclic properties,
- Combined tension-torsion tests to estimate multiaxial fatigue life and critical plane direction, using either proportional or non-proportional loading

Non-proportional load was defined as two sine waves of tension and torsion as in Equations 3-13 and 3-14 [43].

\[
\varepsilon = \varepsilon_a \sin \omega t \quad (3-13) \\
\gamma = \gamma_a \sin(\omega t + 90) \quad (3-14)
\]
Load was set into five biaxial strain ratios ($\lambda$), which is the ratio between shear and normal strains, 0, 0.5, 1, 2 and $\infty$, while zero strain ratio representing pure tension and $\infty$ representing pure shear.

Due to non-proportionality of the loading, and at high strain values, lag occurs between peak stresses and strains as a result of the distortion of hysteresis loops. Fatemi and Socie [43] reported having a lag of 0-30 degrees depending on the strain amplitude. Due to this lag, and in a strain controlled multiaxial fatigue test for instance, two values of stresses are usually reported: first is the stress amplitude value, which is the peak of the stress signal. The other is the stress at the peak strain signal, and the later one was used in the current analysis.

Different researches use different criterion in their work. In their research Fatemi and Socie [43] defined failure as the number of load cycles corresponding to 10% drop in load value, which is common for strain controlled tests. For most cases of torsional loading tests, a 10% drop in load coincided with observation of 10 mm through thickness crack.

### 3.2.2 Inconel 718 Data Set

The second data set used to verify the current model came from Socie, Waill, and Dittmer [44] work with the Inconel 718 alloy. Table 3-2 presents the monotonic and cyclic properties of Inconel 718 as provided in [41]. These values were used as input for *Elrond* in the second verification case.
Table 3-2 monotonic and cyclic properties of Inconel 718 [44]

**Monotonic Properties**

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of elasticity (MPa), $E$</td>
<td>209,000</td>
</tr>
<tr>
<td>Yield Strength (MPa), $Y$</td>
<td>1160</td>
</tr>
<tr>
<td>Modulus of rigidity (MPa), $G$</td>
<td>81,008 (calculated)</td>
</tr>
<tr>
<td>Poisson’s ratio, $\nu$</td>
<td>0.29</td>
</tr>
</tbody>
</table>

**Cyclic Properties**

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fatigue strength coefficient (MPa), $\sigma_f'$</td>
<td>1640</td>
</tr>
<tr>
<td>Fatigue strength exponent, $b$</td>
<td>-0.06</td>
</tr>
<tr>
<td>Fatigue ductility coefficient, $\varepsilon_f'$</td>
<td>2.67</td>
</tr>
<tr>
<td>Fatigue ductility exponent, $c$</td>
<td>-0.82</td>
</tr>
<tr>
<td>Fatemi-Socie material Constant, $k$</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Socie, Waill, and Dittmer [44] performed two types of strain controlled fatigue tests. Uniaxial fatigue tests to determine uniaxial cyclic properties of the material, and
biaxial fatigue tests to determine critical plane and fatigue life. Both were conducted using thin-walled tubes with inner diameter of 25 mm and thickness of 2 mm. Material cyclic fatigue shear properties were calculated from material cyclic fatigue axial properties along with modulus of rigidity using Equations 3-8 to 3-12.

3.3 Elrond, Drilling Version

Fatigue tests are usually strain controlled ones, but in drilling operations, it is more common to control loads and inclinations than strains. Therefore, Elrond software was modified accordingly to adopt to oil industry parameters and conditions, as well as the recommended system of units. As mentioned before, and to calculate multiaxial fatigue life using Fatemi-Socie equation [45], normal stress is calculated using axial force (which is the drill string buoyant weight carried by the DP segment of interest) and bending moment. Bending moment in drilling operations comes mainly from wellbore curvature, as presented in the following section.

3.3.1 Bending Moment and Wellbore curvature

When the drill string is drilling an inclined section, and trying to hold its direction, the job called “Build and Hold” and the tool face angle will not change and drilling bit direction nearly remains constant. As the drilling pipe segment drills in that curved section, Lubinski [10] showed that bending moment in the drill pipe segment depends on the curvature of the DP segment and tension in the pipe segment as in chapter 2. Disregarding the gravity effect, Lubinski [10] calculated the bending moment in the curved drill pipe
segment using equations 2-2 to 2-5. These relations were used in Elrond drilling version to estimate the bending moment

### 3.3.2 Subroutine Stresses

To comply with oil & gas industry standards and units’ system, a subroutine called *Stresses* was added to the program Elrond. *Stresses* has three main functions:

- Convert between SI and metric units.
- Calculate bending moment from wellbore curvature as described in the above section.
- Calculate stress and strain matrices from the input.

The subroutine calculates stresses and strains from torque, bending and axial tension in the pipe segment using the following power law [19] in Equation 3-16:

\[
\gamma_a = \frac{\tau_a}{G} + \left(\frac{\tau_a}{K_o'}\right)^{1/n_o'}
\]  

(3-15)

Few tests have been made to estimate cyclic shear material properties, so many attempts have been made to relate their values to the more common cyclic axial ones. Li, Zhang, Sun, Li, and Li [50] related the cyclic shear strength coefficient and cyclic shear strain exponent to their axial peers and verified their results using several material specimens. They started by comparing the elastic and plastic terms Coffin-Manson relation presented in Equations 3-6, 3-7 with their equivalent terms in Ramberg-Osgood relation presented in Equation 3-17, 3-18 Both in the axial and shear forms respectively [50].
\[
\frac{\Delta \varepsilon}{2} = \frac{\Delta \sigma}{2E} + \left( \frac{\Delta \sigma}{2K'} \right)^{1/n'} \tag{3-16}
\]

\[
\frac{\Delta \gamma}{2} = \frac{\Delta \tau}{2\sigma} + \left( \frac{\Delta \tau}{2K_o'} \right)^{1/n_o'} \tag{3-17}
\]

Then, and in the light of Von Mises criterion; they calculated the cyclic shear strength coefficient and the cyclic shear strain hardening exponent from the cyclic axial strength coefficient and cyclic axial strain hardening exponent respectively using Equations 3-19, 3-20 [50].

\[
n_o' = n'
\tag{3-18}
\]

\[
K_o' = 3 \left( \frac{1 + n'}{2} \right) K'
\tag{3-19}
\]

Figures 3.9, 3.10 show the changes made to the main program flow chart and the new subroutine \textit{Stresses} respectively.
Start

- Material Cyclic properties
- Material Mechanical properties
- Axial force and Torque
- **Dogleg Severity**
- Pipe size

**STRESSES**

Estimate shear strain and normal stress on each plane

Critical Plane Direction \( \theta_C \)

Critical plane calculator

Calculate L.H.S

Numerical Solver

Number of Life Cycles \( N_f \)

End

Figure 3.9 *Elrond* Main Window, Default Version
3.3.3 Material Selection and Mechanical Properties

In drill string design, steel family AISI 41xx are common to use, and in this analysis steel AISI 4142 was adopted. Pipe body OD was 5 in (127 mm, and ID was 4.267 in (108 mm). The API fatigue limitations [11] and Lubinski’s work [10] were all based on drill pipes grade E, with a minimum limit of yield strength of 75,000 psi (517 MPa), and
ultimate tensile strength of 100,000 psi (689 MPa). These values were lower than yield and ultimate strength of the AISI 4142 used here. So, to make *Elrond* results comparable with the API standards; yield and ultimate strength values suggested by the API [11] were used in this analysis. Also, as some of the cyclic properties of the material- specially the fatigue strength coefficient value- are linked to yield and ultimate strength, they were also modified. The ASM Handbook [51] discussed a relation between material ultimate strength and its fatigue strength coefficient as in Equation 3-21.

\[
\sigma_f' = \sigma_u + 345 \quad (3-20)
\]

Where both \(\sigma_f'\) and \(\sigma_u\) are both in MPa. Table 3-3 presents mechanical and cyclic properties of the steel AISI 4142 as presented by the ASM Handbook [51], except for the yield strength, ultimate strength and fatigue strength coefficient, which were modified to match API values for the same pipe grade.

**Table 3-3 Mechanical Monotonic and Cyclic Properties Used in the Drilling Case Study**

<table>
<thead>
<tr>
<th>Monotonic Properties</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of elasticity (MPa), E</td>
<td>206,000</td>
</tr>
<tr>
<td>Yield Strength (MPa), Y</td>
<td>517</td>
</tr>
<tr>
<td>Ultimate Strength (MPa), U</td>
<td>689</td>
</tr>
</tbody>
</table>
While Poisson’s ratio and Fatemi-Socie’s material constants were not provided in the ASM handbook [51], the values of 0.29 and 1 were assumed respectively. Value of 1 were recommended by Stephens, Fatemi, Stephens, & Fuchs [19] if test data were not available.

All other aspects of the program remained the same. After the maximum shear strain and maximum normal stress acting on the maximum shear strain are calculated using the CriticalPlanCalc; fatigue life is calculated accordingly using the SOLVER subroutine as explained before.
3.3.4 Cumulative Fatigue Assessment

The API had an approximate method to estimate the cumulative fatigue damage in a drill pipe segment after drilling certain interval. This method was developed by Hansford and Lubinski [52]. The method calculates fatigue life of the DP segment of interest using Lubinski’s [10] method, calculates the number of revolutions required to drill this segment with a certain rotary speed, then use Equations 3-22, 3-23 to calculate the damage percentage.

\[
\begin{align*}
    f &= \frac{B}{N_f} \\
    B &= \frac{60 \cdot R \cdot d}{V}
\end{align*}
\]  

(3-21)  

(3-22)

Where:

B is the revolutions expended in an interval

R is rotary table speed, rpm

d is length of the dogleg interval, ft

V is drilling rate, ft/hr.

Then percentage of fatigue life expended in a certain interval is plotted vs. dogleg severity and axial force.

To compare Elrond results with the API current standards, fatigue life values were calculated using Elrond and cumulative life for several dogleg severity values were calculated. Results were plotted with results from Hansford and Lubinski [52] work. Rotary
speed, length of dogleg and drilling rate were used as per values used in the API RP7G [11] and presented in Table 3-4.

Table 3-4 Initial Conditions Used in Calculating Cumulative Fatigue Damage [11; 52]

<table>
<thead>
<tr>
<th>Condition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotary Speed, rpm</td>
<td>100</td>
</tr>
<tr>
<td>Dogleg length, ft.</td>
<td>30</td>
</tr>
<tr>
<td>Drilling Rate, ft./hr.</td>
<td>10</td>
</tr>
</tbody>
</table>
Chapter 4

Results

In this chapter, results from all the cases described in chapter (3) are presented. The chapter is divided into four main parts:

- Results from the validation case using steel 1045HR,
- Results from the validation case using Inconel 718,
- Results from field cases simulating realistic drilling cases,
- Cumulative fatigue life assessment.

4.1 Validation Case 1, Steel 1045HR

4.1.1 Fatigue Life Estimation

In this case study, a set of strain-controlled, proportional and non-proportional biaxial fatigue tests are conducted on thin-walled tubes made from hot rolled 1045 steel, with known cyclic properties, Fatigue life and critical plane directions are determined. The time variation of axial strain $\varepsilon$ and shear strain $\gamma$ are prescribed by Equations 3.13, 3.14. Tests are conducted at five biaxial strain ratios $(\lambda = \gamma/\varepsilon)$ of 0, 0.5, 1, 2 and $\infty$, where $\lambda = 0$ corresponds to pure tension, $\lambda = \infty$ corresponds to pure shear and $\lambda = 0.5$, 1, 2 represents cases of biaxial fatigue tests.
Three sets of data for fatigue life are compared here: observed experimental life from [40], fatigue lives predicted by Elrond, and fatigue lives predicted by Fatemi and Socie in [40].

Figures 4.1 and 4.2 compare observed fatigue lives with lives predicted in [40], and lives predicted by *Elrond* respectively using logarithmic scales. The solid line denotes observed lives as a reference and the two dashed lines represent a factor of 2 bands.

![Figure 4.1 Observed vs. Predicted Life, Fatemi-Socie Results [43]](image-url)
Figure 4.2 Observed Data [43] vs. predicted life, Elrond results

In general, Elrond predicted lives were more conservative than Fatemi-Socie experimentally observed ones, and most of the lives predicted by Elrond were within a factor of two from the observed values and close to Fatemi-Socie predicted lives. It was noticed that the model gave better match with experimental data for non-proportional loading than with proportional cases. That can be related to the physical mechanism behind the model, and the fact that the model considers both, material hardening and hysteresis loops distortion resulting from non-proportionality.
The second step of validation was to compare *Elrond* results with Fatemi-Socie numerical results directly. Figures 4.3 and 4.4 show selected cases from *Elrond* and Fatemi-Socie work plotted versus their experimental results. In Figure 4.3, with a biaxial strain ratio – shear to normal strain ratio – of 0.5, *Elrond* gave very close results to Fatemi-Socie predicted fatigue lives in both proportional and non-proportional loading, while in Figure 4.4, with strain ratio of 1, *Elrond* resulted in better matching with observed life than Fatemi-Socie predicted values did. It is noticed also that the model yielded better match with experimental results when the strain ratio is dominated by shear, while cases with low strain ratios or pure normal strain resulted in poor matching. These results were expected as the model was built for ductile materials; where crack initiation and growth is along the maximum shear planes. For more brittle materials, another version of the critical plane approach (Smith, Watson, and Topper model [46]) was recommended by Stephens, Fatemi, Stephens, and Fuchs [19].

Differences were noticed between fatigue lives estimated by *Elrond* and corresponding lives estimated by Fatemi-Socie [43]. These differences can be attributed to differences between Elrond numerical solver and the numerical solver Fatemi-Socie used to solve their fatigue equation. They [43] did not indicate the numerical method or the initial guesses they used to solve their equation.
Figure 4.3 Comparison Between Predicted Life from Elrond and Predicted Life from Fatemi-Socie’s Work, Both vs. Experimental Results – $\lambda=0.5$
Figure 4.4 Comparison Between Predicted Life from Elrond and Predicted Life from Fatemi-Socie’s Work, Both vs. Experimental Results – \( \lambda=1 \)

4.1.2 Critical Plane Direction

Brown and Miller [39], as reported by Socie [40], differentiated crack propagation into two main stages: stage I, which is the early stage where crack nucleate and propagate on shear slip bands, and stage II where crack shifts to a plane perpendicular to maximum principal stress plane. Socie [40] noticed that some materials experience that shift and show the two stages of crack propagation before final failure. Other materials do not reach this late stage; and the final failure crack coincides with the critical plane directions estimated numerically. Carpinteri, Spagnoli, and Vantadori [53] also discussed this discrepancy.
between the defined “critical plane direction” and the final crack direction on the macroscopic level. They stated that the direction of the critical plane generally does not agree with the direction of the fatigue final fracture on a macroscopic scale.

In their work, Fatemi and Socie [43] did not report the critical planes directions estimated by their numerical model and published only the experimental final crack directions; which may not coincide with the estimated critical plane if the material went into stage II before final failure. According to the experimental results reported in [43], and for proportional loading tests, all cracks initiated on plane of maximum shear strain. For pure tension samples ($\lambda=0$) microcracks initiated on maximum shear planes, then in their later life, microcracks either linked or grew into a direction normal to maximum principal stress. In their pure torsion loading ($\lambda=\infty$) crack initiation and growth were on the specimens’ longitudinal direction. Elrond critical plane directions matched Fatemi and Socie experimental results in both cases, with critical plane directions of 135° and 180° to the specimen axis in pure tension and torsion respectively as shown in Figure 4.5. For experimental combined in-phase tension-torsion loading, cracks generally initiated and grew on maximum shear planes, then switched direction to that perpendicular to the maximum principal stress direction. Elrond results matched also these cases, and critical plane directions are illustrated in Figure 4.5.
For the non-proportional loading cases, Fatemi and Socie [43] stated that in their experiments, and for strain ratio of 0.5, several non-failure cracks were noticed perpendicular to the maximum principal stress direction, with a major failure-crack on a plane closely aligned with the maximum shear strain directions (45°, 135°). This result agrees with critical plane direction results estimated from Elrond and presented in Figure 4.6, where the critical plane direction was 59° and 159° for strain ratios 0.5 and 1 respectively. For cases with biaxial strain ratio of 2, critical plane direction estimated by Elrond was 167° to the specimen axis. Fatemi and Socie experimental results for these
cases stated that many small cracks formed on the maximum shear strain direction, then these cracks linked to form the final failure cracks parallel and normal to the specimen axis.

![Specimen Axis](image)

\( \lambda=0.5, 59^\circ \)

\( \lambda=1, 159^\circ \)

\( \lambda=2, 167^\circ \)

**Figure 4.6 Estimated Critical Plane Directions, Non-Proportional loading**

Figures 4.7, 4.8 and 4.9 presents variation of shear and normal stress values over plane direction, and the vertical black line is the direction of the critical plane. For all three biaxial values tested, the critical plane direction followed the direction of the maximum shear and normal stresses. That was expected knowing that maximum shear is the responsible for activating slip bands. There was a difference between the predicted critical plane direction and the direction of plane with maximum shear and normal stresses, this
difference was minimum for small strain ratios ($\lambda=0.5$) and increased for larger strain ratios ($\lambda=2$).

Figure 4.7 Direction of Maximum Shear and Normal Stresses and Critical Plane Direction, $\lambda=0.5$
Figure 4.8 Direction of Maximum Shear and Normal Stresses and Critical Plane Direction, $\lambda = 1$
Figure 4.9 Direction of Maximum Shear and Normal Stresses and Critical Plane Direction, \( \lambda = 0.5 \)

4.2 Validation Case2, Inconel 718

Socie, Waill, and Dittmer [44] tested thin walled cylinders of the Inconel alloy under proportional biaxial strain-controlled conditions. Observed experimental fatigue lives are compared to three versions of the critical plane approach which are: Maximum Plastic Shear strain model, Lohr and Ellison model, and Kandil, Brown and Miller Model. As none of the models they used was the Fatemi-Socie model used in the current research; only Elrond results were compared to the observed lives in Figure 4.10, which compares
predicted fatigue lives from *Elrond* to the observed experimental lives from [41]. In [41], fatigue lives were reported at three different stages of cracking as follows: 0.1 mm, 1 mm and at final separation. Lives corresponding to 1 mm crack provided the best match with all numerical models, and these were the lives adopted in Fig. 4.10. The solid line represents the observed data, and the two dashed lines represents a factor-of-two bands. All the fatigue lives calculated by *Elrond* fall into that band except two data points from the pure tension tests with low strain amplitude. That again emphasis on Fatemi-Socie model limitation to pure normal strain loading conditions.
4.2.1 Critical Plane Direction

Three biaxial strain ration of \((0, \sqrt{3}, \infty)\) are used in [41], and the crack directions were reported for each strain ratio. Cracks in pure tension loading \((\lambda = 0)\) were observed to be closely aligned with the maximum shear stress direction \((45^\circ, 135^\circ)\), which was the values predicted by Elrond for this type of loading. Under combined loading \((\lambda = \sqrt{3})\), cracks were observed to be inclined with an angle around \(160^\circ\). Elrond estimated the critical plane inclination for the same case to be \(165^\circ\) to the specimen axis. For pure torsion, vertical and horizontal cracks were visible on specimen surface. Elrond predicted the
critical plane direction to be 180 of the specimen axis, which matches with the experimentally observed direction. Figure 4.11 shows the critical plane directions for the three biaxial loading ratios as predicted by Elrond.

![Figure 4.11 Predicted Critical Plane Directions](image)

### 4.3 Downhole Drilling Conditions

#### 4.3.1 Fatigue Life Estimation

As mentioned before, the drilling operation chosen for this analysis was “Build and Hold” operation, where the drill pipe’s segment of interest is in a curved section. In this case the drill pipe segment is subjected to tension from the buoyant weight of the drill
string below it, bending moment due to the wellbore curvature, and axial moment -torque- that is applied on the drill pipe at the surface. Both, tension and bending moment can be assumed constant in this case, but in directional drilling, stick-slip is usually present, and therefore the axial torque gets fluctuating.

Figure 4-12 Shows the elastic, plastic and total normal strains for selected cases from the downhole drilling results according to Coffin-Manson relation.

Figure 4.12 Normal strains according to Coffin-Manson Relation

Figures 4.13, 4.14 and 4.15 show fatigue life as predicted by Elrond versus wellbore curvature (dogleg severity) for various level of axial load and limits of the fluctuating axial
torque. The three figures compare three cases with exact same loading except for axial torque. Assuming axial torque to be sinusoidal, three amplitudes were examined between 10 and 30 k-lb. ft. (13,558 - 40,675 N-m). It was observed that the increase in axial torque amplitude reduced fatigue life for the same axial and bending loading conditions. Figure 4.16 shows same results but in 3-D, the fatigue life in this graph was presented as a surface, separating between safe and non-safe load values. As expected, fatigue life decreased dramatically with the increase of axial load and wellbore curvature.

Each line in Figures 4.13, 4.14 and 4.15 represents fatigue life vs. wellbore curvature at certain axial force and torque ranging from 100 – 250 k lb. (444.8 -1112 k N). Lines seem to converge for higher wellbore curvature values and diverge for lower ones. Meaning that higher wellbore curvature values (dogleg severity) results in amplifying the axial load effect and decreasing fatigue life at higher slope. This effect is even more clear in Figure 4.16 with the fatigue surface, the surface slope was more steep for higher values of dogleg severity and axial force, and more relaxed at the lower values.
Figure 4.13 Estimated Fatigue Life for Different Well Curvatures and Axial Forces, $T_x=10-20$ klb-ft (13,558-27,116 N-m)
Figure 4.14 Estimated Fatigue Life for Different Well Curvatures and Axial Forces, $Tx = 15-30$ klb-ft (20,337-40,675 N-m)
Figure 4.15 Estimated Fatigue Life for Different Well Curvatures and Axial Forces, $Tx=10-30$ klb-ft (13,558-40,675 N-m)
To show the effect of torque amplitude; in Figure 4.17 fatigue lives vs. dogleg severities were plotted for three different torque amplitudes, and using the lower and higher values of axial force. Torque amplitude of 20 k lb. -ft. (27,116 N-m) significantly reduced fatigue life compared to amplitudes of 10 or 15 k lb.-ft. (13,558 or 20,337 N-m). Higher torque resulted in even greater reduction in fatigue life at higher wellbore curvature values as the lines converged. The Fatemi-Socie model used in Elrond was developed for low cycle fatigue conditions, where plasticity has a significant portion of deformation behavior. Some fatigue lives predicted by Elrond were higher than 1,000,000 cycles, which is usually
Fatigue life can be presented in hours of drilling. Figures 4.18, 4.19 present same results as Figure 4.13 and 4.14 but fatigue life is presented in hours rather than fatigue cycles, assuming drill string rotary speed of 100 rpm. In Figures 4.18 and 4.19, cases with lower loading -axial force and bending- have significantly high fatigue life, that they can be considered non-failing cases, their fatigue life values were added for clarification and
comparison purposes. Cases with very high axial force or wellbore curvature have very short life; and higher grade of drill pipe is recommended in such loading cases to insure success of the drilling job. Figure 4.18 shows a comparison between torque amplitudes of 10 and 15 k lb.-ft. (13,558 and 20,337 N-m), for the lowest and highest cases of axial loading and different wellbore curvature values. Figure 4.20 shows comparison between fatigue life at two different axial torque amplitudes, 10-20 k lb.-ft. (13,558-27,116 N-m) and Tx= 15-30 klb-ft (20,337-40,675 N-m). Increasing the torque amplitude by 50% resulted in 74% reduction in fatigue life, and doubling the torque from 10 k lb.-ft. (13,558 N-m) to 20 k lb.-ft. (27,116 N-m) – which was not plotted- resulted in a 94% reduction in fatigue life. These reductions were calculated for 100 k lb. (444.8 k N) axial force and 10 deg./100ft dogleg severity.
Figure 4.18 Fatigue Life, Hours to failure, Tx=10-20 klb-ft (13,558-27,116 N-m)
Figure 4.19 Fatigue Life, Hours to failure, $T_x=15-30$ klb-ft (20,337-40,675 N-m)
4.3.1.1 Special Case, Vertical Wellbore

A special case was examined, where the wellbore is totally vertical; and now bending moment due to curvature is affecting the drill pipe. Cases were examined with axial force between 100-250 klbf (444.8-111.2 KN) and torque amplitudes of 10-20, 15-30 and 10-30 klb-ft (13,558-27,116, 20,337-40,675 and 13,558-40,675 N-m). All the cases examined did not fail, except for the cases with torque amplitude of 10-30 klb-ft (13,558-
40,675 N-m) as shown in Figure 4.21. these results indicate that bending moment has a greater effect than torque on fatigue life.

![Figure 4.21 Fatigue Life under Axial Force and Torque Load Only](image)

**Figure 4.21 Fatigue Life under Axial Force and Torque Load Only**

### 4.3.2 Critical Plane Direction

The other main output of *Elrond* is the prediction of critical plane direction with respect to pipe axis. Figures 4.22 and 4.23 shows critical plane direction for axial torque amplitude of 10 and 15 k lbf-ft. (13,558 and 20,337 N-m). The critical plane direction started farther from the pipe axis in lower well-curvature cases, and moved towards pipe
axis with higher curvature values. The overall change in critical plane direction with loading was limited. Over the whole tested loading range of 150 k lb. (667 KN) axial force, 5 deg./100ft dogleg severity and 15 k lb.-ft. (20,337 N-m) of axil torque, the critical plane direction moved between 130° and 160° from the pipe axis.

Figure 4.22 Critical Plane Direction, Tx= 10-20 klb-ft (13,558-27,116 N-m)
4.4 Cumulative Fatigue Assessment

Cumulative fatigue damage expended in a certain interval was calculated using fatigue life from the software Elrond, then using Hansford and Lubinski [52] technique described in chapter 3 (section 3.3.3). Results are presented in Figure 4.24, 4.25 and 4.26 for axial torque amplitudes 10-20, 15-30 and 10-30 k lb.-ft. (13,558-27,116, 20,337-40,675 and 13,558-40,675 N-m) respectively and axial force of 100, 150, 200 and 250 k lb. (445, 667, 890 and 1112 kN). The abscissa represents the percentage of fatigue life expended in drilling 30ft (9.13 m), while the ordinate is the axial load carried by the DP segment. Elrond...
results – in blue color- were plotted at different dogleg severities, while API results at the corresponding axial force and dogleg severity were plotted in different shades of red color. For higher values of axial force like 200 and 250 k lb. (890 and 1112 kN), significant percentage of fatigue life would be expended to drill the section; and moving to a higher grade of pipes with stronger properties would be recommended to successfully finish the job.

It was noticed that the API results [11] are more conservative than Elrond results, especially for low torque amplitude values. As the API fatigue life calculation method is based on uniaxial fatigue criterion, and does not consider the effect of axial shear on fatigue life; it was difficult to know the values of torque which the API charts were developed at for proper comparison with Elrond or any multiaxial fatigue life estimation criterion. Also, as discussed earlier in chapter 2, Lubinski’s estimation of bending stress was too conservative due to the fact that he used a modified Goodman diagram [8] and lower bound of Bachman[14] experimental results [13]

But one of the reasons API results are more conservative is the way Lubinski estimated bending stress at first place. Lubinski [10]-as reported by Vaisberg [8]- started with Goodman diagram to estimate relation between bending stress and axial force the DP segment subjected to. But then he used a modified Goodman diagram with a lower endurance limit to account for slip marks and wear on the pipe body. He also assumed a cutoff on the mean stress at 67 ksi [8]. In addition, Grondin and Kulak [13] stated that Lubinski’s fatigue curve characterize lower bound of test results performed by Bachman [14].
Figure 4.24 Cumulative Fatigue Life, Tx= 10-20 klb-ft (13,558-27,116 N-m)
Figure 4.25 Cumulative Fatigue Life, Tx = 15-30 klb-ft (20,337-40,675 N-m)
Figure 4.26 Cumulative Fatigue Life, $T_x = 10$-$30$ klb-ft ($13,558$-$40,675$ N-m)
Chapter 5

CONCLUSION AND FUTURE DIRECTIONS

5.1 Conclusions

The main objective of the research was to develop a technique to predict drill pipes fatigue life in real life drilling operations and under non-proportional multiaxial loading conditions. The critical plane approach is most suitable for analyzing multiaxial fatigue problems where loading is non-proportional, and Fatemi-Socie numerical model has two main advantages: considering hardening due to load non-proportionality, and normal mean stress effect.

The software \textit{Elrond} was built to calculate the fatigue life under proportional/non-proportional multiaxial fatigue conditions, and critical plane direction. \textit{Elrond} gave good match with experimental data from previous strain controlled fatigue tests. The software matched better with loads with high biaxial strain ratio, and poor matching with pure normal or mostly normal strain cases. These results relate back to the physical crack opening mechanism the approach was based on, where cracks initiate and propagate on planes of maximum shear, and normal stresses act as the driving force in crack propagation.

The drilling operation chosen for study in this research was “Build and Hold”. Where the drill string is drilling directional section, and the DP segment of interest is located into curved section of the wellbore and subjected to fluctuating bending stress and axil torque. It was observed that the increase in torque amplitude reduced fatigue life for the same axial and bending loading conditions. Increasing the torque amplitude by 50%
resulted in 74% reduction in fatigue life, and doubling the torque from 10 k lb.-ft. (13,558 N-m) to 20 k lb.-ft. (27,116 N-m) resulted in a 94% reduction in fatigue life. It was noticed that the relation between fatigue life, bending moment (dogleg severity) and torque was not linear, as the lines of life converge with higher loads; resulting in greater loss of life.

Change of critical plane direction was minimum in response of load change. Over the whole tested loading range of 150 k lb. (667 kN) axial force, 5 deg./100ft dogleg severity and 15 k lb.-ft. (20,337 N-m) of axil torque, the critical plane direction moved between 130° and 160° from the pipe axis.

To compare results to the API fatigue design guidelines [11], cumulative fatigue damage was calculated using Hansford and Lubinski’s technique [52]. The API results were generally more conservative than Elrond ones, for the following three causes:

- Absence of torque values in API charts
- Modified Goodman diagram to account for DPs slip marks
- Lubinski used lower band of fatigue data

### 5.2 Recommendations

To update and enhance drill pipes fatigue design guides numerous areas need further investigation. Some suggested areas are the following:

- Further research into contact problem between wellbore and DP body
- Downhole history of the drill string tools that include loading history
- Linking fatigue life calculators to drill string 3-D simulators or downhole data from sensors for better input of loading
- Further research into non-proportional multiaxial fatigue failure mechanisms
References


[34] Leese, G. Engineering Significance of Recent Multiaxial Research. doi:10.1520/STP24527S


Appendix A

ELROND CODE

• MAIN PROGRAM

```matlab
%% Input of mechanical and cyclic properties
clc;
format long;
% Steel AISI 4142
% % mechanical and cyclic properties, pipe dimensions
OD_US=5; %OD(in)
ID_US=4.276; %ID(in)
YS_US=75000; %Yield Strength (psi)
E=206000; %modulus of elasticity (MPa)
neu=0.29; %Poisson’s ratio
sigmaf=1034; %2017; %fatigue strength coefficient (Mpa)
b=-0.085; %fatigue strength exponent
epsf=0.85; %fatigue ductility coefficient
c=-0.90; %fatigue ductility exponent
K=2359; %Cyclic strength coefficient (Mpa)
n=0.11; %Cyclic strain hardening Exponent
km=1; %Fatemi_socie material constant
% % Loading input
Fx_US=250; %Axial force in DP segment (klb)
dogleg=4; %curvature(dogleg)(deg/100ft)
Tmin_US=10; %min value of axial torque in X-direction (klb-ft)
Tmax_US=30; %max value of axial torque in X-direction (klb-ft)

%% Calculating Cyclic shear properties
OD=OD_US*0.0254;
ID=ID_US*0.0254;
A=(pi/4)*(OD^2-ID^2);
Iy=pi*(OD^4-ID^4)/64;
Ix=pi*(OD^4-ID^4)/32;
G=E/(2*(1+neu)); %Modulus of rigidity (MPa)
Tauf=sigmaf/(sqrt(3)); %fatigue shear strength coefficient Mpa
bo=b; %fatigue shear strength exponent
gf=epsf*(sqrt(3)); %fatigue shear ductility coefficient
co=c; %fatigue shear ductility exponent
Ko=(3^((1+n)/-2))*K; %Cyclic shear strength coefficient (Mpa)
no=n; %Cyclic shear strain hardening exponent

[sigmaXa,sigmaXm,eXa,tauXYa,gXYa,YS]=Stresses(OD_US,ID_US,E,G,K,n,Ko,no,Fx_US,dogleg,Tmin_US,Tmax_US,YS_US); % calling function Stresses
```
theta=1:1:360;
g=(gXYa*cosd(2*theta)-eXa*sind(2*theta))';
sigma=(((sigmaXm+sigmaXa)/2)*(1+cosd(2*theta))+tauXYa*sind(2*theta))';
LHS=(g.*(1+km*(sigma/YS)));
[gmax , sigmamax, thetaC] = CriticalPlanCalc(g,sigma,theta');

LHSmax=(abs(gmax)*(1+km*(sigmamax/YS)));

Nf=Solver(Tauf,G,bo,gf,co,LHSmax)%% Calling function Solver

%% Output
CPlane=thetaC
thetaplot=theta';
SUBROUTINE Stresses

function
[sigmaXa, sigmaXm, eXa, tauXYa, gXYa, YS]=Stresses_curvature(OD_US, ID_US, E, G, K, n, Ko, no, Fx_US, dogleg, Tmin_US, Tmax_US, YS_US)
clc;
format long;
OD=OD_US*0.0254; % OD in m
ID=ID_US*0.0254; % ID in m
A_US=(pi/4)*(OD_US^2-ID_US^2); % cross section area (in^2)
A=(pi/4)*(OD^2-ID^2); % cross section area (m^2)
Iy_US=pi*(OD_US^4-ID_US^4)/64; % modulus of Inertia (in^4)
Iy=pi*(OD^4-ID^4)/64; % modulus of Inertia (m^4)
Ix=pi*(OD^4-ID^4)/32; % modulus of Inertia (m^4)
EUS=E*145.038; % Modulus of elasticity (psi)
YS=YS_US/145.038; % yield strength (Mpa)
Fx=Fx_US*4448.2216; % Axial force in DP segment (N)
Tmin=Tmin_US*1355.82; % min value of axial torque in X-direction (N-m)
Tmax=Tmax_US*1355.82; % max value of axial torque in X-direction (N-m)
KT=sqrt(Fx_US*1000/(EUS*Iy_US));
L=180; % length of Type2 pipes (in)
My_US=((2*pi/432000)*(dogleg*EUS*Iy_US*L*KT/(tanh(L*KT))))/12000 % bending moment in Y-direction (klb-ft)
My=My_US*1355.82; % bending moment in Y-direction (N-m)

wt=1:1:360;
sigmaX=((Fx/(A))+(My*(OD/2)*sind(wt)/Iy))/1000000; % normal stress in X-direction (MPa)
sigmaXma=max(sigmaX);
sigmaXmi=min(sigmaX);
sigmaXm=(sigmaXma+sigmaXmi)/2;
sigmaXa=(sigmaXma-sigmaXmi)/2;
eXa=(sigmaXa/E)+(sigmaXa/K)^(1/n)); % normal strain amplitude in x-direction

TXYa=(Tmax-Tmin)/2;
tauXYa=(TXYa*(OD/2)/Ix)/1000000; % shear stress amplitude XY-direction
gXYa=(tauXYa/G)+(tauXYa/Ko)^(1/no); % shear strain amplitude in XY-direction
end
SUBROUTINE CriticalPlaneCalc

function [gmax, sigmamax, thetaC] = CriticalPLanCalc(g, sigma, theta)

%get max values
[amax, I] = max(abs(g(:)));

p = [0, 0, 0, 0];
c = 0;

%loop over g to get indices of max two points
%points vector contains the indices of the two max point as follows
[x1, y1, x2, y2]
for i = 1:size(g, 1)
    for j = 1:size(g, 2)
        if (abs(g(i, j)) == amax)
            c = c + 1;
            p(c) = i;
            c = c + 1;
            p(c) = j;
        end
    end
end

x1 = p(1);
y1 = p(2);
x2 = p(3);
y2 = p(4);

%find equivalent points in matrix sigma and get the larger
if (sigma(x1, y1) > sigma(x2, y2))
    sigmamax = sigma(x1, y1);
    gmax = g(x1, y1);
    thetaC = theta(x1, y1);
else
    sigmamax = sigma(x2, y2);
    gmax = g(x2, y2);
    thetaC = theta(x2, y2);
end
function cr=Solver(Tauf,G,bo,gf,co,LHSmax)
iter=1;
errs=0.00001;
erra=1;
cl=100; % first initial value guess
cu=cl+500; %second initial value guess
% calculate function lower and upper bands
fl=(Tauf/G)*((2*cl)^bo)+ gf*((2*cl)^co)-LHSmax;
fu=(Tauf/G)*((2*cu)^bo)+ gf*((2*cu)^co)-LHSmax;
fprintf('Iteration=%.5f,cl=%.8f,cu=%.8f,fl=%.8f,fu=%.8f\n',iter,cl,cu,fl,fu);
while fl*fu>0; %seeking function sign change
cl=cu;
cu=cu+500;
fl=(Tauf/G)*((2*cl)^bo)+ gf*((2*cl)^co)-LHSmax;
fu=(Tauf/G)*((2*cu)^bo)+ gf*((2*cu)^co)-LHSmax;
iter=iter+1;
fprintf('Iteration=%.5f,cl=%.8f,cu=%.8f,fl=%.8f,fu=%.8f,ea=%.8f\n',iter,cl,cu,fl,fu,erra);
end
%bisection solver
while abs(erra)>errs
    cr=(cl+cu)/2;
    fr=(Tauf/G)*((2*cr)^bo)+ gf*((2*cr)^co)-LHSmax;
    iter=iter+1;
    erra=(fr);
    fprintf('Iteration=%.5f,cl=%.8f,cu=%.8f,fl=%.8f,fu=%.8f,cr=%.8f,ea=%.8f\n',iter,cl,cu,fl,fu,cr,erra);
    if fl*fr<0 ;
        cu=cr;
    elseif fl*fr>0;
        cl=cr;
        fl=fr;
    end
end
end